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16. Abstract

Annual increases in automobile ownership, vehicular traffic and vehicle miles traveled have resulted in congestion problems, which in turn impact mobility, quality of life and air quality as well as waste fuel. The Clean Air Act Amendment of 1990 and ISTEA provisions have encouraged the exploration of alternatives to traditional capacity expansion approaches, such as demand management and congestion pricing. Congestion pricing involves charging for the use of the facility only during heavy congested periods. This encourages motorists to use the facility when costs are lower (less congested), use other modes such as transit, or to forego the trip completely. In addition to its potential as a source of new revenue, congestion pricing could contribute to reductions in fuel consumed. It would be compatible with the provisions of the 1990 Clean Air Act, because it would assist non-attainment areas to comply with stipulated standards. Technical feasibility has been established in Norway and Singapore, however, little is known regarding current levels of acceptability in the United States. Therefore, more information is needed to assess the viability of this alternative in Texas and determine its effectiveness and impact on congestion and fuel consumption.

This work builds on efforts to characterize travel attitudes and response to different congestion pricing schemes. A critical issue being addressed is that of public acceptability. Models of user response were developed based on survey data as well as behavioral experiments. These models have been incorporated in a methodology built on a unique dynamic traffic assignment capability developed at the University of Texas to predict network level impacts on congestion and fuel consumption.

This study provides an analysis of current attitudes and behavior of travelers in Texas, and provides useful data to characterize public acceptability and user response. The methodology is designed to identify candidate locations for congestion pricing in Texas and the associated energy savings at these locations.

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User's Response to Pricing in a Traffic Network

by

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and
Hani S. Mahmassani

SWUTC/99/465620-1

Combined final report for the following SWUTC projects:

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EXECUTIVE SUMMARY

Congestion pricing seeks to achieve more efficient utilization of the traffic system by means of pricing the use of the roads or access to restricted zones of a city. Congestion pricing involves charging a price that depends on the level of congestion prevailing in the selected facility or restricted zone. Pricing is considered by economists as a more efficient mechanism than queues and delays to allocate use of limited transportation system capacity, especially during peak periods. It also provides a mechanism to charge drivers for the full cost that they impose on the rest of the system, including external effects on the environment.

However, several serious obstacles slowed the widespread implementation of road pricing as a congestion control measure, chief among them has been the lack of public acceptance, as well as equity considerations, concern about effectiveness, and lack of sufficient number of success stories.

On the other hand, the technological barriers that had previously acted as a strong deterrent against road pricing are no longer binding, as existing Automatic Vehicle Identification technologies allow various non-intrusive ways of charging and collecting fees from users.

Although there is a large body of work in the area of congestion pricing, virtually no study has focused on the effect of previous studies of congestion pricing in a traffic network in a dynamic environment. Most previous studies are limited to analysis of simplified networks. This report addresses an important gap by developing a methodology to estimate the users' response to the implementation of a congestion-pricing scheme in a traffic network. In the analysis, different pricing schemes are applied in a traffic network in such a way to reflect the use of second-best congestion pricing options, and the users' reaction observed. The analysis includes the development of a methodology for the determination of the optimal prices to charge.

A dynamic traffic simulation-assignment procedure (DYNASMART, developed at the University of Texas at Austin) constitutes the principal methodological approach in this investigation. This procedure incorporates user behavior rules in response to prices and information within a network traffic simulator. The simulator provides the performance indices needed to evaluate the network effects of congestion pricing. DYNASMART was modified for this study to incorporate consideration of constant or variable prices, and to provide information about revenues.

To illustrate the methodology developed in this study, and its ability to provide insight into various congestion pricing schemes under different behavioral response rules, the following scenarios are defined on the basis of the pricing schemes evaluated:

(1) Constant prices on specific links; this scenario would reproduce the pricing of single facilities of a traffic network, such as freeways or bridges.

(2) Constant prices in a zone of an urban traffic network; this scenario would reproduce the application of cordon prices in specific areas of a traffic network such as the schemes applied in Singapore or in Norway.
(3) Time-dependent, concentration-based prices in a traffic network; this scenario would base prices in the level of congestion rather than simply on time of the day.

Furthermore, the methodology is developed for three user behavioral rules in the network assignment approach followed:

(1) Users follow the current best route at any node of the network along their journey.
(2) Drivers are loaded onto the network to achieve user equilibrium (UE).
(3) Longer-term effects of pricing are investigated using a day-to-day dynamics framework. This scenario would reproduce the effects of adaptation of drivers to conditions in which prices are applied.

The results of extensive simulation-experiments suggested several substantive conclusions regarding the effectiveness of different pricing schemes. Under the assumption of inelastic demand, and for the limited number of pricing schemes considered, constant prices applied in a limited number of links of a network or in a restricted zone, could affect negatively the operation of the entire network, regardless of the assignment criteria considered. Although such schemes could improve traffic conditions in the restricted zone or in the affected links by reducing the number of vehicles using the tolled arcs, vehicles that are priced off the tolled links may have to follow longer routes with higher travel times. Higher tolls increase revenues only due to the number of vehicles that belong to the captive population. However, there may be cases where the drivers that remain using a given road greatly benefit from the reduced number of users, and the operation of the whole network also improves.

Variable prices, unlike constant prices, show different effects for the schemes considered here, depending on the assignment criteria followed. Under very congested conditions, when vehicles are assigned to the prevailing shortest route, total travel times are smaller for tolls updated at short intervals and low densities. Higher density levels or larger update times increase the total travel time. Total trip distances are for all the cases higher than for the no toll condition. However, this increase in trip distances is not particularly large. Revenues behave in a more predictable way, decreasing as the toll update intervals are lengthened or density levels for tolling are increased.

When a user equilibrium assignment criterion is followed, the use of variable tolls reduces the total travel times for congested conditions. In the case of limited congested conditions, the total travel times are very similar to the user equilibrium with no toll operation. Trip distances are not very much affected by the use of variable tolls at any of the load factors considered. They only increase marginally at medium and congested conditions. Revenues decrease, as in the case of the assignment to the prevailing shortest route, as the density threshold level is increased.

For the day to day assignment rule set, variable tolls, for the loading factor and the value of the parameters considered, do not have a significant effect on the travel times or traveled distances. These increase, but only marginally. Revenues also increase as the iterations progress but remain generally at a very low average level for the cases considered in the experiments conducted here. Essentially, the tolls did not influence drivers decisions significantly, as they were kept fairly low and the effect of travel times are less important than the schedule delay.

The most important contribution of this research is the development of a methodology for the study of the effects of constant and variable pricing in a road network under different traffic assignment criteria including current best path, user equilibrium and day to day dynamics. More generally, the methodology can evaluate various pricing schemes under different user behavior rules governing the response to pricing, information and pricing control.

This study has also developed an improved simulation-based numerical approximation to the global marginals; this approach incorporates intertemporal terms in previously estimated local marginals. Nonetheless, obtaining an exact expression for the global marginals and consequently for the optimal prices, remains a difficult and challenging task.

Within the methodology developed, the research has also shown how prices can be incorporated into a dynamic traffic simulator to reproduce a congestion pricing scheme, and how different assignment criteria can be followed to analyze the effect of pricing. This can be particularly helpful to evaluate proposed schemes before they are actually implemented.

ABSTRACT

Annual increases in automobile ownership, vehicular traffic and vehicle miles traveled have resulted in congestion problems, which in turn impact mobility, quality of life and air quality as well as waste fuel. This study explores alternatives to traditional capacity expansion approaches, such as demand management and congestion pricing. Congestion pricing involves charging higher tolls for the use of a facility during heavy congestion periods, thus encouraging motorists to use the facility when costs are lower (less congested), consider other modes of transit, or forego a trip completely. Congestion pricing could contribute not only to a reduction in fuel consumption, but also provide a source of additional revenue. It could facilitate meeting the requirements of the 1990 Clean Air Act, because it would assist non-attainment areas in complying with current stipulated standards. Although technical feasibility has been established in Norway and Singapore, little is known about its acceptability in the United States. This study assesses the viability of this alternative in Texas and determines its possible effectiveness and impact on congestion and fuel consumption.

This study builds on efforts to characterize travel attitudes and responses to different congestion pricing schemes, as well as the critical issue of public acceptability. Models of user response were developed based on survey data as well as behavioral experiments. These models were incorporated in a methodology built on a unique dynamic traffic assignment capability developed at the University of Texas to predict network level impacts on congestion and fuel consumption. The simulation-assignment approach extends the DYNASMART model to allow consideration of a wide range of pricing schemes differentiated on the basis of spatial extent (selected individual links, entire sub-areas such as a city's core area), temporal applicability, and charging basis (i.e., how toll amount is set, such as by congestion-dependent formula).

Extensive numerical experiments are performed to evaluate the impact on network performances of a wide range of pricing schemes, with both constant and variable tolls, and variation by time-of-day and as a function of prevailing congestion. Pricing schemes are also evaluated jointly with various strategies to supply real-time information to tripmakers via advanced traveler information systems. The results illustrate situations where judicious application of differentiated pricing produces both improved travel conditions and additional revenues; on the other hand, it is also shown that application of tolls without proper consideration of potential system-wide effects could contribute to worsening performance. Recommendations and tools for the determination of beneficial congestion prices are provided.

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CHAPTER 1: INTRODUCTION

MOTIVATION AND PROBLEM STATEMENT

With continuing increase in automobile use, induced in part by economic and population growth, traffic congestion is recognized as a pressing problem for most metropolitan areas around the world. Its negative consequences, such as the deterioration of air quality and related health problems, noise, travel delays and economic losses due to the inefficient use of the roads, are experienced by almost all network road users during peak hours, and by non-users as well. Although some researchers, such as Button and Pearman (1983), believe that road congestion may be an effective way to allocate that scarce resource, congested conditions can be accepted only up to a point where the value of the service received by using the road is higher than the value of the lost time in queue to receive that service and of all the other associated costs. Traffic congestion, and particularly its associated environmental costs, is a serious concern that requires major attention and concerted action.

In its simplest form, congestion arises because the available capacity cannot serve the desired demand during a certain period. This imbalance may be a daily occurrence (recurrent congestion) or due to unusual occurrences such as traffic accidents or lane closures (non-recurrent congestion). The result is a degradation of the level of service provided to the users. Its effect is accentuated by unpredicted fluctuations in demand (such as especial events) or supply (e.g., traffic accidents or malfunctioning equipment). It is also accentuated by the fact that the service rate itself (i.e., effective capacity) of transportation facilities typically diminishes with increasing congestion (e.g., bumper-to-bumper traffic on freeways and gridlock).

Two broad kinds of measures can be proposed when dealing with traffic congestion problems. On the supply side, measures such as new construction, upgrading of existing facilities to increase road network capacity and implementation of better traffic controls, are typically proposed. Demand management constitutes the other category of measures, which include flexible work schedules, increasing parking fees, fuel taxation or vehicle license fees, high occupancy vehicle lanes, information technologies (telematics) to improve network efficiency and congestion pricing. Effectively, a combination of both types of measures is required to make serious headway in dealing with congestion.

In order to have a significant impact, supply side measures require typically high levels of investment. The construction of new roads with higher standards, additional lanes, overpasses and other forms of physical capacity additions are financially demanding and are often politically unacceptable. It is widely accepted that any contemplated increase in the capacity of the road

network generally falls short of the actual increases in the demand, and that sooner than later the roads become congested again. On the other hand, demand side measures are generally less expensive. Their application is aimed to reduce the number of cars on the road network or redistribute them temporally, so as to reduce congestion severity by means of more efficient usage.

Taking the available infrastructure as given and assuming that it is operating at the limit of its efficiency for the given demand pattern, the available capacity can be viewed as a scarce resource that needs to be allocated among competing users. At present, queues are the primary means of incurring and distributing the cost of congestion, in other words, the primary mechanism to "clear the market". Queues directly translate into time delays and associated irritation, out of pocket costs such as wasted fuel, and poor health due to lower air quality resulting from greater emission levels. These costs, which may not be fully perceived by the trip-makers, would normally contribute to reduce the demand for the limited capacity, by discouraging certain trip-makers from competing for the facility at certain peak times. In other words, the demand levels that actually materialize are lower than they would have been without the perception by the users of the congestion costs. However, experience indicates that the resulting "equilibrium" demand levels do not eliminate congestion. Hence the interest in alternative methods of clearing the market, such that the equilibrium demand imposes less congestion on the system.

The principal mechanism by which economic markets clear is the price mechanism. In free markets, prices at which transactions take place reflect an equilibrium between supply and demand. In transportation systems, this mechanism has traditionally been shunned for a variety of philosophical, political and operational reasons. According to economic theory, a price can be set so as to discourage a sufficient number of users from using a facility at peak times, and shift them to less congested (and thereby cheaper) portions of the network, off-peak times, less congested modes, or to give up the trip altogether. Thus, one could control the level of congestion by pricing the use of the facilities at different times of the day. Pricing would work by affecting changes in both the spatial distribution of demand in the network (by inducing users to take alternate routes) and its temporal distribution (by shifting certain trips from peak to off-peak periods).

Congestion pricing has received renewed attention from policy-makers in recent years because of its potential environmental benefits. National environmental groups, such as the World Resources Institute, see congestion pricing as an important element of strategies aimed at cleaner air and energy conservation (Governing, 1993). Congestion pricing seeks to reduce

traffic demand by means of pricing the use of the roads or the access to restricted zones of a city. Congestion pricing involves charging a constant price for the use of a facility during congested periods or different prices that reflect the level of congestion. Congestion pricing could also be used as a significant source of new revenue (Bernstein and Muller, 1993), since the congestion charges could be in addition to other current forms of taxation such as fuel taxes or annual licenses.

In theory, as shown in the next chapter, congestion pricing is the first-best solution to deal with congestion (Emmerink et al., 1994). Some of the arguments in favor of pricing as a congestion-reduction (and hence fuel saving and air pollution reduction) tool include:

(1) It is more efficient, in economic terms, than queuing as a market clearing mechanism. Only those economically efficient trips are undertaken. Economists believe that through pricing, rather than delays, the use of the scarce resource presumably goes to the highest-value users.

(2) Users do not perceive their own true costs of using the facility, nor those that they impose on the rest of the system. Congestion pricing internalizes the cost of those external effects of driving. A common feature of traffic congestion is that the marginal cost imposed by each additional user on all other users tends to increase non linearly. Pricing allows such costs to be passed on to the user, thereby influencing his/her decisions not only on the basis of his/her own perceived costs, but also according to the cost imposed on the system.

(3) It reduces non-essential travel, energy consumption and the environmental impacts of driving such as: noise, air pollution, visual intrusion and accidents.

(4) It improves travel times and operating costs by reducing the number of vehicles in the facility or by inducing their temporal redistribution.

(5) It improves transit productivity since demand may be shifted to these systems.

(6) It reduces demand for new roads.

(7) It is applied only when needed without affecting non-peak hour traffic like some other traffic control measures.

(8) It provides a potential source of revenues for the operating agency, taxing externalities rather than economic activity, at a time when infrastructure rehabilitation needs far outstrip available resources. This has become an increasingly popular feature of congestion pricing tied to the need to finance other transport policies. These could include additional forms of traffic restraint, new or improved transit systems and in some cases the construction of new roads (Milne, 1992).

(9) It provides the opportunity to privatize part of the road network operations, and thus widens the scope of government policy options.

On the other hand, several serious obstacles have acted against the widespread implementation of road pricing as a congestion control measure, among them:

(1) Public acceptance: free access to roadways is taken for granted by most users, particularly in areas where urban form has evolved around the personal automobile. Roads are built with taxpayers' money, and drivers pay tax on every gallon of fuel purchased, so why pay again to use a public facility? Jones (1991) lists, according to a number of surveys conducted in England, congestion pricing as being the least popular of the measures to control congestion among the public at large. Support increased among professionals and politicians showed caution. Jones (1991) also mentions that support for congestion pricing increases when it is considered as a part of a more comprehensive package that could include the use of revenues for the improvement of alternative transportation modes and the physical environment. In other words, explicitly linking revenues from pricing to specific popular uses tends to increase public acceptance.

(2) Equity considerations: congestion pricing would allocate the use of limited capacity according to ability to pay, rather than according to need or to general notions of fairness such as first-come, first-served. Trips of drivers with low value of time will be first affected regardless of their social value. Additionally, it might induce regional inequality since a congestion charge is only paid in congested areas. People living in these areas would have to pay higher prices for mobility than those living in rural uncongested areas.

(3) Lack of alternatives: particularly in freeway-oriented cities, alternatives to the automobile may not be available or acceptable to the common driver, highlighting the equity issue for those who are priced out of the use of the road space.

(4) Operational considerations: traditional methods for collecting fees from drivers typically rely on stopping vehicles at toll facilities, thereby further contributing to delay and creating operational difficulties. Another concern is the lack of privacy of these operations. Vehicles can be identified at toll stations, affecting the drivers' privacy. Most of the concerns regarding these matters can now be alleviated with Automatic Vehicle Identification (AVI) technologies, which have provided much of the renewed interest in operational road pricing schemes.

(5) Concerns about unproven effectiveness: with very few congestion pricing schemes actually in place (the major showcase remains Singapore's downtown access control plan), it can not be said that congestion pricing will solve the problem of congestion.

(6) Although congestion pricing maximizes (in theory) social welfare, it does not lead to a strict Pareto improvement. The revenues obtained are in theory large enough to render each individual better off. However, there is no agreement about a feasible redistribution scheme. With no redistribution, individuals tolled and tolled off the roads are worse off, making "government" the winning party.

(7) Congestion pricing might be perceived as serving different objectives by different groups of society. The government may perceive it as an instrument to increase social welfare and government resources; environmentalists may view it as a means to stimulate carpooling and the use of public transit, and hence to reduce pollution and the

need for new roads; businessmen may perceive road pricing as a funding sources for new highways and continued economic development. The interests of the above mentioned groups do not necessarily converge.

The answers to the above concerns hinge on two key interrelated issues: (1) the manner in which trip-makers respond to different pricing schemes, and (2) public acceptability. The former determines the effectiveness of pricing in reducing congestion and saving fuel. Pricing will succeed in inducing spatial and temporal shifts in demand only if users are willing to change routes, modify departure times, switch mode of travel, engage in carpools, participate in telecommuting programs, or forego certain trips altogether. Success will depend on the extent and nature of the users' response to the set prices and the particular schemes at which certain changes are likely to occur. Similarly, the potentially objectionable aspects of pricing, such as reduced mobility for lower-income users, also hinge on the users' responses. These are also key elements in the determination of the prices that must be charged in a particular situation to achieve the desired operational objectives.

The second issue, namely public acceptance, is equally critical to the effectiveness of any pricing scheme. As unpopular traffic jams are, measures that violate people's basic ideas of fairness or impose undue hardship on some are not likely to be viable approaches to congestion reduction in environments where political leadership is subject to constant scrutiny and electoral review every two or four years.

User response, effectiveness, public acceptability, and viability all depend on the features of the particular pricing scheme that is devised and adopted. Devising fair, equitable, operationally convenient schemes remains an important challenge, one that is interdependent with the above-mentioned elements.

The practical difficulties for the general application of congestion pricing such as pricing all the links in a network, the provision of perfect information to the users, or the estimation of the correct tolls to charge in a dynamic assignment, have made second-best options the most attractive way to introduce congestion pricing (Emmerink, 1996). Second-best options consider pricing only a limited number of links in the traffic network, pricing only certain classes of users, the use of constant prices, the use of step tolls with known schedules of application and variable tolls based on a local estimation of the marginal cost imposed by additional drivers using the network.

Technical feasibility for congestion pricing using electronic road pricing has been established in a pilot project in Hong Kong. However, little is known regarding its effects on network flows,

travel times, delays or revenues. Therefore, more information is needed to assess the viability of this demand management alternative and determine its effectiveness and impact on congestion.

Another mechanism that could similarly induce temporal and spatial shifts in the demand pattern is information. This is one of the basic motivations of Advanced Traffic Information Systems (ATIS), a collection of Intelligent Transportation Systems (ITS) user services. User response is still not well understood in this case, nor is the resulting effectiveness. Complementarity and substitution effects between pricing and information strategies have been explored but, still more research is needed in order to determine the role that each can play in a concerted strategy for reducing congestion (see Emmerink et al., 1995).

In summary, the technological barriers that had previously acted as a strong deterrent against road pricing are no longer binding, as existing AVI technologies allow various non-intrusive ways of charging and collecting fees from users. Furthermore, the political climate, largely driven by growing popular concern over excruciating traffic congestion and the desire for better air quality, appears conducive for concerted action to address urban traffic congestion. A diverse array of measures is being seriously considered for this purpose, including market-like mechanisms such as pricing, as a means to rationally allocate the use of limited capacity, as well as for its potential as an additional revenue source to meet transportation needs. However, many questions remain to be answered in order to determine the effectiveness and viability of pricing as a general-purpose congestion control tool, as well as its suitability for particular locations. The principal challenges for studies in this area are to (1) define the proper role of congestion pricing, among the array of both demand-side and supply-side approaches for congestion management, (2) determine the impact of particular pricing schemes, and (3) devise fair equitable and practical schemes that meet with public acceptability and achieve the desired objectives.

Although there is a large body of work in the area of congestion pricing, virtually no study has focused on the actual effect of the application of congestion pricing in a traffic network in a dynamic environment. Most works are limited to the analysis of simplified networks with only one origin-destination (OD), linked by a single arc in which a bottleneck occurs or two parallel routes and that do not consider intersections. The users' response to congestion pricing schemes in a real traffic network remains a major area for investigation. No model has been developed to predict the effect of congestion pricing on travel times, speeds, delays or distance traveled. This report addresses that gap by developing a methodology to estimate the users' response to the implementation of a congestion pricing scheme in a traffic network. In the analysis, different pricing schemes will be applied in a traffic network in such a way to reflect the use of second-best

congestion pricing options, and the users' reaction observed. The analysis includes the development of a methodology for the determination of the optimal prices to charge.

This work continues on the line of the work by Ghali and Smith (1993), Milne (1992), Milne, May and Van Vliet (1993) and Emmerink (1996) for the determination of the effects of second-best congestion pricing in a traffic network using a dynamic traffic simulator.

OBJECTIVES

The main objective of this research is to develop the methodology to evaluate the effect that application of a congestion pricing scheme would have on the users of a road network. In particular, this research is concerned with two types of congestion pricing schemes: application of constant prices, and of variable prices on selected links, possibly to control access to a subarea of a traffic network. Would the drivers follow a different route? Would the travel times, distances, delays or operating speeds be drastically affected by the use of congestion pricing? These are the kind of questions that the evaluation methodology developed in this research attempts to answer.

To illustrate the methodology developed in this study, and its ability to provide insight into various congestion pricing schemes under different behavioral response rules, the following scenarios are defined on the basis of the pricing schemes evaluated:

(1) Constant prices on specific links; this scenario would reproduce the pricing of single facilities of a traffic network, such as freeways or bridges.

(2) Constant prices in a zone of an urban traffic network; this scenario would reproduce the application of cordon prices in specific areas of a traffic network such as the schemes applied in Singapore or in Norway.

(3) Time-dependent, concentration-based prices in a traffic network; this scenario would base prices in the level of congestion rather than simply on time of the day.

Furthermore, the methodology is developed for three user behavioral rules in the network assignment approach followed:

(1) Users follow the current best route at any node of the network along their journey.

(2) Drivers are loaded onto the network to achieve user equilibrium (UE).

(3) Longer term effects of pricing are investigated using a day-to-day dynamics framework. This scenario would reproduce the effects of adaptation of drivers to conditions in which prices are applied.

RESEARCH OVERVIEW

The nature of the evaluation problem of the effect of congestion pricing in a traffic network in a time-dependent framework requires assessment of the network's performance with and without pricing. How is this operation affected by the incorporation of pricing as a characteristic of its links? How different are the travel times, distances, delays and speeds if prices are introduced? Are the revenues sufficient to compensate those priced off the roads? How are the day-to-day travel decisions modified by the imposition of prices?

Although analytic formulations can be proposed to partially address some of the above questions, the current state of the art in the area is still limited and confronts serious difficulties. Ghali and Smith (1993) have shown the non-convexity of the total travel time functions, in a time-dependent framework, a property that precludes their use in a formal mathematical optimization program. They have also shown that no correct expression has been formulated to represent the First In-First out constraint in a multi-commodity network with multiple origins and destinations.

An alternative approach consists of the use of non-analytic instruments to generate and gather the range of information needed to evaluate the user responses to the implementation of a congestion pricing scheme, and the resulting operational impacts at the network level. For this purpose, several instruments and techniques have been identified. Those instruments range from the use of a dynamic traffic assignment simulation model to the full implementation of a congestion pricing scheme in a city or certain facility, as in Singapore in the seventies (Holland and Watson, 1978). Other possible forms include experiments with a limited number of participants, and the application of pilot projects such as those implemented in the 80's in Hong Kong (Dawson and Catling, 1986; Dawson and Brown, 1985; Catling and Harbord, 1985; Harrison et al, 1986; Fong, 1986).

The main objective of this research is to develop a general purpose methodology to evaluate the likely effects of congestion pricing on the network users before the full-scale deployment of such a policy. To achieve it, a dynamic traffic simulator (DYNASMART, a dynamic traffic simulator developed at The University of Texas at Austin) constitutes the principal methodological approach in this investigation. The dynamic traffic simulator provides the information needed for the evaluation of congestion pricing. It can provide, among others, information on travel times, travel distances and routes followed. With small changes to its current structure, it can incorporate constant or variable prices and provide information about revenues. The simulator has been used as an integral part of complex algorithms to solve time-dependent simulation-

assignment models such as UE and system optimal (SO), (Peeta, 1994). The simulator was also used to analyze the problem of day-to-day dynamics of commuter decisions under real-time information (Hu, 1995).

The incorporation of prices in the dynamic traffic simulator was the first step to analyze the effects of congestion pricing. In the initial experiments, different pricing schemes with constant and variable prices were considered, assigning vehicles to the prevailing best path, and assuming that drivers do not have en-route information about travel times on alternative routes, so they cannot switch paths at decision points. Later experiments incorporated the provision of information to all the drivers, assuming that they follow a boundedly rational behavior as described in Mahmassani and Stephan (1988).

To analyze the effect of pricing when vehicles are loaded into the network to achieve a user equilibrium, the time-dependent user equilibrium approach developed by Peeta (1994) was followed. Peeta's formulation was modified to incorporate constant and variable pricing.

Finally the longer term effects of pricing on a traffic assignment were analyzed following the day to day dynamics framework proposed by Hu (1995).

STRUCTURE OF THE REPORT

This report is organized as follows. Chapter two describes the background for the application of congestion pricing. It reviews the existing literature in the area of congestion pricing, from the theoretical formulations to the practical applications in the US and overseas. It introduces the concepts of marginal cost, first-best congestion pricing and second best congestion pricing. The same chapter includes a description of the technology for the application of congestion pricing.

Chapter three presents a new formulation for the determination of optimal prices in a time-dependent case. The chapter reviews the concept of global and local marginals and how they can be used to find optimal prices. It describes how a proposed combination of an analytical and a simulation based approaches can lead to the determination of improved local marginals and the corresponding optimal prices. The chapter concludes with an application example of the proposed methodology.

Chapter four describes in a general form the dynamic traffic simulator used in this research. The same chapter presents the simulation experiments conducted for the determination of the effect of constant and variable prices in a traffic network under the assumption that vehicles are assigned to the current least cost path. It describes the traffic network used, the loading patterns,

information provision, selection of the priced links and assumptions regarding user behavior. Results are analyzed and conclusions drawn.

Chapter five presents the case of vehicles being assigned to the traffic network under a user equilibrium (UE) rule when prices are incorporated. In the introduction, the time-dependent UE with pricing formulation is described and the algorithm for its solution presented. Then, the results of experiments for the application of constant and variable pricing in a traffic network with this assignment criteria are presented.

Chapter six describes the case of day-to-day dynamics when prices are incorporated. As in chapter five, the introduction describes the general framework of the day to day dynamics with pricing and its solution algorithm. Results of the experiments under this assignment rule set are presented.

Chapter seven presents the conclusions of the research.

CHAPTER 2: BACKGROUND AND LITERATURE REVIEW

This chapter reviews several concepts related to congestion pricing. These include: 1) the economic rationale for pricing as a first-best option to control congestion; 2) the objections to first-best congestion pricing; 3) second-best congestion pricing options that could be used as an alternative to first-best congestion pricing; 4) the technological advances that have virtually solved the operational problems that have precluded the application of congestion pricing; 5) the combination of information supply and pricing as complementary policies with a common goal; and 6) the experience with congestion pricing both in the US and abroad. A summary of the chapter is also presented.

ECONOMIC JUSTIFICATION FOR CONGESTION PRICING

In this section, the concept of congestion pricing as a theoretical first-best option to manage congestion is analyzed. In the discussion, following Beckmann et al (1956), the only cost considered is the transportation cost incurred by the road users; costs to provide the service such as those related to the construction or maintenance of the road are not included. This transportation cost is represented by travel times.

Let us consider the situation depicted in Figure 2.1, which reproduces Pigou's explanation for road pricing (Pigou, 1920; Mishan, 1971; Walters, 1961; Beckmann et al, 1956; Field, 1992). For a homogeneous group of drivers, demand for the use of the road can be represented by curve dd. This curve represents also the Marginal Private Benefit (MPB). The assumption of a homogenous group of drivers does not consider that "an important but not essential element in the strategic importance of pricing as a factor influencing investment decisions is the existence of variations in the value of time, not only for different persons at the same time, but for the same individual at different times" (Vickrey, 1969). Other important assumptions are that: (1) individuals behave rationally, i.e., they seek to maximize their utility; (2) there is perfect information for all the decision agents about all costs involved; (3) time is a normal economic good; and (4) congestion pricing is technically feasible and the cost of the operation of the system is relatively low (Emmerink, 1996). Assumptions (1) and (2) are common assumptions in economic analysis. Assumption (3) ensures that congestion is associated with disutility. The last assumption ensures the economic feasibility of the implementation of congestion pricing. The curve labeled MPC = ASC in Fig. 2.1 represents the marginal private cost, or average social cost incurred by a driver, which is an increasing function of the traffic volume because the average speed will be lower and travel times higher the larger the flow. It is assumed that the increase in travel time costs outweighs any savings in operating costs or possible reduced accident risks for

a road operating at reduced speeds. The curve labeled MSC represents the marginal social cost imposed by an additional vehicle using the road.

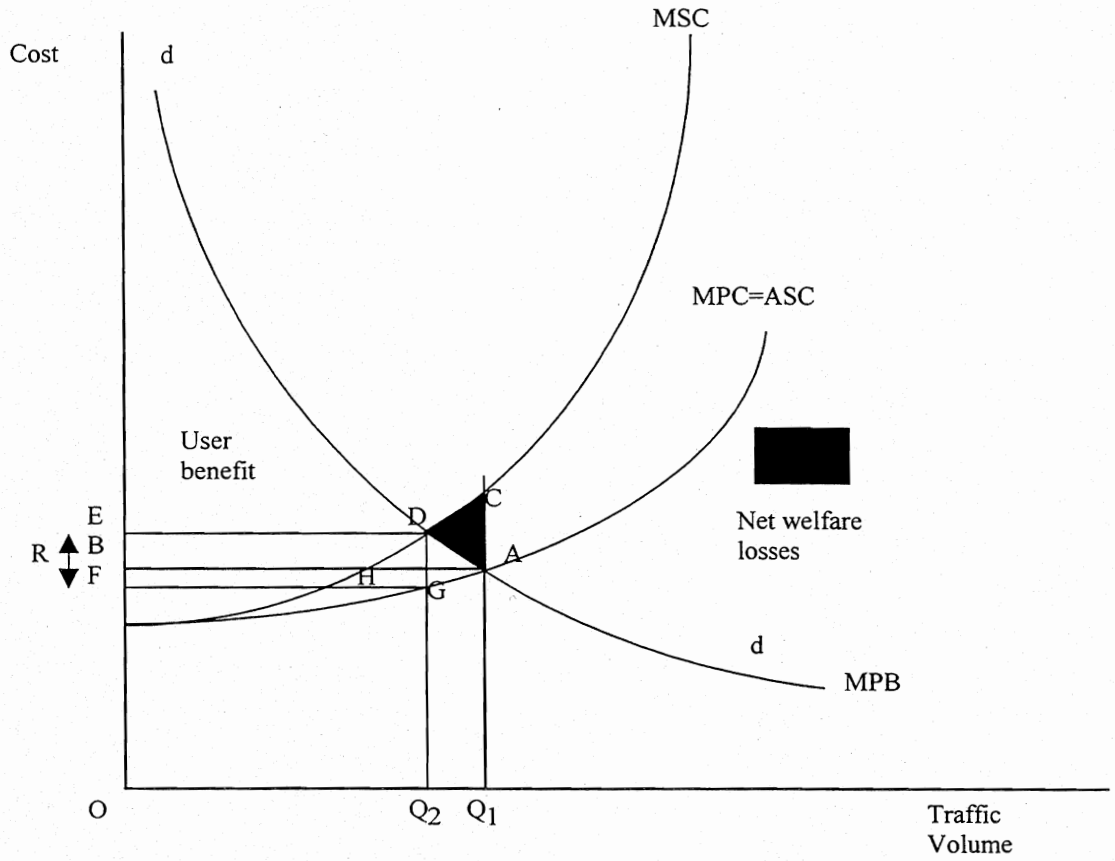


Figure 2.1 A Graphic Analysis of Congestion Pricing

If the road is operated without any kind of price control, every driver "pays" only the experienced cost of his/her trip, represented in Fig. 2.1 by the average cost (under the assumption of identical users and deterministic conditions). The net benefit to the users of the road is represented by the area under the dd curve and above line AB (i.e., the difference between the benefit of using the road and the cost paid for that use, represented by the area

OQ_1AB). However, the part of this benefit represented by the area ADH is paid by society as a whole since the cost of using the road is more than the cost of producing the service (area under MSC). There are also net welfare loses for this kind of operation represented by the triangular area ACD.

Without pricing controls, the market outcome will be at Q_1. The difference between the marginal private cost and the marginal social cost, AC, represents the external congestion costs. These costs include the costs that his/her driving imposes on other motorists by increasing their travel time. However, if a price equal to the difference between the marginal social cost and the average social cost is applied in the form of tolls (R), the external congestion costs will be covered, and drivers incurring these external costs penalized.

With the application of optimal pricing controls, traffic demand is reduced up to the point at which its curve intercepts the MSC curve (it goes from Q_1 in the original situation to Q_2 in the operation with pricing). Total social cost is then completely covered, inefficiencies eliminated and revenue generated. The system is led to an optimal state where total travel time for the drivers is minimal (Wardrop's [1952] system optimum). "Note that congestion pricing does not totally eliminate congestion; it simply reduces congestion to the point where the marginal cost of the congestion is equal to the marginal revenue produced by an optimal user fee." (McMullen, 1993). Clearly, if the road users are identical in terms of relative value of time and monetary tolls, everyone will be worse off if the revenues are not redistributed. Drivers between Q_2 and Q_1 will be priced off the road and will have to choose an option that is inferior for them. Drivers that continue to use the road will see their individual surplus reduced by EB. They will incur a lower average travel time cost, that goes from B to F, as a result of the reduced number of vehicles on the road but, in exchange, they will have to cover the toll EF.

The ethical concern would be the fairness of the toll system. Only those who price their time higher than the price set for the use of the road would be willing to pay, and some may not be able to do so because of their inability to pay. Those with lower buying power would be forced to wait until reduced or no tolls are in effect. However, they will still have the option of carpooling or using transit. They could also be compensated in some other way, since the additional revenue generated by the toll system could be spent in improvements to the transit system, new road construction or even in subsidies to low income driver groups (Small, 1992). If a subsidy is considered, drivers should not receive an amount in proportion to how much they paid. Doing this will provide no incentive to change the drivers' behavior (Litman, 1996).

Bernstein (1993) suggests a way to solve some of the equity concerns. He proposes a scheme under which the application of tolls would be complemented by subsidies. Every driver would pay tolls according to the level of congestion they encounter while driving. Revenues from

these tolls would then be distributed evenly among all drivers. In this form, drivers willing to arrive much earlier or much later to their destinations during the peak period would receive subsidies to compensate for their increased schedule delay, while drivers who arrive closer to their desired arrival times will be paying tolls.

De Corla-Souza (1995) proposes a cashing out approach that would make congestion pricing more palatable for commuters. In this approach, revenues collected from non-commuters are distributed among commuters by way of smart card technology. This will give the latter group the opportunity to use their extra cash to pay tolls, transit fares or keep the money for any other purpose. He does not offer any proof that revenues generated by non-commuters will be sufficient to cover all the commuters' toll payments and leave them with extra cash.

However, there is no agreement about how to redistribute the revenues from congestion pricing. This remains a major problem for the application of congestion pricing.

OBJECTIONS TO CONGESTION-PRICING AS A FIRST-BEST OPTION TO CONTROL CONGESTION

The first objection to congestion pricing as the first-best option to control congestion arises from the non-static nature of congestion. In a static formulation, optimal prices can be calculated by finding the first-order derivative of a well-defined objective function (total travel time). However, congestion is a dynamic phenomenon. Congestion charges should reflect future changes in traffic rather than just the current conditions (Agnew, 1977). To realistically formulate and solve a general dynamic model of congestion is a very difficult task. Current approaches lead to mathematically untractable formulations for which no adequate analytical tools are available. Only approximate solutions to these models have been proposed.

A second objection is based on the conditions to consider for the determination of the correct price to charge. Ran et al. (1993) have distinguished between two different user-optimal dynamic assignment models, one that considers current traffic conditions as the basis for the drivers' decisions, and another that considers actual (experienced) travel conditions. These two assignment approaches would lead to two different estimates of the congestion costs and consequently of the optimal tolls. Emmerink (1996) proposes that prices should be charged according to the anticipated levels of congestion and the effects of a new driver entering the network on the rest of the vehicles. However, the correct estimation of the effects of an additional driver in future traffic conditions is not trivial. Furthermore, unpredicted events such as accidents would rapidly change traffic away from the predicted situation.

A third objection to congestion pricing as a first-best option is based on the inability to estimate all the external costs of driving. Although the effect of additional drivers on travel times

can be calculated, it is not clear how to internalize the costs of air pollution, noise or safety in order to charge drivers for these costs.

Another objection to congestion pricing is directed at the simplified framework typically used for its analysis. This analysis considers simplified networks where interaction between link flows is not considered. The use of traffic controls at intersections is also ignored. Consideration of these additional elements can lead to even more complicated analysis for which mathematical formulations are not yet developed (Ghali and Smith, 1992; Smith and Ghali, 1992)

SECOND-BEST OPTIONS FOR CONGESTION PRICING

The objections to congestion pricing as a first-best option to manage congestion have led viewing it as a second-best option for reducing congestion problems. However, congestion pricing is still considered a better option than alternative measures such as parking management, high occupancy lanes, public transport subsidies, fuel taxation, or vehicle license fees. A number of second-best pricing options have been proposed, several of which are selectively reviewed in this section. This review is not intended to be exhaustive, but merely illustrative; for further analysis and discussion, the reader is referred to Verhoef et al (1994, 1995) and McDonald (1995).

First-Best Optimal Tolls

To explore the effects of the different pricing options, a mathematical framework, as in Verhoef (1996), Verhoef, Nijkamp and Rietveld (1994, 1995), is followed. The determination of optimal tolls for the static case as a basis for the analysis of second-best pricing options is first reviewed. A central controller seeking the optimal tolls that maximize the social welfare is assumed. Three cases are reviewed: a single route connecting one origin-destination pair, multiple routes connecting one origin-destination pair, and finally the case of a general network.

A Single Route. The first case consists of a single road connecting an origin-destination pair, under the assumptions listed in the previous section on economic justification for congestion pricing. Going back to Fig. 2.1, the total benefit for the road users is given by

$$W = \int_0^N D(n)dn - N \cdot c(N) \qquad (2.1)$$

where D(n) is the inverse demand function, N is the total number of drivers and c(N) is the average cost incurred by the N drivers. For an optimal operation of the road, the value of the inverse demand function should be equal to the average cost plus a toll, then

$$D(N) = c(N) + t \qquad (2.2)$$

If the controller desires to find the toll that maximizes total benefits for the system, the problem becomes

$$\text{Max } W = \int_0^N D(n)dn - N \cdot c(N) \tag{2.3}$$

s. t. $D(n) = c(N) + t$

Using the Lagrange Multipliers technique, the following lagrangian is formed

$$L = \int_0^N D(n)dn - N \cdot c(N) + \lambda \cdot [D(N) - c(N) - t] \tag{2.4}$$

With first order conditions being

$$\frac{\partial L}{\partial N} = D(N) - c(N) - Nc'(N) + \lambda(D'(N) - c'(N)) = 0 \tag{2.5}$$

$$\frac{\partial L}{\partial t} = -\lambda = 0 \tag{2.6}$$

$$\frac{\partial L}{\partial \lambda} = D(N) - c(N) = 0 \tag{2.7}$$

where the prime symbol stands for the first derivative.

Combining (2.7),(2.6) in (2.5) leads to

$$t - Nc'(N) = 0$$

or

$$t = Nc'(N) \tag{2.8}$$

which is the well-known solution that the optimal toll should be equal to the marginal cost.

Multiple Parallel Routes. In this case a simple network with multiple parallel routes connecting one origin-destination is assumed. There are demand and cost interdependencies among the different routes. It is assumed that the drivers consider the alternative routes as perfectly comparable. Then a single demand function is considered. The total number of drivers is given by adding the drivers on each of the individual routes. As in the case of a single route, drivers are assumed to be identical in terms of their value of travel time. According to Wardrop's first principle, routes used will be those with a minimal cost and the costs will be equal for all the routes used. Cost for each of the routes is defined as the average cost plus any tolls.

The mathematical formulation of the problem is:

$$\text{Max } W = \int_0^N D(n)dn - \sum_{i=1}^{k} N_i \cdot c_i(N_i)$$

$$\text{s. t. } D(N) = c_i(N_i) + t_i \quad \forall i = 1,...,k \tag{2.9}$$

and $N = \sum_{i=1}^{k} N_i$

where k is the number of parallel routes that are actually used at equilibrium. Routes with higher than the minimal cost will not be used and therefore will not satisfy the equation.

The lagrangian is formed as

$$L = \int_0^N D(n)dn - \sum_{i=1}^{k} N_i c(N_i) - \sum_{i=1}^{k} \lambda_i [D(N) - c(N_i) - t_i] \quad (2.10)$$

The first order conditions are

$$\frac{\partial L}{\partial N_i} = D(N) - c_i(N_i) - N_i c_i'(N_i) + \lambda_i (D'(N) - c_i'(N_i)) = 0 \quad \forall i = 1,...,k \quad (2.11)$$

$$\frac{\partial L}{\partial t_i} = -\lambda_i = 0 \quad \forall i = 1,...,k \quad (2.12)$$

$$\frac{\partial L}{\partial \lambda_i} = D(N) - c_i(N_i) - t_i = 0 \quad \forall i = 1,...,k \quad (2.13)$$

Combining the corresponding set of equations from (2.11), (2.12) and (2.13) leads to

$$t_i = N_i c_i'(N_i) \quad (2.14)$$

a similar solution to the single route case. The optimal price to charge for a given route depends on the number of vehicles using the route.

A General Network. Using a different notation, Yang and Huang (1998) confirm earlier results by Dafermos and Sparrow (1971) and show that for a network with multiple origin destinations pairs an multiple routes between those origin destinations the optimal toll is similar to the cases presented above or

$$t_{ij} = N_{ij} c_{ij}'(N_{ij}) \quad (2.15)$$

where t_{ij} is the toll for link ij that depends on the flow N_{ij} and on the first derivative of the cost associated to that flow $c_{ij}'(N_{ij})$.

Second-Best Pricing Options

Second-best congestion pricing options still use pricing controls, but the tolls used are not the optimal ones. They are considered to be imperfect substitutes to first-best optimal pricing. Here, the cases of (1) two different classes of users sharing the same route (Verhoef, 1996); (2) two different routes, one tolled and one untolled, connecting an OD pair (Verhoef, Nijkamp and

Rietveld, 1994) and; (3) a common congestion toll are reviewed (Verhoef, Nijkamp and Rietveld, 1995).

Two Different Classes of Users Sharing the Same Road. This case reflects conditions under which one group of users is exempted from paying the toll, e.g., public transit, carpools or low income. Drivers in the tolled group are priced off the road, thereby reducing congestion, and freeing up capacity that can be used for the non-tolled users. Non-tolled drivers entering the road increase congestion. Total welfare is reduced since users from the non-tolled groups, and consequently less willing to pay, can use the road, while users from the tolled groups, who are more willing to pay, are priced off the road. Drivers from the different groups will share the road. They will have different demand functions but will experience a unique cost function based on the total number of drivers. This case is formulated as the following mathematical program (Verhoef, 1996):

$$\text{Max } W = \int_0^{N_1} D_1(n_1)dn_1 + \int_0^{N_2} D_2(n_2)dn_2 - Nc(N) \qquad (2.16)$$

$$\text{s. t. } D_1(N_1) = c(N) + t$$

$$D_2(N_2) = c(N)$$

$$\text{and } N = N_1 + N_2$$

where $D_1(N_1)$ and $D_2(N_2)$ are the inverse demand functions for the tolled and untolled groups, c(N) is the common cost function and t is the toll paid by the tolled drivers.

The lagrangian is formed as

$$L = \int_0^{N_1} D_1(n_1)dn_1 + \int_0^{N_2} D_2(n_2)dn_2 - Nc(N) + \lambda_1[D_1(N_1) - c(N) - t] + \lambda_u[D_2(N_2) - c(N)] \qquad (2.17)$$

The first order conditions are:

$$\frac{\partial L}{\partial N_1} = D_1(N_1) - c(N) - Nc'(N) + \lambda_1[D_1'(N_1) - c'(N)] - \lambda_2 c'(N) = 0 \qquad (2.18)$$

$$\frac{\partial L}{\partial N_2} = D_2(N_2) - c(N) - Nc'(N) - \lambda_1 c'(N) + \lambda_2[D_2'(N_2) - c'(N)] = 0 \qquad (2.19)$$

$$\frac{\partial L}{\partial f} = -\lambda_1 = 0 \qquad (2.20)$$

$$\frac{\partial L}{\partial \lambda_1} = D_1(N_1) - c(N) - t = 0 \qquad (2.21)$$

$$\frac{\partial L}{\partial \lambda_2} = D_2(N_2) - c(N) = 0 \qquad (2.22)$$

Solving for λ_2 with (2.20) and (2.22) in (2.19)

$$\lambda_2 = \frac{Nc'(N)}{D_2'(N_2) - c'(N)} \qquad (2.23)$$

Combining (2.23), (2.21) and (2.18) leads to

$$t - Nc'(N) - \left[\frac{Nc'(N)}{D_2'(N_2) - c'(N)}\right] c'(N) = 0 \text{ or}$$

$$t = Nc'(N)\left[\frac{D_2'(N_2)}{D_2'(N_2) - c'(N)}\right] \qquad (2.24)$$

Unlike the optimal toll for the case where all users are tolled, the magnitude of the toll depends now on the slope of the inverse demand curve for the untolled users. Two extreme cases are of special interest. First, if the demand for the untolled group is perfectly elastic, ($D_2'(N_2) = 0$), the term in brackets is zero. The second-best optimal toll for this case is no toll at all. If a toll is applied, it will result in pricing off the road drivers more willing to pay than the untolled drivers and replacing them by drivers less willing to pay. The second case is that of a demand curve perfectly inelastic ($D'(N) = -\infty$), which results in a second-best optimal toll equal to $t = Nc'(N)$ which is the toll for the first-best optimal case. However, the magnitude of the toll depends now on the total number of tolled and untolled drivers. In some way, capacity is being reduced by the presence of the untolled drivers, making the operation of the road less efficient. Tolled drivers pay a higher fee than they would be paying if the untolled drivers were kept off the road.

Two Routes, One Tolled, One Untolled. This is the case of two congested routes, one tolled and one untolled, connecting an origin-destination pair in which the central controller desires to maximize efficiency knowing that one of the routes will have to be kept untolled. Demand and costs, as in the case of optimal tolls for parallel routes, are assumed to be interdependent. Then, drivers on each of the routes will have a common inverse demand function D(N), where N is the total number of drivers in the system ($N=N_t+N_u$), N_t is the number of drivers using the toll route, and N_u is the number of drivers using the untolled route. Average costs are represented by $c_t(N_t)$ and $c_u(N_u)$ for toll road users and untolled road users

respectively. As in the case of multiple parallel routes, the average cost for using the untolled route should be equal to the average cost of using the toll route plus any toll. Then the problem facing the central controller is formulated as the following mathematical program (Verhoef, Nijkamp and Rietveld, 1994):

$$\text{Max } W = \int_0^N D(n)dn - N_t c_t(N_t) - N_u c_u(N_u) \tag{2.25}$$

$$\text{s. t. } D(N) = c_t(N_t) + t$$

$$D(N) = c_u(N_u)$$

The lagrangian is formed as:

$$L = \int_0^N D(n)dn - N_t c_t(N_t) - N_u c_u(N_u) + \lambda_t[D(N) - c_t(N_t) - a] + \lambda_u[D(N) - c_u(N_u)] \tag{2.26}$$

The first order conditions are

$$\frac{\partial L}{\partial N_t} = D(N) - c_t(N_t) - N_t c_t'(N_t) + \lambda_t[D'(N) - c_t'(N_t)] + \lambda_u D'(N) = 0 \tag{2.27}$$

$$\frac{\partial L}{\partial N_u} = D(N) - c_u(N_u) - N_u c_u'(N_u) + \lambda_t D'(N) + \lambda_u[D'(N) - c_u'(N_u)] = 0 \tag{2.28}$$

$$\frac{\partial L}{\partial f} = -\lambda t = 0 \tag{2.29}$$

$$\frac{\partial L}{\partial \lambda_1} = D_1(N_1) - c(N) - t = 0 \tag{2.30}$$

$$\frac{\partial L}{\partial \lambda u} = D(N) - c_u(N_u) = 0 \tag{2.31}$$

Solving for λ_u with (2.29) and (2.31) in (2.28)

$$\lambda_u = \frac{N_u c_u'(N_u)}{D'(N) - c_u'(N_u)} \tag{2.32}$$

Combining (2.30), (2.32) and (2.27) leads to

$$t - N_t c_t'(N_t) + \left[\frac{N_u c_u'(N_u)}{D'(N) - c_u'(N_u)}\right] D'(N) = 0 \text{ or}$$

$$t = N_t c_t'(N_t) - \left[\frac{N_u c_u'(N_u)}{D'(N) - c_u'(N_u)}\right] D'(N) \qquad (2.33)$$

The second-best optimal toll equals the first-best toll plus an additional term. The value of term in brackets can go from zero to one, depending on the value of the slope of the inverse demand function. If total demand for the road is completely elastic ($D'(N) = 0$), then the toll should be equal to the first-best optimal toll. If total demand is perfectly inelastic ($D'(N) = -\infty$), then the optimal second-best toll is given by

$$t = N_t c_t'(N_t) - N_u c_u'(N_u)$$

In this case, the only action available to the central controller is to change the route split by setting the toll accordingly. The toll should be set equal to the difference between the marginal external congestion costs for tolled and untolled users. If marginal external costs are higher on the untolled route than in the tolled route, it may be the case that the optimal toll should in fact be negative. It would imply a subsidy for those using the toll road in such a way to compensate those using the toll road to reduce overall congestion costs.

A Common Congestion Toll. This case can be seen as that of a central controller seeking a second-best optimal common fee for those drivers using a radial system of roads converging to a city center (Verhoef, Nijkamp and Rietveld, 1995). The demand and cost functions are independent for each of the groups. Costs for a particular group are defined as the average costs for that group multiplied by the number of drivers on that route. The problem is formulated as the following mathematical program:

$$\text{Max } W = \sum_{g=1}^{G} \int_0^{N_g} D_g(n_g)dn_g - \sum_{g=1}^{G} N_g c_g(N_g) \qquad (2.34)$$

$$\text{s. t. } D_g(N_g) = c_g(N_g) + t \quad \forall g = 1,...,G$$

The lagrangian is formed as:

$$L = \sum_{g=1}^{G} \int_0^{N_g} D_g(n_g)dn_g - \sum_{g=1}^{G} N_g \cdot c_g(N_g) + \sum_{g=1}^{G} \lambda_g \cdot \left[D_g(N_g) - c_g(N_g) - t\right] \qquad (2.35)$$

The first order conditions are

$$\frac{\partial L}{\partial N_g} = D_g(N_g) - c_g(N_g) - N_g \cdot c_g'(N_g) + \lambda_g \cdot \left[D_g'(N_g) - c_g'(N_g)\right] = 0 \qquad 2.36)$$

$$\frac{\partial L}{\partial f} = -\sum_{g=1}^{G} \lambda_g = 0 \tag{2.37}$$

$$\frac{\partial L}{\partial \lambda_g} = D_g(N_g) - c_g(N_g) - t = 0 \tag{2.38}$$

from (2.38) in (2.36)

$$\lambda_g = \frac{f - N_g \cdot c_g'(N_g)}{D_g'(N_g) - c_g'(N_g)} \tag{2.39}$$

then

$$\sum_{g=1}^{G} \lambda_g = \sum_{g=1}^{G} \left[\frac{t - N_g \cdot c_g'(N_g)}{D_g'(N_g) - c_g'(N_g)} \right] = 0 \tag{2.40}$$

or

$$\sum_{g=1}^{G} \left[\frac{t}{D_g'(N_g) - c_g'(N_g)} \right] - \sum_{g=1}^{G} \left[\frac{N_g \cdot c_g'(N_g)}{D_g'(N_g) - c_g'(N_g)} \right] = 0 \tag{2.41}$$

or

$$t \cdot \sum_{g=1}^{G} \left[\frac{1}{D_g'(N_g) - c_g'(N_g)} \right] = \sum_{g=1}^{G} \left[\frac{N_g \cdot c_g'(N_g)}{D_g'(N_g) - c_g'(N_g)} \right] \tag{2.42}$$

Solving for f leads to

$$t = \frac{\sum_{g=1}^{G} \left[\frac{N_g \cdot c_g'(N_g)}{D_g'(N_g) - c_g'(N_g)} \right]}{\sum_{g=1}^{G} \left[\frac{1}{D_g'(N_g) - c_g'(N_g)} \right]} \tag{2.43}$$

The common fee is a weighted average of the external congestion tolls for each of the groups.

Evaluation Of Second-Best Pricing Policies. Verhoef et al (1995) have proposed that the following index should be used to evaluate the effects of second-best pricing policies. They propose what they called index of relative welfare improvement as:

$$\omega = \frac{W_p - W_0}{W_R - W_0} \tag{2.44}$$

where W_p is the social welfare (social benefits minus social costs) of second-best policies, W_R is the social welfare of first-best congestion pricing and W_0 is the social welfare under no intervention. By definition, $W_0 \leq W_p \leq W_R$. Welfare for first-best congestion pricing is at least equal to the welfare for second-best policies. Welfare for second best policies is at least as high as the welfare for no intervention. To avoid 0/0 values, $W_0 < W_R$ is assumed (i.e., not all demands are perfectly inelastic and externalities exist). The value of the index goes from 0, when it is not possible to increase welfare by using second-best policies, to 1, when second best policies achieve the same welfare improvement of first-best congestion pricing.

Extension To The Time-Dependent Case

The number of studies addressing the determination of optimal tolls in the time-dependent case is much smaller than for the static case. Most works consider very simple networks with only one origin-destination pair linked by a single arc where a bottleneck occurs; others consider two parallel routes. For these simple cases, analytical solutions are derived. Vickrey (1969) analyzed a pure bottleneck problem. Henderson (1974) and Agnew (1977) assume speed flow functions to derive time-dependent tolls. Ben Akiva et al (1984) and Arnott et al (1993) reviewed the case of bottlenecks with elastic demand.

Arnott et al (1990), based on Vickrey's work (1969), analyze the case of a very simple network with one origin and one destination, connected by a single link, at which a bottleneck occurs. Drivers have different departure times and total demand is inelastic. Drivers try to minimize their scheduled delay (i.e., the difference between their desired arrival time and their actual arrival time at the destination). They conclude that by altering the frequency distribution of departure times, congestion tolls generate efficiency gains not considered before, but which make the application of pricing much more attractive.

Using the same type of network as Arnott et al (1990), Else (1981) had previously suggested that the use of the number of vehicles on the road instead of the traffic flow would provide a better basis for the determination of the marginal social costs. In the same paper, Else notes the difficulties that the estimation of an optimal congestion tax would have due to the dynamic nature of the problem.

Carey and Srinivasan (1993) derive expressions, which include static and intertemporal componentes, for the time-dependent marginals in a simplified network with multiple origins and a single destination.

The theoretical estimation of optimal tolls in a time-dependent framework for a general network remains an open question. Alternative means have to be used to analyze the effect of pricing in a time-dependent environment. The most promising tool is the use of a dynamic traffic

simulator that reproduces, as far as possible, the operational conditions of a traffic network when pricing is introduced. Work on this line by Mahmassani and others (Mahmassani et al, 1994; Hu. and Mahmassani, 1995; Jayakrishnan, Mahmassani and Hu, 1994; Peeta, 1994) have shown promising results.

TECHNOLOGY FOR THE APPLICATION OF CONGESTION PRICING

It has been argued that the practical implementation of congestion pricing might worsen the problem it is intended to solve. Toll collection or enforcement at entrance points of a restricted zone or facility could cause additional congestion because vehicles have to come to a complete stop under conventional methods of toll collection.

However, recent advances in the area of Automatic Vehicle Identification (AVI) make possible the collection of tolls without slowing down the traffic. Radio Frequency Identification Technology consists of a passive identity card posted inside the windshield or a small, inexpensive and robust solid state device called "electronic license plate" placed underneath the vehicle (device used in the Kong Electronic Road Pricing Pilot Project; Dawson and Catling, 1986). The identity card is similar to the electronic license plate. These devices do not require any electrical connection and once placed in site do not need manual intervention, and are virtually maintenance free. When the vehicle passes through a collection point, the card is made active by an outside power source that sends a signal. The signal is sent back and read by an overhead or pavement-embedded automatic reader that identifies the unique vehicle number. The reader passes the information to roadside cabinets that contain microcomputers. The information is then decoded and sent to a central control that checks the identification number allowing the vehicle to cross the tolling point. The central control keeps record of the entrance point and, when the vehicle exits the tolled facility, the identification number is read again and the corresponding toll applied. If the vehicle is not identified or the card has any problem a message is displayed and the vehicle directed to an attended booth. The complete operation is done in fractions of a second without the vehicle having to stop. A record of the different transactions is kept by the central control. A statement is sent to the vehicle's owner at the end of a period, commonly once every month. Payment of the bill can be made by mail, charged to a credit card or directly at the operating company's office. The account can be prepaid in the form of a debit account.

If the vehicle does not have the required card, it will be directed to an attended toll booth where the toll is paid automatically when using correct change or manually when change is needed. If the vehicle passes the collection point without paying, the vehicle plate number can be photographed by a closed-circuit (CCTV) enforcement system. A ticket can be sent to the owner

or an enforcement official can be dispatched to catch the intruder. The technology is designed so that only the number plate can be identified without knowing about the driver or passengers in the vehicle.

An important concern has been the privacy of users. If the system can identify the vehicle, it would then be possible to track the movement of a person, thereby infringing upon his/her privacy. This problem has been solved using an advanced form of the identity card, the read and write cards. These cards have two memory components, one permanent that contains the account information, vehicle identification number and classification. The other can be overwritten to keep variable information such as records of the balance and of the entrance and exit points so the correct toll is deducted (AT/*Comm*, 1992). Read and write cards have a display element that requires electrical connections and a power source making the device more expensive than the read-only card. The read and write cards allow the system not only to identify the vehicle, but also to automatically deduct the amount of the toll from a prepaid account. The current balance is displayed inside the vehicle so the driver knows it. When the funds reach a pre-specified low level the driver is advised to add to the account or go to an attended toll booth.

Although read and write cards are, by their components, more expensive than the read only cards, the whole system using read and write cards should be less expensive. The toll collection system is decentralized eliminating the need for central control and associated communication system. The whole system requires audit equipment that compares the loop counts with the transactions registered so that any inconsistency or equipment failure is detected.

INFORMATION AND PRICING COMPLEMENTARITY

Congestion pricing is widely accepted has the first-best solution to reduce congestion. More recently, the provision of traffic information has come to be regarded as a second-best tool to solve the same problem. Information on traffic conditions is provided to drivers so they can improve their travel decisions on mode, route and departure time choices. The information provided helps to direct traffic flows towards the system optimum. Information is also seen as an inexpensive way to improve the capacity of transport networks without having to build new facilities. However, the potential benefits of providing traffic information have been questioned due to the effect that increased capacity would have on latent demand. It is claimed that the additional capacity provided by the use of information would be quickly used by an increase in the number of vehicle miles traveled (VMT) in the network. El Sanhoury and Bernstein (1995) consider that the use of congestion pricing can be an important factor for the success of traffic information systems and vice-versa. Pricing can act then as deterrent for that latent demand and help lead the traffic to the system optimum. Pricing, via tolling and subsidies, gives an incentive

to make drivers follow route guidance provided by the information systems in congested situations when substantial gains to the system can be achieved by affecting a reduced number of drivers. Congestion pricing may be more attractive if used in combination with traffic information. Revenues from congestion pricing could be spent on intelligent transportation systems (ITS) technologies, making people feel that they are receiving something besides reduced congestion in return for the tolls they are paying.

For El Sanhoury and Bernstein (1995), information and pricing are complementary and should be applied together. They share similar technology requirements for implementation. In-vehicle units can be used for several purposes: to collect traffic information, to receive traffic information and to serve as road pricing devices. Information systems can be used to assess the correct variable road pricing and drivers can be informed about price levels by the information systems (Emmerink, Nijkamp and Rietveld, 1995).

EXPERIENCE WITH CONGESTION PRICING

US Experience with Congestion Pricing

US experience with congestion pricing has been very limited due in large part to strong public opposition to any kind of movement restraint. Higgins (1986) provides an account of the limited demonstration projects contemplated in the seventies. He describes how, in 1976, then Secretary of Transportation William T. Coleman offered limited funding for the implementation of a Singapore-type pricing scheme to cities where decision makers seemed to be concerned with traffic problems. None of the largest cities were included due to the high cost of transit improvements needed to complement the application of congestion pricing. In the Singapore-type scheme, the vehicles would be provided with stickers that would allow the drivers to enter restricted zones during pre-specified times.

Of the cities that received the offer, only three were interested in further discussion about the implementation of a congestion pricing scheme: Madison, Wisconsin; Berkeley, California; and Honolulu, Hawaii. Other cities were more interested in demonstration of auto free zones and some others considered that practical, technical, political, and financial problems would affect the possible application of congestion pricing. Concerns about businesses in downtown areas were also raised.

After preliminary work for the three cities, the outcome was the same: no further study was recommended, and proposed application of congestion pricing was abandoned.

Higgins (1986) claims that the main reason for such lack of interest in the demonstration projects was the absence of sufficient understanding of congestion pricing by the general public. This misunderstanding was exacerbated by the media that raised important concerns about

freedom of movement, effect on local businesses and fairness of the system. The consequence was public overreaction to any further proposal in the area. With respect to decision maker support, Higgins suggests that stronger support from local politicians was needed but not achieved because of their lack of interest in the subject.

Los Angeles Airport. Even though, there was a lack of general interest for congestion pricing projects in the 1980's, a successful scheme was tested at the Los Angeles Airport to control the number and time that buses and taxis spent in the airport area. Lampe (1993) describes the scheme as follows: at Los Angeles airport, 60,000 vehicles use the central terminal area every day. There are about 500 commercial carriers that operate some 5,500 vehicles. Commercial vehicles compete with private vehicles for curb passengers, creating curbside congestion while waiting for additional passengers. The airport authority imposed an access charge for commercial vehicles based on an honor system, where the operators reported the number of times they entered the central terminal area.

However, authorities were not convinced that the honor system worked effectively. In 1989, after evaluating then available technologies, they decided to install an AVI system to reduce traffic congestion and to maximize revenues collected from commercial operators. The system's installation was completed in September 1990.

The AVI system consists of electronic tags and readers. Forty-one antennas were mounted on existing overhead structures of the central terminal area. Tags were installed in all the 5,500 commercial vehicles. They are counted each time they enter the zone and the corresponding fee is assessed. The system is capable of charging different tolls according to the vehicle type.

Since the system was implemented congestion has been reduced by 20 percent and revenue collection has gone up by more than 250 percent when compared to the honor system previously used.

Recent developments show renewed interest of U. S. Policy-makers in congestion pricing. By the end of 1992, the Federal Highway Administration, under a program authorized by the Intermodal Surface Transport Efficiency Act (ISTEA), invited applications from state and local governments for funding for up to five Congestion Pricing Pilot Programs. The ISTEA provided up to $25 million a year. The main requirements that needed to be satisfied by the proposals were that they (ITE Journal, January 1993):

- "Indicate a clear intent to use congestion charges to modify driver behavior;
- Include comprehensive applications of congestion pricing, including the use of road pricing;
- Include congestion pricing as a part of a program for addressing congestion, air quality, and energy goals;

- Demonstrate public and private involvement in the development of the program;
- Demonstrate the likelihood of early implementation;
- Indicate that the pricing project will not have major adverse effects on alternative routes or modes;
- Include plans for monitoring and evaluating proposed projects;
- Incorporate the use of advanced electronic toll and traffic management technologies;
- Include sound financial and management plans for pilot projects; and
- Be likely to add to the base of knowledge of congestion pricing applications."

The initial deadline for the submission of proposals was extended twice (ITE Journal, July 1994). Of 16 applications received from urban areas in nine states, only one met the conditions of the original solicitation, a proposal to raise peak period-period tolls on the Oakland-San Francisco Bay Bridge to control demand (ITE Journal, August 1994).

The Oakland-San Francisco Bay Bridge. This was the first of the projects under the ISTEA program for the demonstration of congestion pricing. Its planning phase started in the fall of 1993. The project tried to find the most feasible ways to reduce congestion on the bridge through the use of congestion-pricing (Frick et al, 1996). The current toll of $1 for using the bridge would be increased to $2 or $3 during the peak period with the intent of shifting demand to the off peak or to transit, reducing in this form air pollution and congestion. (TRB, 1994).

The San Francisco Bay Area can be considered as an ideal place for testing congestion pricing. There exists an extensive base of support for air quality and public transportation issues. The San Francisco-Oakland Bay Bridge Corridor is one of the most traveled in the US with about 485,000 person trips daily. The number of trips during the morning peak period is 135,000. Traffic congestion is, as such, a recurrent problem on the bridge. The Bay bridge corridor offers multiple ridership alternatives useful to a congestion pricing program. These include heavy rail, buses, ferries, carpools, vanpools and a shuttle service for bicycle users.

Tolls before the congestion pricing program were collected manually only from westbound travelers during the peak period (5:00 a.m. to 10:00 a.m. and 3:00 p.m. to 6:00 p.m.). The charge was $1.00 for non-commercial vehicles and from $3.00 to $10.50 for commercial vehicles, depending on the number of axles. Passenger cars with three or more passengers were exempt from payment. Caltrans started to implement an electronic toll collection (ETC) system using AVI technology (TRB, 1994).

The main challenge detected in the planning phase was the lack of public acceptability for congestion pricing. Some citizens saw the idea of implementing congestion pricing as a way to bail out cash-short transit agencies by means of a new tax increase. Some elected officials did not want to take risks due to fears of electoral repercussions. After a year of discussions with a number of interested parties, the management board adopted a proposal in November 1994. The proposal included a $3 toll for westbound vehicles with less than three occupants between the hours of 6 a.m. and 9:30 a.m. and between 3 p. m. and 6 p. m. Cars with three or more occupants would be exempted from paying the toll. Revenues in the estimated amount of $22 million were to be used to increase transit, ride sharing and other mobility alternatives. The proposal also included a provision for a "lifeline" toll discount for low income drivers that will continue paying the $1 toll. Due to the particular characteristics of the traffic network, it was expected that the implementation of the congestion pricing program would not create significant spillover effects on other facilities.

Due to federal congressional changes, the project lost the $23.5 million from a Federal Highway Administration (FHWA) grant. The management board was required to reshape the proposal to consider the lack of federal funding for transit improvements. In 1994, changes in the state legislature made the board reconsider the presentation of the proposal to the state congress the approval of which is mandatory to implement any change in the toll levels. Members of the board consider that the proposal did not have any chance to pass in the 1995 session. They decided to work in a reformulation of the proposal to be presented to the state congress in 1997 (Frick et al, 1996).

Interstate 15 in the San Diego Area. California distinguishes itself as a US leader in congestion pricing not only by the San Francisco Bay Bridge project, but also the Interstate 15 project in the San Diego area, which opened in December, 1996, for a three year trial period. Another project is already in operation in the Riverside Freeway in Southern California, and one more is planned for the California, Route 57 (TRB, 1994; Finch, 1996).

The congestion pricing element of the Interstate 15 project in San Diego comprises the 12.9 km (8 mi) of the two-lane reversible high-occupancy-vehicle (HOV) facility constructed in the median. The project's main objective is to optimize the use of the HOV lanes and to reduce corridor congestion using a market approach that charges a premium price for single-occupancy vehicles using the HOV lanes. The demonstration project was approved to operate under the same FHWA program that provided funds to the San Francisco Bay Bridge. In 1992, the Federal Transit Administration (FTA) also provided funds for transit development and congestion pricing demonstration. State legislation restricted the congestion pricing demonstration project to be applied only to the reversible lanes, that the revenues were used for transit improvements, and

that the presence of single occupancy vehicles, paying a fee, in the high-occupancy lanes did not negatively affect the number of HOV´s using these lanes.

The San Diego project considers that congestion pricing should be used as a tool to achieve region-wide objectives, such as traffic congestion relief, improved air quality, and improved mobility. It should be implemented in stages based on technical analysis, public involvement, and political acceptance. When the I-15 lanes were opened to solo drivers in December, 1996, monthly ExpressPass were sold in a first come first served basis. By the end of the Summer, 1997, ExpressPasses were replaced by windshield mounted electronic transponders. In the Spring of 1998, varying tolls based on the level of congestion on the HOV lanes and time of day were introduced (FHWA, 1997; fall 1997; 1998).

The Riverside Freeway. Similar to the San Diego project, on the Riverside Freeway (SR 91), the first commercial test of congestion pricing in the US started in December, 1995. Peak-period tolls, adjusted according to the number of vehicle occupants, time of day and amount of traffic are being applied in the 16 km (10 mi), between the Riverside–Orange County lane and the Newport-Costa Mesa Freeway, of newly and privately constructed express lanes. To allow solo drivers to use the express lanes they must pay $2.50 during rush hours and as little as 25 cents during off-peak times. Vehicles with three or more passengers pay no toll. All toll transactions are conducted electronically. Drivers not willing to pay can continue using the heavily congested old lanes. (Finch, 1996; FHWA, 1997)

The Twin Cities in Minnesota. Contrary to the single facility projects in the rest of the US, the Minnesota Department of Transportation and the Metropolitan Council of the Twin Cities of Minneapolis and St. Paul have been studying road pricing alternatives for several years at the statewide and metropolitan level. They have been considering three different types of road pricing: tolls, congestion pricing and mileage-based taxes, the latter as a substitute for the current state gasoline tax.

Lari and Buckeye (1996) describe the public outreach effort made as a part of an undergoing study to define the selection of road pricing options for future demonstration and testing. The outreach effort involved five major elements: Citizens jury, focus groups, opinion leader interviews, interactive video information survey and statewide random telephone survey. They conclude that the use of a number of outreach techniques improves the quality of the information. In the study, they identified a lack of understanding by Minnesotans regarding road pricing concepts. Implementation of a pricing project by the MnDOT has been put on hold until greater public support is developed. (FHWA, Spring 1998)

Other Projects. Besides of the projects listed above, nine different places are part of the Congestion Pricing Pilot Program overseen by FHWA. Congestion pricing is on different stages

of planning or implementation in Houston, Texas, (Katy Freeway); Boulder, Colorado; bridges in Fort Meyers, Florida; Maine Turnpike; Portland, Oregon; Seattle, Washington; Sonoma County, California; Los Angeles, California and Tappan Zee Bridge in New York, NY. The projects go from the use of variable tolls on existing roads or bridges to incentive tolling in HOV lanes to parking pricing (Munnich et al, 1997; FHWA, Spring1998).

Congestion pricing is now seen in the US as a possible element of the deployment of ITS in places where policy recommends its application. The Automatic Vehicle Identification/Electronic Toll Collection technologies will allow the implementation of demand management programs based on road pricing to switch or limit travel demand. This can be achieved without increasing congestion around toll plazas. Toll levels can be changed to spread travel demands among competing facilities (Pietrzyk, 1994; IVHS America, 1992).

International Experiences with Congestion Pricing

International experience with congestion pricing has been considerably more extensive than in the US. A congestion pricing scheme has been in operation in the central area of Singapore since 1977. A pilot study for a similar system was conducted in Hong Kong in 1985. Some European cities are now applying a form of congestion pricing in central areas (Bergen, Oslo and Trondheim in Norway, Stockholm in Sweden), France is using an intercity congestion pricing program on one of its roads (A-1 expressway from Paris to Lille), and others are considering its implementation (Randstad in the Netherlands, London and Cambridge in England, Seoul in Korea, Stuttgart in Germany and New Zealand,.)(Gomez-Ibanez and Small, 1994; FHWA, Fall 1997; Spring 1998).

Singapore. The best documented experience in congestion pricing is Singapore's central Area License Scheme (ALS). There, as mentioned by Morrison (1986), "...the relative isolation of the region from outside traffic makes administration and enforcement easier". Besides, the percentage of commuters affected by the application of the ALS was relatively small. The public transit system had enough capacity to accommodate those who left their cars parked. Morrison also makes note of the political acceptability of government actions in Singapore. Government is seen as acting in the interest of the general public and the single level of government makes things much easier than in a multilevel government.

The Singapore system consisted of daily or monthly stickers that were needed to enter the restricted zone. The stickers were initially sold in especially designated places for about US $1.30 a day. The restricted zone consists of the areas with congestion problems, leaves diversion routes for automobiles with destinations outside the restricted zone and minimizes the number of entry points. The restricted times were initially from 7:30 A.M. to 9:30 A.M., but were

extended after implementation until 10:15 A.M. due to the congestion that developed after 9:30 A.M.

In addition to the stickers, a Park-and-Ride scheme and parking policies were also implemented. The Park-and-Ride provided ten thousand parking spaces outside the restricted zone with special shuttle buses serving these parking lots. Parking fees were increased by one hundred per-cent at public parking lots within the restricted zone. The fee structure was modified to encourage short-term use.

General fiscal measures such as increased registration fees or gas taxes were not used since they do not discourage the use of the automobile in specific zones or times; vehicle metering would have required special equipment that was not available in the needed number; the application of street tolls would have required complicated collection facilities.

Among the benefits reported in conjunction with the Singapore scheme, it is worth noting the reduction in the number of cars entering the restricted zone by about 73 per cent; the large increase in occupancy of the vehicles due to the exemption granted to car pools; the number of taxis entering the restricted zone fell to about one third of the pre-scheme level; the mean speeds increased by about 22 per cent during the restricted hours compared to the evening peak.

The effect of the scheme on area businesses is not entirely clear. Interviews with local store managers, bankers, wholesalers and property agents showed that they did not consider the scheme responsible for the reduction in activity. Some companies were directly affected since they had to buy licenses for company cars. Taxi drivers complained about the low level of activity during the morning hours.

Recent evaluation of the Singapore Congestion Pricing Scheme (Field, 1992) shows that although traffic conditions in the restricted zone improved, diversion of traffic to avoid the central area caused increased congestion on streets just outside the restricted zone.

Hong Kong. Another well documented and successful experience (Dawson & Catling, 1986), at least in the pilot stage, in congestion pricing is the project developed in Hong Kong in the years 1983 to 1985. The project was the first to apply extensively Electronic Road Pricing (ERP) technologies tied to the then recent advances in microelectronics. It consisted of a fully operational subset of a complete system. The technological components used were an electronic license plate fixed underneath the vehicle; electronic loops embedded in the pavement that transmitted signals each time a vehicle crossed a tolling point; roadside cabinets that contained microcomputers to manage the information generated by the electronic loops and modems for communication to the central control. For purposes of enforcement a CCTV system was installed. The TV system provided pictures of the plates of the vehicles trying to cheat the system for later prosecution.

The central control included an accounting system that was able to bill vehicles for the use of the roads in the selected priced zones. Monthly statements were generated by the central control and bills, similar to a credit card statement, sent to the vehicle owners. The system offered diverse means of payment (mail, direct debit) and assured confidentiality by containing a single total for the month. No vehicle record was kept longer than necessary to ensure payment. It is claimed that the accuracy of the ERP system was above 99%.

The Hong Kong congestion pricing scheme was seen by the local government as an efficient alternative to the high car ownership taxes that were implemented in 1982. The traffic problems in the urban areas during working days were so critical that the authorities were forced to increase the annual license fees and the first registration tax. Although these measures reduced, in the short term, the number of vehicles on the roads, they were expected to lose effectiveness over time given the fast-growing economy.

However, all the advantages of ERP shown in the pilot stage in Hong Kong were not sufficient to convince the local authorities of the desirability of its full implementation. Local opposition and the success of the other traffic restraint measures delayed the application of ERP.

Borins (1988) formulated hypotheses about the reasons why the ERP system was not further implemented in Hong Kong. He offered three possible explanations of that failure. The first is that the time in which the ERP was put in practice was a time when other political concerns were much more important for the Hongkonese. The second explanation is the lack of ability of the Transport Branch of the Hong Kong Government to introduce effectively electronic road pricing. The third is that electronic road pricing, even with its economic advantages, will have difficulty gaining acceptance in any democratic society since it will likely be rejected if a referendum were held. He concluded that some combination of the three explanations can be attributed for such a failure and if no attention is given to them, and especially to the third one, congestion pricing will be shelved as an economical but not practical congestion management tool.

The Scandinavian Toll Rings. In 1986, Bergen, the second largest city in Norway, with a population of about 200,000, opened a toll ring around the central business district (CBD). The main purpose of the toll ring was not to divert traffic to alternative routes or to modify trip-making behavior, but to help finance a major program of road construction. This has been a common characteristic of the other Norwegian toll rings.

Bergen's abrupt topography concentrates the built-up area in certain corridors. The cost of road construction is high and the land available for new roads is scarce in the central part of the city. The topography also helps to facilitate the installation of toll booths. Access to the CBD is covered by only six toll stations, on the main roads to the CBD. No suitable alternative access

route is provided to the drivers, forcing them to incur the tolls. Tolls are charged to all motor vehicles, except buses and motorcycles, going to the CBD. Tolls are charged Monday to Friday from 6:00 a.m. to 10:00 p.m. except on official holidays. Tolls can be paid by single tickets bought at the toll booths, prepaid tickets in booklets of 20, and by monthly, semiannual or annual passes placed on the windshield. There are reserved lines for vehicles with passes so they do not have to stop. The toll ring in Bergen is technologically simple, but more complicated systems were not considered due to the lack of time and personnel for implementation.

The main reason for the toll ring was the need for supplementary funds to upgrade the road system. Before the toll ring, no income from taxes was earmarked for the construction of roads. In the case of Bergen, without the toll ring, the construction of the roads recommended in the Master Plan for Roads would have taken at least 30 years. To accelerate road construction, collecting tolls was seen as the best solution. Public support was gained by emphasizing the choice between having good roads in 12 vs. 30 years. The toll ring was linked to the completion of specific projects (Larsen, 1987).

In 1990, Oslo became the second Norwegian city to implement a toll ring. This was the first large European metropolitan area (700,000) where a toll ring was attempted. Oslo was also the first place that implemented electronic pricing on a massive scale. The toll ring is part of extensive transportation improvements where new capacity, safety and environmental improvements, and public transit were considered.

As in Bergen, Oslo's topography, with traffic entering the city concentrated in three corridors, helped to set up the toll ring. Only 19 toll stations and four street closures were needed to set up the ring. Although it was possible to construct the ring further away from or closer to the CBD, it was believed that the selected location optimized affected population, revenue and costs.

Three forms of payment are used: manual collection, payment to a coin machine, and electronic payment. Transit buses, emergency vehicles, motorcycles and disabled people are exempt from payment. The electronic payment system uses a passive electronic transponder contained in a small plastic box attached to the rear-view mirror. The transponder receives a signal from the toll station and reflects its identification number. The identification number is checked against account record and a picture of the license plate is taken for verification. No information is kept longer than necessary for enforcement or accounting purposes, giving users a great deal of privacy.

In late 1991, Trondheim, the third largest city in Norway, opened a more complex and flexible system than Oslo's. As in Bergen and Oslo, the purpose of the ring was to collect funds for road and public transportation improvements. The small traffic volumes in Trondheim make necessary the use of automated operations. Users have the options of paying at a coin machine

or be part of any of several electronic payment schemes that permit drivers cross the toll plazas without stopping. The tolls vary during the day: the maximum charge is from 6:00 a.m. to 10:00 a.m., slightly lower from 10:00 a.m. to 5:00 p.m., and free after 5:00 p.m. or on weekends.

Trondheim's toll structure can be considered close to congestion pricing, though toll differentials are so small that they do not affect significantly peak/off-peak traffic volumes. The use of electronic toll collection clearly allows for the future implementation of congestion pricing.

The interest in Sweden for the use of toll rings is based more on concerns for the environment than for reducing congestion or raising money for road construction. Several Swedish cities, Stockholm, Gothenberg and Malmo, are considering the implementation of toll rings.

France. In 1992, France implemented an intercity congestion pricing program. The place selected for this program was the A-1 expressway, which runs less than 200 km (124 mi) between Paris and Lille. The expressway suffers from serious congestion on Sunday afternoons and evenings due to the large number of drivers returning home to Paris from the countryside. The idea of implementing varying toll rates to control congestion received mixed reviews. High ranking officials of the Ministries of Finance and Equipment were in favor while technical staff of the Ministry of Equipment was opposed, arguing that it would create more congestion because drivers would drive faster immediately before the peak toll hours or slowly before the end of the peak to avoid payment.

Tolls are collected as follows. From 4:30 to 8:30 p.m. drivers pay a "red tariff" that is about 25 % higher than the normal toll for longer trips. Before and after the "red tariff" period - specifically from 2:30 to 4:30 p.m. and from 8:30 to 11:30 p.m.- the toll is 25 % lower than the normal rate. The toll schedule is designed in such a way that the revenue generated in the peak hours compensates the loses of the off-peak hours. The variable toll program distributed traffic flows more uniformly during the Sunday afternoon and evening hours.

Randstad. Contrary to the limited scope of the Scandinavian toll rings, the plans for the application of congestion pricing in the Netherlands included a large metropolitan area. The Randstad region covers 5,800 Km^2 with a population of about 6 million people. The region includes the country´s four largest metropolitan areas: Rotterdam, Amsterdam, The Hague and Utrecht. There, the local concern about traffic congestion and the national about automobile air pollution lead to the consideration of a sophisticated electronic system with characteristics such as low cost, ease of use, reliability, flexibility concerning location and time of day, and capability for multi-lane operation at highway speeds. The original scheme was conceived as a multiple-cordon system where cordon lines were defined and 140 charge points located.

In 1990, the plan was deemed to be too radical to win parliamentary approval. Critics voiced their concerns about the efficiency of the technology proposed, the privacy and the spillover effects. The response from the government was to reduce the scope of the system with a reduced number of toll sites and toll collection options. The new proposal was revised in 1992 but, its later implementation was delayed due to the 1994 national election. The government coalition lost the election and the new Minister of Transport and Public Works is believed to be against road pricing (Gomez-Ibanez and Small, 1994). However, the political climate has changed since 1994. The increased congestion has made politicians realize the need for some kind of policy to restrain road usage (Emmerink, 1996). A revised proposal will come into effect in 2001.

In the revised system, a smart card, that may be used for other transactions, will be placed in the cars. The charge for driving into any of the four cities between 6:00 a.m. and 10:00 a.m. will be set at about 15 ecus ($17). The price will be only 3 ecus at any other time. A video camera system will be used to catch non-payers and bills will be sent by the tax office. The location of the toll plazas will make it almost impossible to enter the restricted zones without passing a toll point. No agreement has been reached regarding the use of the toll revenues. Two positions remain: (1) to return the revenues to the general public via reduced taxes or to spend the money on public transport (The Economist, 1997).

London. A number of studies have been conducted to explore the application of congestion pricing in the Greater London region. The 1960's Smeed Government Commission report on traffic problems is recognized as the first to strongly support congestion pricing as a means to control congestion. The report even suggested some principles for implementation. Although its recommendations were not put to practice, the report has fueled a long debate about the merits of congestion pricing for the British cities.

During the 1970's, the Greater London Council (GLC) studied the use of supplementary licensing in which a special permit would be required to drive a vehicle in a certain area of central London. Supplementary licensing was found to significantly improve traffic in its area of application (Gomez-Ibanez and Small, 1994).

In 1985, the London planning Advisory Committee (LPAC) replaced the GLC and continued the studies on congestion pricing. The first of its commissioned studies, known as TASTE I and conducted by the MVA consultancy firm, showed significant advantages for the use of congestion pricing over other methods to improve traffic circulation in London. The pricing scheme consisted in a charge of £5.00 per day for driving anywhere within central London during the morning peak. Further studies found even greater improvement to traffic circulation by the use of electronic road

pricing with a structure that allowed for greater flexibility in setting charges for different kinds of trips.

In 1988, the LPAC recommended the implementation of a transportation strategy that included traffic restraint and pricing measures in central London. The plan also included heavy investments in rail systems, comprehensive traffic management of roads, no more road construction and improved bus service.

Additional studies by the UK Department of Transport reveal a surprising potential public support for road pricing. This support increases when pricing is a part of more complete packages to solve transportation problems. Support for road pricing also increases when the charging scheme is easy to understand, charges are predictable, pricing is applied in a limited zone and, in particular there is a clear understanding of the use of the revenues collected.

Cambridge. There have been plans to take congestion pricing to its theoretical limits in Cambridge. Charges would vary in real-time to consider the level of congestion experience. Real-time congestion pricing would be implemented by using an in-vehicle meter. Charges would be deducted from a prepaid card. The meter would be made active when passing select entry points in the city perimeter. It would be deactivated when leaving it. Enforcement beacons would check for the validity of the in-vehicle equipment and photograph the plates of the violators and notify the police. Visitors would use special daily passes.

The implementation of the congestion pricing proposal was put on hold after the May 1993 local elections in which a new government coalition took power. This new government is divided in respect to the application of congestion pricing to control congestion.

Stuttgart. Similar to the pilot project in Hong Kong, the city of Stuttgart in Germany has conducted a field trial. In the MobilPASS project, southern approaches to Stuttgart were covered with roadside debiting stations. A limited number of drivers were hired as participants. They were provided with smart cards to pay for tolls. In the project, the effects of variable road pricing charges on the behavior of drivers were investigated. The research paid special attention to the interaction between the charging schemes and the reduction in number of trips, changes in the mode of transport, route changes, time shifts, carpooling and trip chaining (Hug et al, 1997). Results from the project indicate that the use of variable tolls can reduce traffic peaks and the likelihood of congestion, price differences between routes can redistribute the traffic and that changes keep a direct relation to the difference in rates. One interesting finding is that some of the participants in the trial kept their modified behavior after the trial ended. The level of demand for transit remained higher after the trial.

SUMMARY

This chapter has reviewed several topics relevant to the conceptual basis and application of congestion pricing. These topics range from the early theoretical discussion of the rationale for congestion pricing as a means to eliminate inefficiencies in the operation of a road, to examples of practical applications of congestion pricing in the US and abroad.

The chapter has reviewed how the inefficient operation of a road under congested conditions can be made efficient using congestion pricing. The optimal prices to charge would be equal to the cost of the externalities imposed by additional drivers on a road.

The objections to congestion pricing that have precluded its general were presented. It has been seen that second-best pricing options, which avoid some of the objections, are easier to implement though they do not lead to the first-best optimum.

The determination of optimal tolls in a time-dependent framework is an unsolved problem. Analysis of congestion pricing in time-dependent framework has to be limited to second-best pricing options. These may consist of constant tolls or variable tolls related to the levels of congestion.

The technology that makes possible the practical implementation of congestion pricing has been reviewed. It has been shown that the available electronic technology is ready to solve the problems that in the past were seen as major drawbacks for the application of congestion pricing.

The complementary roles of congestion pricing and information were also briefly discussed. Both approaches can benefit from the use of the other. The implementation of congestion pricing requires information dissemination for correct application. Information dissemination strategies, when coupled with some sort of pricing as a means to control latent demand, improve the efficiency of the operation of the road network.

Finally, the growing role of congestion pricing in the US and abroad has been discussed. Political barriers and public acceptability concerns that had prevented its application are becoming less rigid. More places are now on various stages of study or implementation of congestion pricing.

CHAPTER 3: DETERMINATION OF OPTIMAL PRICES

INTRODUCTION

This chapter explores the determination of optimal tolls in a time-dependent framework. The first part discusses different concepts of marginal travel times. Global and local marginals are described. The second part describes a traffic simulation based approach for the determination of the local marginal travel times in a time-dependent problem. The third part discusses how an additional term that reflects inter-temporal effects can be incorporated in a simulation based approach. The fourth part presents a numerical example. Finally, conclusions are presented.

BACKGROUND

Chapter two has shown that the optimal toll for the static case should be equal to the cost that the additional driver imposes on the rest of the drivers already using the facility. This cost, also known as externality and represented by travel times, is defined as the difference between the marginal and the average costs. Then, to determine the optimal tolls, marginal and average costs should be known.

One problem with the static models is that they ignore the changing nature of congestion and the effects that travel decisions have on later or earlier periods. They predict the same optimal tolls whether congestion is building-up or dissipating from a peak, though these situations are drastically different.

Congestion is a dynamic process and should be analyzed as such. A time-dependent framework where time is discretized in small intervals is followed here. The following sections review the concepts of marginals and of the determination of optimal tolls in a time-dependent framework.

Definition of Variables and Notation

The following notation is used to represent variables throughout this chapter:

A = set of arcs in the network

N = set of nodes in the network

S = set of destination nodes

q = any node in the network

s = a destination node

T= total duration for which assignments are to be made

j = subscript for a link (or arc) in the network $j \in A$

t = subscript denoting current time interval, t=1,..., T

c = length of the time interval t

v = link flow vector

v_j = flow on arc j for the static case

v_{tj} = flow on arc j at time t

v_{tj}^s = actual rate of flow with destination s from arc j at time t

x = vector of link states (number of vehicles on link)

X_j = number of vehicles on arc j for the static case

X_{tj} = number of vehicles on arc j at the beginning of period t

X^L_{tj}= number of vehicles on arc j at the beginning of period t doing turning movement L,

L = 1: left turn movement

L = 2: straight and other movements

X_{tj}^s = number of vehicles with destination s on arc j at the beginning of period t d_{tj}^s = number of vehicles with destination s which enter link j in period t

d_{tj} = total number of vehicles which enter link j in period t

z(.) = total travel cost

T(.) = total travel time

T´(.) = first derivative of the travel time function

$T_j(.)$ = link travel time experienced by vehicles using arc j for the static case

$T_{tj}(.)$ = link travel time experienced by vehicles that enter link j at time t

$T_{tj}^L(.)$ = estimated link trip time for vehicles that enter link j at time t and perform movement L

$T_{tj}^s(.)$ = link travel time for vehicles with destination s that enter link j at time t

mmt(.) = marginal travel time

smmt(.) = static component of the marginal travel time

ext (.) = externality

sext (.) = static component of the externality

α = percentage of the straight-going vehicles affected by an additional left-turning vehicle

β = percentage of the left-turning vehicles affected by an additional straight-going vehicle

Q = magnitude of the candidate queue

C = capacity of the link

ΔTE^L_{tj} = difference on travel times for link j at the beginning of time intervals t-1 and t for

vehicles performing movement L

ΔTF^{L}_{tj} = difference on travel times for link j at the beginning of time intervals t and t+1 for vehicles performing movement L

ΔXE^{L}_{tj} = difference on the number of vehicles performing movement L for link j at the beginning of time intervals t-1 and t

ΔXF^{L}_{tj} = difference on the number of vehicles performing movement L for link j at the beginning of time intervals t and t+1

I^{s}_{tq} = exogenous net inflow or input at node q in period t with destination s

$g_j(.) = \text{congestion function for link j}$

Global and Local Marginals

The marginal travel time, defined for a given link for a given time of entry (of flows) onto that link, captures the total additional travel time incurred by the entire system as a consequence of an additional vehicle entering the network at the entrance of the link at the given time and exiting the network at the end of the same link. Depending on the scope (spatial and temporal) over which the additional travel time is included, Ghali and Smith (1993) have distinguished two different classes of marginal costs for dynamic transportation networks: global and local marginals, with global marginals encompassing the entire network over the entire planning horizon, while local marginals somehow restricting this scope spatially and/or temporally, as discussed hereafter.

Global Marginal

The global marginal associated with a given link for a given entry time is the increase in the system-wide travel times due to the entry of a new vehicle on that link at the corresponding entry time. It represents the totality of the delay, regardless of its location, caused by the entry of an additional vehicle on that link. Global marginals may be positive or negative. Negative global marginals will arise when the benefits of delaying some vehicles exceed the benefits of not delaying others, reflecting the non-linear nature of traffic interactions on a link and in a network.

Mathematical Expressions for the Global Marginal. In the static case, the marginal cost (savings) of a vehicle entering (leaving) the network at a particular link is defined as the change in total systemwide cost per unit increase (reduction) in the flow on that link. Mathematically, it is expressed as:

$$\text{mmt(j)} = \frac{dz(\mathbf{v})}{dv_j}, \quad \forall\ j \tag{3.1}$$

where mmt(j) is the link marginal travel time; z(.) is the total cost, a function of the vector of link flows **v**, and v_j is the link flow in arc j. When the system cost z(**v**) is the total travel time, it can be expressed as:

$$z(\mathbf{v}) = \sum_j v_j T_j(\mathbf{v}), \quad j \in A \tag{3.2}$$

where A is the set of arcs j in the network, and $T_j(\mathbf{v})$ is the travel time for the vehicles using arc j. As defined, the marginal travel time in equation (3.1) and the travel times in equation (3.2) depend not only on the flow for the link under study, but also on the flows for all of the other links in the network, i.e., all the spatial interactions are being considered. Combining equations (3.1) and (3.2), the link marginal travel time can be expressed as:

$$\text{mmt(j)} = T_j(\mathbf{v}) + v_j \frac{\partial T_j(\mathbf{v})}{\partial v_j} + \sum_b v_b \frac{\partial T_b(\mathbf{v})}{\partial v_j} \tag{3.3}$$

where b corresponds to all the arcs in the network but j.

The marginal effect of an additional vehicle entering (leaving) in the system wide travel time has three components: (1) $T_j(\mathbf{v})$, the travel time experienced by the vehicle using link j, (2) the additional travel time (or travel time savings) $v_j \frac{\partial T_j(\mathbf{v})}{\partial v_j}$ that the vehicle entering (leaving) the network at link j has on the rest of the vehicles already using link j, and (3) the additional travel time (or travel time savings) $\sum_b v_b \frac{\partial T_b(\mathbf{v})}{\partial v_j}$ that the vehicle entering (leaving) the network at link j has on the vehicles already using any other link in the network.

In an analogous form to the static case, equation (3.1) can be extended to the time-dependent case. The global link marginal travel time mmt(t,j) is the travel time increment (reduction) due to an additional vehicle entering (leaving) the network on link j at time t. If the vector **x** represents the number of vehicles on each link, X_{tj} the number of vehicles on link j at the beginning of period t, **T**(**x**) the system-wide travel time, and if z(**x**), the objective function, is the total travel time, then

$$z(\mathbf{x}) = \mathbf{T}(\mathbf{x}) \tag{3.4}$$

and

$$\text{mmt}(t,j) = \frac{d\mathbf{T}(\mathbf{x})}{dX_{tj}}, \quad \forall\, t, j \tag{3.5}$$

Equation (3.5) can be expanded to consider the effects of the additional vehicle on link j and on the rest of the links as:

$$mmt(t,j) = T_{tj}(\mathbf{x}) + \sum_{\tau=1}^{T} d_{\tau j} \frac{\partial T_{\tau j}(\mathbf{x})}{\partial X_{tj}} + \sum_{\tau=1}^{T} \sum_{b \in B} d_{\tau b} \frac{\partial T_{\tau b}(\mathbf{x})}{\partial X_{tj}} \quad \forall\, t, j \qquad (3.6)$$

where $\sum_{\tau=1}^{T} d_{\tau j} \frac{\partial T_{\tau j}(\mathbf{x})}{\partial X_{tj}}$ represents the effects on the vehicles using link j at any time; $\sum_{\tau=1}^{T} \sum_{b \in B} d_{\tau b} \frac{\partial T_{\tau b}(\mathbf{x})}{\partial X_{tj}}$ represents the effects on the vehicles using any other link at any time; B is the set of all the links in the network except link j; $d_{\tau j}$ is the number of vehicles that enter link j at time τ.

Further expansion of Equation (3.6) leads to:

$$mmt(t,j) = T_{tj}(\mathbf{x}) + d_{tj} \frac{\partial T_{tj}(\mathbf{x})}{\partial X_{tj}} + \sum_{\tau=1}^{t-1} d^{\tau j} \frac{\partial T_{\tau j}(\mathbf{x})}{\partial X_{tj}} + \sum_{\tau=t+1}^{T} d^{\tau j} \frac{\partial T_{\tau j}(\mathbf{x})}{\partial X_{tj}}$$

$$+ \sum_{b \in B} d_{tb} \frac{\partial T_{tb}(\mathbf{x})}{\partial X_{tj}} + \sum_{\tau=1}^{t-1} \sum_{b \in B} d_{\tau b} \frac{\partial T_{\tau b}(\mathbf{x})}{\partial X_{tj}} + \sum_{\tau=t+1}^{T} \sum_{b \in B} d_{\tau b} \frac{\partial T_{\tau b}(\mathbf{x})}{\partial X_{tj}}, \quad \forall\, t, j \qquad (3.7)$$

where $d_{tj} \frac{\partial T_{tj}(\mathbf{x})}{\partial X_{tj}}$ is the delay that the new vehicle inflicts on vehicles entering link j at time t; $\sum_{\tau=1}^{t-1} d_{\tau j} \frac{\partial T_{\tau j}(\mathbf{x})}{\partial X_{tj}}$ represents the delay to vehicles entering link j before time t; $\sum_{\tau=t+1}^{T} d_{\tau j} \frac{\partial T_{\tau j}(\mathbf{x})}{\partial X_{tj}}$ is the additional delay caused by the new vehicle to vehicles entering link j after time t; $\sum_{b \in B} d_{tb} \frac{\partial T_{tb}(\mathbf{x})}{\partial X_{tj}}$ represents the effects of the additional vehicle on vehicles entering links other than j at time t; $\sum_{\tau=1}^{t-1} \sum_{b \in B} d_{\tau b} \frac{\partial T_{\tau b}(\mathbf{x})}{\partial X_{tj}}$ includes the effects on vehicles entering other links than j before time t; $\sum_{\tau=t+1}^{T} \sum_{b \in B} d_{\tau b} \frac{\partial T_{\tau b}(\mathbf{x})}{\partial X_{tj}}$ represents all the additional delays caused by the new vehicle on other links than j after time t. In general, it can be expected that the magnitude of the

partial derivatives, and consequently the effect of the additional vehicle on the rest of the vehicles using the network, decreases with the order of the temporal interactions and with the distance to link j (Peeta, 1994).

The exact numerical calculation of the global link marginals requires two simulations to obtain the total system travel time under two scenarios: (1) with an additional vehicle entering the link under study at time t, and (2) without the additional vehicle entering the link under study at time t. This gives an idea of the high computational cost that the exact determination of the global marginals for each time and interval would imply. On the other hand, the complexity of the above equations for the calculation of the global marginals, such as Equation (3.7), and the difficult calculations of the partial derivatives, particularly because the functions $T_{tj}(\mathbf{x})$ are not analytically specified, dictate that the estimation of the global marginals be reduced to only an approximation. Marginals are calculated taking into account only some of the terms of the global marginal's expression, thereby neglecting terms that are believed to be relatively less important than others.

Local Marginals

Local marginals consider only part of the effects that the additional vehicle has on the rest of the vehicles using the network at any time. As a consequence, only some of the terms of Equation (3.7) are considered. One case of local marginals is to consider only the effects that the additional vehicle entering the network has on the vehicles entering link j at time t and ignoring any other additional temporal or spatial effects. The mathematical expression for the marginal travel time for this case considers only the first two terms of Equation (3.7) as:

$$\mathrm{mmtL}(t,j) = T_{tj}(x) + d_{tj}\frac{\partial T_{tj}(x)}{\partial X_{tj}} \qquad (3.8)$$

If inter-temporal effects are taken into account then the third and fourth terms of the right hand side of Equation (3.7) will be added as:

$$\mathrm{mmtLI}(t,j) = T_{tj}(x) + d_{tj}\frac{\partial T_{tj}(x)}{\partial X_{tj}} + \sum_{\tau=1}^{t-1} d_{\tau j}\frac{\partial T_{\tau j}(x)}{\partial X_{tj}} + \sum_{t+1}^{T} d_{\tau j}\frac{\partial T_{\tau j}(x)}{\partial X_{tj}} \qquad (3.9)$$

When spatial interactions are ignored, the discussion of global versus local marginals is only relevant in the time-dependent case. For the steady-state case, both marginals would be the same since an additional vehicle entering the network at link j and leaving at the end of the same link would affect only the travel time on that link. The rest of the links would not be affected since their flows and travel times remain the same. The increase in total travel time, given by Equation (3.8), would be exactly the same for the link under study as for the whole network. In the time-

dependent case the new vehicle affects the exit flow from its own exit time. The new vehicle has taken capacity that was available to other vehicles, affecting later entry-flows and delays in link j.

One approach that has been used to find the local marginals consists in using numerical techniques to estimate the marginals from the time varying pattern of traffic flows in a network produced by a traffic simulation model. In the next section this approach is reviewed.

NUMERICAL ESTIMATION OF THE LOCAL MARGINALS USING A TRAFFIC SIMULATION MODEL

Using a traffic simulator to capture traffic interaction in the network, Mahmassani et al (1994) provide a solution to the general time-dependent network traffic assignment problem with multiple origins and destinations. Their solution seeks to determine system-optimal paths for vehicles through an iterative procedure which relies on finding the least marginal time paths at each iteration. Path marginals are approximated using the estimated link marginals, taken at the appropriate link entry time along the path for a given time of departure from the path origin. Mahmassani et al's work divides moving vehicles on a link into left turning and straight or right turning vehicles and calculates the different effect that an additional vehicle would have on the network depending on its desired movement at the next intersection and on the desired movement of the following vehicles. They also consider the effect of the entering vehicle when the capacity of the link under study is lower than the demand of the upstream links.

The main purpose of their derivation was to differentiate the spatial effect of the vehicles according to their desired movement at the downstream node; spatial effects for the upstream nodes are also captured by adding the queuing time of upstream vehicles. Mahmassani et al's work estimates the first two terms of the right hand side of Equation (3.7) and part of the fifth term of the same equation. However, no inter-temporal effects for the current link are incorporated. Their approach, based on numerical traffic simulation results, does not use nor explicitly specify the underlying link performance functions since they are not needed.

The expressions proposed for the calculation of the marginals are:

$$\mathrm{mmt}(t,j,1) = T^1_{tj}(x) + \frac{\partial T^1_{tj}(x)}{\partial X^1_{tj}} \cdot X^1_{tj} + \frac{\partial T^2_{tj}(x)}{\partial X^1_{tj}} \cdot X^2_{tj} \cdot \alpha + \frac{\partial T^2_{tj}(x)}{\partial X^1_{tj}} \cdot \mathrm{Max}\{0,(Q-C)\} \qquad (3.10)$$

for a left turning vehicle, and

$$mmt(t,j,2) = T^2_{tj}(x) + \frac{\partial T^2_{tj}(x)}{\partial X^2_{tj}} \cdot X^2_{tj} + \frac{\partial T^1_{tj}(x)}{\partial X^2_{tj}} \cdot X^1_{tj} \cdot \beta + \frac{\partial T^1_{tj}(x)}{\partial X^2_{tj}} \cdot Max\{0,(Q-C)\} \quad (3.11)$$

for straight or right turning vehicle. In Equations (3.10) and (3.11): $mmt(t,j,1)$ is the link marginal travel time on link j at time simulation interval t for vehicle performing a left turn movement.

$T^1_{tj}(x)$ is the estimated trip time when there are X(t, j,.) vehicles on link j at time t and the additional vehicle is going to turn left.

$\frac{\partial T^1_{tj}(x)}{\partial X^1_{tj}} \cdot X^1_{tj}$ is the trip time increment to the left-turning vehicles X^1_{tj} due to the additional left-turning vehicle on link j at t.

$\frac{\partial T^2_{tj}(x)}{\partial X^1_{tj}} \cdot X^2_{tj} \cdot \alpha$ is the trip time increment to the straight-going vehicles X^2_{tj}, due to the additional left-turning vehicle on link j at t. α is the percentage of the straight-going vehicles influenced by the additional left-turning vehicle. Currently, α=100%.

$\frac{\partial T^2_{tj}(x)}{\partial X^1_{tj}}$ is the trip time increment to the straight-going vehicles, due to the additional left-turning vehicle on link j at t.

Max {0,(Q-C)} is positive only when the demand in the upstream candidate queue is greater than the capacity on the current link. Otherwise, there will be no marginal effect on the upstream links. Q represents the magnitude of the candidate queue. C represents the capacity of the current link.

$mmt(t,j,2)$ is the link marginal travel time on link j at time simulation interval t for vehicle performing a straight or right turn movement.

$T^2_{tj}(x)$ is the estimated trip time when there are X(t, j,.) vehicles on link j at time t and the additional vehicle is going to go straight or turn right.

$\frac{\partial T^2_{tj}(x)}{\partial X^2_{tj}} \cdot X^2_{tj}$ is the trip time increment to the straight-going vehicles X^2_{tj}, due to the additional straight-going vehicle on link j at t

$\frac{\partial T^1_{tj}(x)}{\partial X^s_{tj}} \cdot X^1_{tj} \cdot \beta$ is the trip time increment to the left-turning vehicles X^1_{tj}, due to the additional straight-going vehicle on link j at t. β is the percentage of the left-turning vehicles influenced by the additional straight-going vehicle. Currently, β = 100 %.

$\frac{\partial T^1_{tj}(x)}{\partial X^2_{tj}}$ is the trip time increment to the left-turning vehicles, due to the additional straight-going vehicle on link j at t.

The link marginal travel time is made up of two parts, the marginal effect on the current link, represented by the first three terms of the right hand side of equations (3.10) and (3.11), and the marginal effect on the upstream links of the current link, represented by the fourth term of equations (3.10) and (3.11). The first terms of equations (3.10) and (3.11) correspond to the first term of the right hand side of Equation (3.7) (the global link marginal's expression). The second and third terms of equations (3.10) and (3.11) correspond to the second term of Equation (3.7). The fourth terms of Equation (3.10) and (3.11) are an approximation of the fifth term of Equation (3.7) in that they are the effects for the upstream links only.

The corresponding time-dependent externalities for Mahmassani et al's work are:

$$ext(t,j,1) = mmt(t,j,1) - T^1_{tj}(x) = \frac{\partial T^1_{tj}(x)}{\partial X^1_{tj}} \cdot X^1_{tj} + \frac{\partial T^2_{tj}(x)}{\partial X^1_{tj}} \cdot X^2_{tj} \cdot \alpha + \frac{\partial T^2_{tj}(x)}{\partial X^1_{tj}} \cdot Max\{0,(Q-C)\} \quad (3.12)$$

$$ext(t,j,2) = mmt(t,j,2) - T^2_{tj}(x) = \frac{\partial T^2_{tj}(x)}{\partial X^2_{tj}} \cdot X^2_{tj} + \frac{\partial T^1_{tj}(x)}{\partial X^2_{tj}} \cdot X^1_{tj} \cdot \beta + \frac{\partial T^1_{tj}(x)}{\partial X^2_{tj}} \cdot Max\{0,(Q-C)\} \quad (3.13)$$

With ext(t,j,1) being the externality on link j at time simulation interval t for vehicle performing a left turn movement, and ext(j,t,2) being the externality on link j at time simulation interval t for vehicle performing a right turn or straight movement.

Like the marginals, the externalities are made up of two parts, the externality on the current link, represented by the first two terms of the right hand side of equation (3.12) and (3.13), and

the externality on the upstream links of the current link, represented by the third term of equation (3.12) and (3.13).

The link marginal travel times in equations (3.10) and (3.11), and the corresponding externalities, can be obtained only if the partial derivatives in those equations can be solved. The way to approximate those partial derivatives is through finite differences using the values of travel times and the number of vehicles for three successive simulation intervals t-1, t and t+1. Time interval E is [t-1, t] and time interval F is [t, t+1]. Then:

$$\Delta TE^2_{tj} = T^2_{tj}(x) - T^2_{t-1,j}(x) \tag{3.14}$$

$$\Delta TF^2_{tj} = T^2_{t+1,j}(x) - T^2_{tj}(x) \tag{3.15}$$

$$\Delta TE^1_{tj} = T^1_{tj}(x) - T^1_{t-1,j}(x) \tag{3.16}$$

$$\Delta TF^1_{tj} = T^1_{t+1,j}(x) - T^1_{tj}(x) \tag{3.17}$$

$$\Delta XE^2_{tj} = X^2_{tj} - X^2_{t-1,j} \tag{3.18}$$

$$\Delta XF^2_{tj} = X^2_{t+1,j} - X^2_{tj} \tag{3.19}$$

$$\Delta XE^1_{tj} = X^1_{tj} - X^1_{t-1,j} \tag{3.20}$$

$$\Delta XF^1_{tj} = X^1_{t+1,j} - X^1_{tj} \tag{3.21}$$

Equations (3.14) to (3.21) are related to the partial derivatives in the following way:

$$\Delta TE^2_{tj} = \Delta XE^2_{tj} \cdot \frac{\partial T^2_{tj}(x)}{\partial X^2_{tj}} + \Delta XE^1_{tj} \cdot \frac{\partial T^2_{tj}(x)}{\partial X^1_{tj}} \tag{3.22}$$

$$\Delta TF^2_{tj} = \Delta XF^2_{tj} \cdot \frac{\partial T^2_{tj}(x)}{\partial X^2_{tj}} + \Delta XF^1_{tj} \cdot \frac{\partial T^2_{tj}(x)}{\partial X^1_{tj}} \tag{3.23}$$

$$\Delta TE^1_{tj} = \Delta XE^2_{tj} \cdot \frac{\partial T^1_{tj}(x)}{\partial X^2_{tj}} + \Delta XE^1_{tj} \cdot \frac{\partial T^1_{tj}(x)}{\partial X^1_{tj}} \tag{3.24}$$

$$\Delta TF^1_{tj} = \Delta XF^2_{tj} \cdot \frac{\partial T^1_{tj}(x)}{\partial X^2_{tj}} + \Delta XF^1_{tj} \cdot \frac{\partial T^1_{tj}(x)}{\partial X^1_{tj}} \tag{3.25}$$

The left hand side of Equation (3.22) is the link travel time increment for the straight-going vehicles on link j from interval t-1 to t. This increment is the sum of two effects: (i) the additional link travel time for the straight-going vehicles due to additional straight-going vehicles, ΔXE^2_{tj}, from t-1 to t, and (ii) the additional link travel time for the straight-going vehicles due to the additional left-turning vehicles, ΔXE^1_{tj}, from t-1 to t. Equations (3.23) to (3.25) are similarly interpreted. After obtaining the slopes (partial derivatives) from equations (3.22) to (3.25), the static externalities in equations (3.12) and (3.13) can be obtained.

DETERMINATION OF ADDITIONAL INTER-TEMPORAL COMPONENT

Carey and Srinivasan (1993), based on Merchant and Nemhauser's work (1978a, 1978b) proposed expressions for the marginals in a time-dependent assignment problem for a network with multiple origins and a single destination. Their marginals include static and inter-temporal components. The static part correspond to the marginals in the equivalent static assignment problem. The inter-temporal component is consequence of the effect that current conditions in a network have in later periods. Carey and Srinivasan's work considers that travel times are function only of the number of vehicles on the current link at the current time. By means of constraints to the objective function, they incorporate spatial and inter-temporal effects on the current and downstream links. Additionally, Carey and Srinivasan's work makes no distinction on the effects that the desired movement of vehicles has on the estimation of the marginals.

Here, Carey and Srinivasan's time-dependent formulation is extended to a multiple origin multiple-multiple destination case and show how additional terms can be estimated and added to Mahmassani et al's expressions incorporating intertemporal and spatial effects on the downstream nodes. These new terms will provide a better approximation of the global marginals. The formulation is presented as follows:

Assume that time-dependent demands are loaded into a traffic network. The network is represented by a directed graph G = {N, A}, where N is the set of nodes N={1,2,...,q,...}; A represents the set of directed arcs joining the nodes. A = {1,2,..., j,...}. Let A(q)={j ∈ A | j points out of node q} and B(q)={j ∈ A | points into node q}. S = {1,2,..., s,...} is the set of destination nodes. The overall planning period is divided into T equal time intervals t=1,..., T.

The exogenous net inflow or input at node q in period t with destination s is I^s_{tq}. The number of vehicles on arc j at the beginning of the t-th time interval with destination s is represented by X^s_{tj}, and d^s_{tj} represents the number of vehicles with destination s admitted onto arc j during the t-th time period. The total number of vehicles on arc j at the beginning of the t-th

interval is given by $X_{tj} = \sum_s X^s_{tj}$. The travel costs are assumed proportional to travel time and expressed in time units, then $c \cdot X^s_{tj}$ represents the travel cost incurred by the X^s_{tj} vehicles on arc j during period t; c is the length of the time period.

It is assumed that each arc has associated capacity or congestion functions $g_j(\sum_s X^s_{tj})$ that represents the uncontrolled rate of flow from arc j at period t, and a variable v^s_{tj} that represents the actual rate of flow with destination s from arc j at time t. Then $\sum_s v^s_{tj}$ represents the total rate of flow from arc j at time t. It is also assumed that $g_j(\sum_s X^s_{tj})$ is a concave, non-negative, non decreasing function, starting from the origin $g_j(0) = 0$. No specific form is proposed for this function.

From the above definitions, the system optimal multiple destination time-dependent assignment problem is stated as the solution to the following mathematical program:

Program C

$$\text{Minimize } Z(X^s_{tj}) = \sum_{t=1}^{T} \sum_{j \in A} \sum_s c\, X^s_{tj} \qquad (3.26)$$

subject to:

$$g_j(\sum_s X^s_{tj}) \geq \sum_s v^s_{tj}, \quad \forall\, j \in A,\ t = 1,...,T-1 \qquad (3.27)$$

$$X^s_{t+1,j} = X^s_{tj} - v^s_{tj} + d^s_{tj}, \quad \forall\, j \in A,\ s \in S,\ t = 1,...,T-1 \qquad (3.28)$$

$$\sum_{j \in A(q)} d^s_{tj} = I^s_{tq} + \sum_{j \in B(q)} v^s_{tj}, \quad \forall\, q \in N,\ s \in S,\ t = 1,...,T-1 \qquad (3.29)$$

$$X^s_{1j} = \bar{X}^s_{1j} > 0, \quad \forall\, j \in A,\ s \in S \qquad (3.30)$$

$$X^s_{tj} \geq 0, \quad \forall\, j \in A,\ s \in S,\ t = 2,...,T \qquad (3.31)$$

$$(v^s_{tj}, d^s_{tj}, I^s_{tq}) \geq 0, \quad \forall\, q \in N,\ j \in A,\ s \in S,\ t = 1,...,T \qquad (3.32)$$

Constraints (3.27) ensure that the actual outflow v_{tj} from each arc in the network is less than or equal to the capacity of the outflow of that link. Users that remain in link j can provide savings to the system since congestion downstream would be worse if those vehicles were allowed to leave. Carey (1987) shows that the optimal value for those flow controls will usually be zero.

Constraints (3.28) are the number of vehicles conservation equations and they state that the number of vehicles with destination s on an arc in one period equals the number of vehicles with destination s in the previous period, minus the outflow plus the inflow. For the artificial arcs j=d that point out of the destinations, constraints (3.28) reduce to $X^s_{t+1,d} = X^s_{td} + d^s_{td}$ for all t=1,..., T-1, so that X^s_{td} represents the cumulative number of vehicles that has arrived to the destination s up to time t.

Constraints (3.29) are the node balance equations (the outflow from the node is equal the inflow to the node). For the destination nodes (q=s), (3.29) reduces to $d^s_{tj} = \sum_{j \in B(q)} v^s_{tj}$ for $t = 1,...,T-1$.

Constraints (3.30) ensure non zero initial conditions. Constraints (3.31) and (3.32) are non-negative constraints.

The lagrangian for program C is formed as:

$$\mathbf{L}(X^s_{tj}, v^s_{tj}, d^s_{tj}, \lambda^s_{tj}, \mu^s_{tj}, \eta_{tj}) = \sum_{t=1}^{T} \sum_{j \in A} \sum_{s} c \cdot X^s_{tj} - \eta_{tj}[g_j(\sum_s X^s_{tj}) - \sum_s v^s_{tj}] +$$

$$- \lambda^s_{tj}[X^s_{t+1} - X^s_{tj} + v^s_{tj} - d^s_{tj}] - \mu^s_{tq}[\sum_{j \in A(q)} d^s_{tj} - I^s_{tq} - \sum_{j \in B(q)} v^s_{tj}] \qquad (3.33)$$

where η_{tj}, λ^s_{tj}, μ^s_{tj} are the Lagrange multipliers associated with constraints (3.27), (3.28) and (3.29), respectively. If $g'_j(\sum_s x^s_{tj})$ denotes $dg_j(\sum_s x^s_{tj}) / dx^s_{tj}$, the Kuhn-Tucker conditions will given by constraints (3.27) to (3.32) plus

$$c - \eta_{tj}\, g'_j(\sum_s X^s_{tj}) + \lambda^s_{t-1,j} + \lambda^s_{tj} \geq 0, \ (X^s_{tj} \geq 0), \ \forall\ j \in A, s \in S \qquad (3.34)$$

$$c - \lambda^s_{T-1,j} \geq 0, (X^s_{Tj} \geq 0), \ \forall\ j \in A, s \in S \qquad (3.35)$$

$$\eta_{tj} - \lambda^s_{tj} + \mu^s_{tq} \geq 0, (v^s_{tj} \geq 0), \ \forall\, j \in B(q), q \in N, s \in S \qquad (3.36)$$

$$\lambda^s_{tj} - \mu^s_{tq} \geq 0, \ (d^s_{tj} \geq 0), \ \forall\, j \in A(q), \ q \in N, s \in S \qquad (3.37)$$

$$\eta_{tj} \geq 0, [g_j(\sum_s X^s_{tj}) \geq \sum_s v^s_{tj}], \ \forall\, j \in A \qquad (3.38)$$

for all periods t=1,..., T-1, and complementary slackness between the pairs of inequalities (3.34) to (3.38).

The Lagrange multipliers are interpreted as follows. μ^s_{tq} is the Lagrange multiplier associated with the node constraint and for nodes $j \in A(q)$ can be considered as the additional cost of having an additional vehicle entering the network at node q and traveling from there to the destination node (i.e. increasing by one unit the exogenous inflow I^s_{tq}). Assuming that the inflow to link j $d^s_{tj} > 0$, then from the complementary slackness in Equation (3.37)

$$\lambda^s_{tj} - \mu^s_{tq} = 0, \quad \forall\, j \in A(q), q \in N, s \in S \qquad (3.39)$$

or

$$\lambda^s_{tj} = \mu^s_{tq}, \quad \forall\, j \in A(q), \ q \in N, s \in S$$

then λ^s_{tj} will have the same interpretation as μ^s_{tq} for $\forall\, j \in A(q), s \in S$. Then, λ^s_{tj} is the marginal cost of having an additional vehicle with destination s entering the network through link j at node q and going from there to the destination. For nodes $j \in B(q)$ μ^s_{tq} is interpreted as the marginal savings of a vehicle leaving the network at the end of an arc. Then $(\lambda^s_{tj} - \mu^s_{tq}) \forall j \in B(q), s \in S$ is the marginal cost of using arc j at time t for a vehicle with destination s, or:

$$\text{mmt(t,j,s)} = (\lambda^s_{tj} - \mu^s_{tq}) \quad \forall\, j \in B(q), s \in S \qquad (3.40)$$

where mmt(t,j,s) is the marginal cost for a vehicle with destination s, that enters the network through link j at time t.

An expression for mmt(t,j,s) is found as follows. If the outflow with destination s from arc j is positive ($v^s_{tj} > 0$) and the optimal flow control is zero then, by complementarity in (3.36)

$$\eta_{tj} - \lambda^{s}_{tj} + \mu^{s}_{tq} = 0 \quad \forall\, j \in B(q), q \in N, s \in S$$

or

$$\eta_{tj} = mmt(t, j, s) \tag{3.41}$$

and by (3.27)

$$g_j(\sum_s X^s_{tj}) > 0 \tag{3.42}$$

which by the assumed characteristics of $g_j(\sum_s X^s_{tj})$ implies that $X^s_{tj} > 0$ and by complementarity in (3.34)

$$c - \eta_{tj}\, g'_j(\sum_s X^s_{tj}) + \lambda^{s}_{tj} - \lambda^{s}_{t-1,j} = 0 \tag{3.43}$$

Replacing (3.40) in (3.43) and rearranging

$$c - mmt(t, j, s)\, g'_j(\sum_s X^s_{tj}) + \lambda^{s}_{tj} - \lambda^{s}_{t-1,j} = 0$$

$$mmt(t, j, s) = \frac{c}{g'_j(\sum_s X^s_{tj})} + \frac{\lambda^{s}_{tj} - \lambda^{s}_{t-1,j}}{g'_j(\sum_s X^s_{tj})}$$

or remembering that $X_{tj} = \sum_s X^s_{tj}$

$$mmt(t, j, s) = \frac{c}{g'_j(X_{tj})} + \frac{\lambda^{s}_{tj} - \lambda^{s}_{t-1,j}}{g'_j(X_{tj})} \tag{3.44}$$

The use of Equation (3.44) for the determination of the link marginals presents two major problems. The first and most important is the circularity that appears in the second term of the right hand side, to find the value of the link marginals is necessary to know the value of the marginals from the beginning of link j to the destination which are found based on the link marginals. The second problem is the lack of an explicit expression for $g'_j(x_{tj})$. No functional expression is known for $g_j(x_{tj})$ or its derivative. Furthermore, functions that might be derived in a manner that is consistent with known traffic theories will not satisfy the properties required by the math program.

However, Carey and Srinivasan state that, even without solving the dynamic assignment model they propose, the dynamic marginals can be approximated by obtaining estimates of $g_j'(x_{tj})$, λ^s_{tj}, and $\lambda^s_{t-1,j}$ from some other source and plugging them into (3.44). They propose that instead of solving a dynamic model with a time horizon of T periods, T independent system optimal static models can be solved. After solving independently the T system optimal static models, one for each of the periods of the time-dependent model, with an origin-destination demand matrix corresponding to the same period of the time-dependent problem, the time-dependent link marginal cost can be approximated from the results of the T system optimal static models as:

$$\mathrm{mmt}(t,j,s) \approx c \cdot \mathrm{smmt}(t,j) + \Delta\lambda^s_{tj}\ \mathrm{smmt}(t,j) \qquad (3.45)$$

where smmt(t,j)is the marginal cost of traversing arc j measured in the number of time periods c obtained from each of the t static models, and $\Delta\lambda^s_{tj}$ is the difference between marginal travel times from the beginning of link j to the destination s from two consecutive solutions of the system optimal static model. c·smmt(j) is in the same time units as c.

Carey and Srinivasan's approximation, using the results from the T system optimal static models, is explained as follows:

In a link j of a traffic network, let the flow rate v_j, and hence the volume X_j on link j be held constant. Let $T_j(v_j)$ be the time taken to traverse link j when the flow rate is v_j, and let $g_j(X_j)$ be the outflow rate from link j when the volume on link j is X_j. Then the number of vehicles on the link is given by $X_j = v_j T_j(v_j)$, also a constant. The outflow rate is equal to the flow rate or $g_j(X_j) = v_j$, also a constant. Replacing the latter into the former and re-arranging gives the expression for the travel time to traverse arc j (user experienced link traversal travel time) as:

$$T_j(v_j) = \frac{X_j}{g_j(X_j)} \qquad (3.46)$$

Differentiating $X_j = v_j T_j(v_j)$ with respect to v_j and $v_j = g_j(X_j)$ with respect to X_j, leads to $\frac{dX_j}{dv_j} = T_j(v_j) + v_j T_j'(v_j)$ and $\frac{dv_j}{dX_j} = g_j'(X_j)$. But $\frac{dv_j}{dX_j} = \frac{1}{\frac{dX_j}{dv_j}}$ then

$$\frac{1}{g'_j(X_j)} = T_j(v_j) + v_j T'_j(v_j) = mmt(j) \qquad (3.47)$$

that is, $\frac{1}{g'_j(x_j)}$ is equal to the marginal cost of using link j in the static case.

Considering that Equation (3.47) holds exactly for static models, and it holds approximately in each period in the time-dependent model, and replacing it in (3.44), Equation (3.45) follows.

Additionally, subtracting the user experienced cost (Equation 3.46) from the marginal (Equation 3.47) the static externality or optimal static congestion toll can be found as:

$$ext(j) = \frac{1}{g'_j(X_j)} - \frac{X_j}{g_j(X_j)} = v_j T'_j(v_j) \qquad (3.48)$$

To calculate the time-dependent externalities, it is necessary to find the user experienced travel time, which will be then subtracted from the time-dependent marginals to find the time-dependent externalities. Following Carey and Srinivasan's work, and using a similar notation to the one use for the determination of the time-dependent marginals, that expression is found as follows:

Let λ^{su}_{tj} be the user experienced travel time from the beginning of link j to the destination s; μ^{su}_{tq} be the user experienced cost from the end of arc j to the destination s. Then, $T^s_{tj}(v_{tj}) = (\lambda^{su}_{tj} - \mu^{su}_{tq})$ is the user experienced cost of traversing arc j at time t for a vehicle with destination s.

Of the number of vehicles with destination s in link j at the beginning of time interval t (x^s_{tj}), the fraction that leaves arc j during time interval t is $\theta^s_{tj} = \frac{g_j(\sum_s x^s_{tj})}{x^s_{tj}}$ and the fraction that remains in the link is $(1 - \theta^s_{tj})$. The experienced travel time for each of these groups is μ^{su}_{tq} and λ^{su}_{tj} respectively. Then, the average travel time for these two groups is $\lambda^{su}_{tj}(1 - \theta^s_{tj}) + \mu^{su}_{tq}\,\theta^s_{tj}$. Vehicles with destination s that enter link j at time t-1 incur a cost c for using the arc until period t. Then, the sum of these two travel times gives the average travel time for a vehicle entering the link at time t-1 or

$$\lambda^{su}_{t-1,j} = c + \lambda^{su}_{tj}(1 - \theta^s_{tj}) + \mu^{su}_{tq}\,\theta^s_{tj} \qquad (3.49)$$

rearranging leads to

$$T^{s}_{tj}(v_{tj}) = (\lambda^{su}_{tj} - \mu^{su}_{tq}) = \frac{c}{\theta^{s}_{tj}} + \frac{(\lambda^{su}_{tj} - \lambda^{su}_{t-1,j})}{\theta^{s}_{tj}} \qquad (3.50)$$

In (3.50) $\frac{1}{\theta^{s}_{tj}} = \frac{x^{s}_{tj}}{g_j(\sum_{s} x^{s}_{tj})}$

Using the results of the static model in a similar form to the case of the time-dependent marginals, $\frac{1}{\theta^{s}_{tj}}$ is replaced in (3.49) by the travel time from the static model or $\frac{1}{\theta^{s}_{tj}} = T_j(v_{tj})$. Then

$$T^{s}_{tj}(v_{tj}) \approx c \cdot T_j(v_{tj}) + \Delta\lambda^{su}_{tj} T_j(v_{tj}) \qquad (3.51)$$

where $\Delta\lambda^{su}_{tj}$ is the difference between consecutive values of the user experienced travel time from the beginning of link j to the destination s from the T system optimal static models. As in Equation (3.45) $T_j(v_{tj})$ is measured in the number of time periods and $c \cdot T_j(v_{tj})$ is the same time units as c.

Subtracting Equation (3.51) from Equation (3.45) the approximate values of the time-dependent externalities for vehicles with destination s are found as:

$$ext(t,j,s) \approx mmt(t,j,s) - T^{s}_{tj}(v_{tj})$$

$$ext(t,j,s) \approx c \cdot smmt(t,j) + \Delta\lambda^{s}_{tj}\, smmt(t,j) - c \cdot T_j(v_{tj}) - \Delta\lambda^{su}_{tj} T_j(X_{tj})$$

$$ext(t,j,s) \approx c \cdot [T_j(v_{tj}) + X_{tj} T'_j(v_{tj})] + \Delta\lambda^{s}_{tj} [T_j(v_{tj}) + v_{tj} T'_j(v_{tj})]$$
$$- c\, T_j(v_{tj}) - \Delta\lambda^{su}_{tj} T_j(v_{tj})$$

$$ext(t,j,s) \approx c \cdot T_j(v_{tj}) + c \cdot X_{tj} T'_j(v_{tj}) + \Delta\lambda^{s}_{tj}\, T_j(v_{tj}) + \Delta\lambda^{s}_{tj}\, v_{tj} T'_j(v_{tj})$$
$$- c \cdot T_j(v_{tj}) - \Delta\lambda^{su}_{tj} T_j(v_{tj})$$

$$ext(t,j,s) \approx c \cdot v_{tj} T'_j(v_{tj}) + \Delta\lambda^{s}_{tj}\, T_j(v_{tj}) + \Delta\lambda^{s}_{tj}\, v_{tj} T'_j(v_{tj}) - \Delta\lambda^{su}_{tj} T_j(v_{tj})$$

$$ext(t,j,s) \approx c \cdot v_{tj} T'_j(v_{tj}) + \Delta\lambda^{s}_{tj} v_{tj} T'_j(v_{tj}) + (\Delta\lambda^{s}_{tj} - \Delta\lambda^{su}_{tj}) T_j(v_{tj}) \qquad (3.52)$$

where ext(t,j,s) is the time-dependent externality (optimal toll) for a vehicle with destination s entering link j at time t; $v_{tj}T_j'(v_{tj})$ is the static component of the externality; and $\Delta\lambda^s_{tj}X_{tj}T_j'(v_{tj})+(\Delta\lambda^s_{tj}-\Delta\lambda^{su}_{tj})T_j(X_{tj})$ is the inter-temporal effect.

Instead of using the results from the T system optimal static models, a better approximation to the global marginals can be found if the results from the Mahmassani et al's approach are used. Let us take the marginal effect on the current link, represented by the first three terms of the right hand side of Equations (3.10) and (3.11) and define the static component of the marginals as smmt(t,j,1) and smmt(t,j,2) or:

$$\text{smmt}(t,j,1)=T^1_{tj}(x)+\frac{\partial T^1_{tj}(x)}{\partial X^1_{tj}}\cdot X^1_{tj}+\frac{\partial T^2_{tj}(x)}{\partial X^1_{tj}}\cdot X^2_{tj}\cdot\alpha \qquad (3.53)$$

and

$$\text{smtt}(t,j,2)=T^2_{tj}(x)+\frac{\partial T^2_{tj}(x)}{\partial X^2_{tj}}\cdot X^2_{tj}+\frac{\partial T^1_{tj}(x)}{\partial X^2_{tj}}\cdot X^1_{tj}\cdot\beta \qquad (3.54)$$

The right hand sides of equations (3.53) and (3.54) correspond to the first two terms of the right hand side of the global marginals expression (Equation 3.7).

The static component of the externality is defined as the difference between the static component of the marginals and the experienced travel time or:

$$\text{sext}(t,j,1)=\frac{\partial T^1_{tj}(x)}{\partial X^1_{tj}}\cdot X^1_{tj}+\frac{\partial T^2_{tj}(x)}{\partial X^1_{tj}}\cdot X^2_{tj}\cdot\alpha \qquad (3.55)$$

and

$$\text{sext}(t,j,2)=\frac{\partial T^2_{tj}(x)}{\partial X^2_{tj}}\cdot X^2_{tj}+\frac{\partial T^1_{tj}(x)}{\partial X^2_{tj}}\cdot X^1_{tj}\cdot\beta \qquad (3.56)$$

Using equations (3.45) and (3.53) or (3.54), establishing equivalencies using equation (3.7) and adding the queuing marginals calculated by Mahmassani et al., a new expression for the time-dependent marginals is found. The term $\text{mmt}(t,j)$ in equation (3.45) that represents the static component of the marginal is replaced by $\frac{1}{c}\text{smmt}(t,j,1)$ (the corresponding static component of the marginals in Mahmassani et al.) for a left turning vehicle or by $\frac{1}{c}\text{smmt}(t,j,2)$

for a straight or right turning moving vehicle. Then, the time-dependent link marginal cost for a vehicle entering link j at time t and turning left with destination s will be given by:

$$\text{mmt}(t,j,s,1) \approx \text{smmt}(t,j,1) + \frac{1}{c}\Delta\lambda_{tj}^{s1}\,\text{smmt}(t,j,1) + \frac{\partial T_{tj}^{2}(x)}{\partial X_{tj}^{1}} \cdot \text{Max}\{0,(Q-C)\} \quad (3.57)$$

where $\Delta\lambda_{tj}^{s1}$ is the difference between two consecutive values of the marginal travel times from the beginning of link j to the destination from the static model for a vehicle turning left.

For a vehicle going straight or turning right, it will be:

$$\text{mmt}(t,j,s,2) \approx \text{smmt}(t,j,2) + \frac{1}{c}\Delta\lambda_{tj}^{s2}\ \text{smmt}(t,j,2) + \frac{\partial T_{tj}^{1}(x)}{\partial X_{tj}^{2}} \cdot \text{Max}\{0,(Q-C)\} \quad (3.58)$$

where $\Delta\lambda_{tj}^{s2}$ is the difference between two consecutive values of the marginal travel times from the beginning of link j to the destination from the static model for a vehicle going straight or turning right. The marginals would be calculated from t=2 to have the initial values for the $\Delta\lambda$'s. The value for c is the length of the simulation interval.

In a similar form, the experienced travel time to traverse arc j for a vehicle with destination s entering arc j at time t and making movement L will be found using expression (3.50) as:

$$T_{tj}^{Ls} \approx T_{tj}^{L}(x) + \left(\lambda_{tj}^{suL} - \lambda_{t-1,j}^{suL}\right)\frac{1}{c}T_{tj}^{L}(x)$$

or

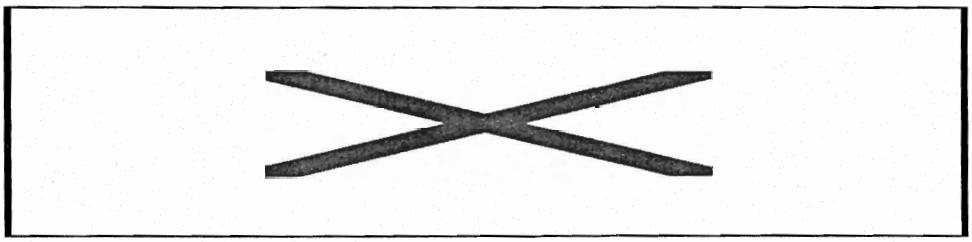
$$T_{tj}^{Ls} \approx T_{tj}^{L}(x) + \frac{1}{c}\Delta\lambda_{tj}^{suL}T_{tj}^{L}(x) \quad (3.59)$$

where λ_{tj}^{suL} is the user experienced travel time at time t from the beginning of link j to the destination for a vehicle with destination s making movement L.

The new expressions for the time-dependent externalities for a vehicle with destination s, entering link j at time t and doing turning movement L (difference between marginals and user perceived cost) are found combining equations (3.57), (3.58) and (3.60) as:

$$ext(t,j,s,1) \approx smmt(t,j,1) + \frac{1}{c}\Delta\lambda_{tj}^{s1} smmt(t,j,1) + \frac{\partial T_{tj}^{2}(x)}{\partial X_{tj}^{1}} \cdot Max\{0,(Q-C)\}$$

$$- T_{tj}^{1}(x) - \frac{1}{c}\Delta\lambda_{tj}^{suL} T_{tj}^{1}(x)$$



for a vehicle turning left, and

$$ext(j,t,s,2) \approx smmt(t,j,2) + \frac{1}{c}\Delta\lambda_{tj}^{s2} smmt(t,j,2) + \frac{\partial T_{tj}^{1}(x)}{\partial X_{tj}^{2}} \cdot Max\{0,(Q-C)\}$$

$$- T_{tj}^{2}(x) - \frac{1}{c}\Delta\lambda_{tj}^{su2} T_{tj}^{2}(x)$$

$$ext(a,t,s,2) \approx sext(t,j,2) + \frac{1}{c}\Delta\lambda_{tj}^{s2} sext(t,j,2) + \frac{1}{c}\left(\Delta\lambda_{tj}^{s2} - \Delta\lambda_{tj}^{su2}\right) T_{tj}^{2}(x)$$

$$+ \frac{\partial T_{tj}^{1}(x)}{\partial X_{tj}^{2}} \cdot Max\{0,(Q-C)\} \qquad (3.61)$$

for a vehicle going straight or turning right.

Algorithm for the Estimation of Time-Dependent Marginals and Externalities

Step 0. Define the link, destination and the assignment interval for which the time-dependent marginals and externalities are to be estimated.

Step 1. Use DYNASMART as a dynamic traffic simulator.

Step 2. From the output files given by DYNASMART extract, for each of the simulation intervals, the traffic flows, directions of the traffic flows and trip times.

Step 3. Calculate the average values for the traffic flow descriptors for each assignment interval.

Step 4. Estimate for the assignment interval, each of the terms of the equations (3.10) and (3.11)(time-dependent marginals for vehicles turning left and making any other movement as estimated in Mahmassani et al.)

Step 5. With the information from step 4, use equations (3.53) and (3.54) to estimate the values of smtt(t,j,L) (first three terms of the time-dependent marginals as estimated by Mahmassani et al.)

Step 6. With the values of the time-dependent marginals estimated in step 4, get the least cost path from the end of the link under study to the desired destination.

Step 7. Get the travel time for the least cost path defined in step 6.

Step 8. Calculate the difference between consecutive values of the least cost paths.

Step 9. Calculate the difference between consecutive values of the travel times that correspond to the least cost paths.

Step 10. Use equations (3.57) and (3.58) and the information from steps 4, 5 and 8 to estimate the values of the time-dependent marginals for the link and destination under study.

Step 11. Use equations (3.55), (3.56), (3.60) and (3.61) and the information from steps 4, 5, 8 and 9 to estimate the values of the time-dependent externalities for the link and destination under study.

APPLICATION EXAMPLES

As an example, Tables 3.1 and 3.2 present the values of the estimation of the marginals and of the externalities using the procedures proposed by Mahmassani et al., and in this work. A simplified traffic network, presented in Figure 3.1, with only 6 nodes and 14 links was used. Nodes 1,3,4 and 6 are the centroids of the demand zones. For the example, 2,823 vehicles were generated, according the load profile presented in Figure 3.3, at nodes 3,4 and 6 and sent to node 1. The length for all the links is 0.5 miles. Free speeds are 20 mph for all the links. Jam densities and maximum bumper to bumper densities are 160 vehicles per mile and 260 vehicles/mile respectively. All the intersections have no signal control. Values of the marginals and of the externalities are for link 2-5. Each assignment interval was of one minute (c=1).

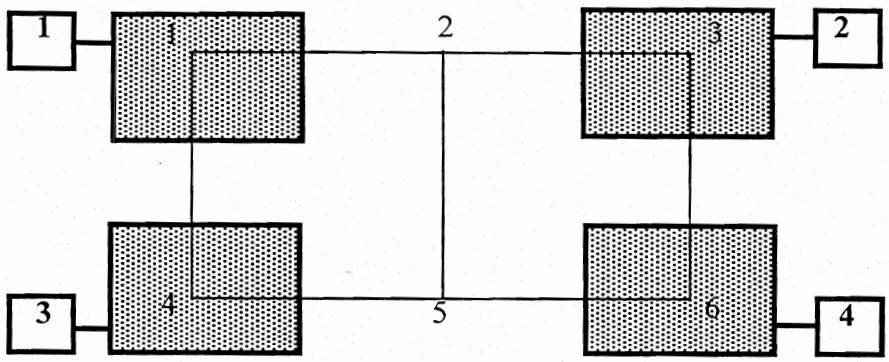


Figure 3.1 Example Traffic Network.

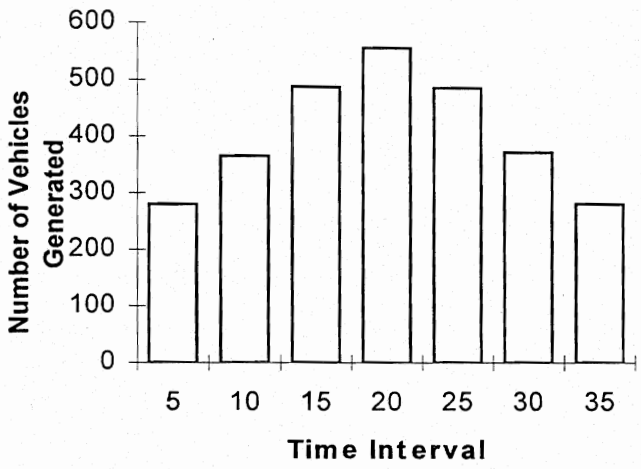


Figure 3.2 Loading Profile for Simulation Experiments.

TABLE 3.1 COMPARISON OF THE LINK MARGINAL TRIP TIMES BETWEEN MAHMASSANI ET AL'S PROCEDURE AND THE NEW PROCEDURE. LINK 10, LEFT TURN AND STRAIGHT MOVING VEHICLES. MARGINAL TIMES IN MINUTES.

	Old Marginals	Old Marginals	New marginals	New marginals
Assign. Intvl.	Left Turn	through.	Left Turn	through.
1				
2	1.757	1.615	1.843	1.694
3	1.757	1.734	1.896	1.871
4	1.757	1.661	2.237	2.114
5	1.757	1.579	2.21	1.986
6	1.757	1.528	1.664	1.447
7	1.757	1.754	1.565	1.563
8	2.093	2.115	1.967	1.988
9	2.775	2.071	3.963	2.957
10	2.591	1.714	3.902	2.581
11	1.908	1.516	2.267	1.801
12	1.836	1.512	1.316	1.084
13	3.192	1.649	2.104	1.087
14	4.567	2.093	4.019	1.842
15	3.795	2.633	4.402	3.054
16	12	2.419	20.304	4.093
17	3.248	1.837	5.45	3.082
18	2.284	1.533	3.168	2.126
19	1.91	1.512	2.363	1.87
20	1.91	1.512	2.26	1.789
21	3.409	1.634	2.598	1.245
22	4.041	1.992	0.55	0.271
23	4.843	2.366	1.884	0.92
24	11.885	2.873	16.354	3.953
25	15.095	2.316	23.76	3.645

TABLE 3.1(CONT'D) COMPARISON OF THE LINK MARGINAL TRIP TIMES BETWEEN MAHMASSANI ET AL'S PROCEDURE AND THE NEW PROCEDURE. LINK 10, LEFT TURN AND STRAIGHT MOVING VEHICLES. MARGINAL TIMES IN MINUTES.

26	2.281	1.982	3.173	2.757
27	1.684	1.622	1.696	1.633
28	1.815	1.511	1.245	1.037
29	1.815	1.511	2.105	1.753
30	1.815	1.511	1.134	0.944
31	2.637	1.604	-0.401	-0.244
32	2.144	1.811	0.941	0.795
33	1.807	1.972	1.807	1.972
34	1.802	1.955	2.197	2.383
35	2.776	1.666	3.573	2.144

TABLE 3.2: COMPARISON OF THE EXTERNALITIES BETWEEN MAHMASSANI ET AL'S PROCEDURE AND THE NEW PROCEDURE. LINK 10, LEFT TURN AND STRAIGHT MOVING VEHICLES. EXTERNALITIES IN MINUTES.

Assign. Intvl.	Old Externality Left Turn	Old Externality Through.	New Externality Left Turn	New Externality Through.
1				
2	0.2	0.06	0.25	0.103
3	0.148	0.127	0.224	0.201
4	0.177	0.083	0.454	0.334
5	0.217	0.041	0.502	0.281
6	0.242	0.015	0.169	-0.046
7	0.133	0.133	0.031	0.031
8	0.337	0.378	0.264	0.303
9	1.009	0.31	1.87	0.871
10	0.994	0.119	2.002	0.683

TABLE 3.2 (CONT'D): COMPARISON OF THE EXTERNALITIES BETWEEN MAHMASSANI ET AL'S PROCEDURE AND THE NEW PROCEDURE. LINK 10, LEFT TURN AND STRAIGHT MOVING VEHICLES. EXTERNALITIES IN MINUTES.

11	0.401	0.011	0.63	0.167
12	0.331	0.009	-0.023	-0.254
13	1.614	0.082	0.732	-0.275
14	2.78	0.348	2.32	0.182
15	1.56	0.717	2.04	1.029
16	10.101	0.556	17.892	1.727
17	1.734	0.325	3.603	1.238
18	0.785	0.036	1.482	0.442
19	0.405	0.009	0.767	0.277
20	0.405	0.009	0.658	0.19
21	1.841	0.073	1.11	-0.236
22	2.29	0.27	-0.706	-0.964
23	2.767	0.481	0.273	-0.542
24	9.495	0.879	13.663	1.708
25	13.208	0.525	21.486	1.487
26	0.786	0.489	1.488	1.074
27	0.192	0.132	-4.97	-5.024
28	0.31	0.008	-0.17	-0.376
29	0.31	0.008	0.5	0.149
30	0.31	0.008	5.117	4.921
31	1.087	0.056	-1.317	-1.159
32	0.496	0.165	-0.323	-0.467
33	0.087	0.256	0.087	0.256
34	0.087	0.244	0.315	0.506
35	1.197	0.089	1.79	0.364

This and the following pages show the graphical representation of the values in tables 3.1 to 3.2.

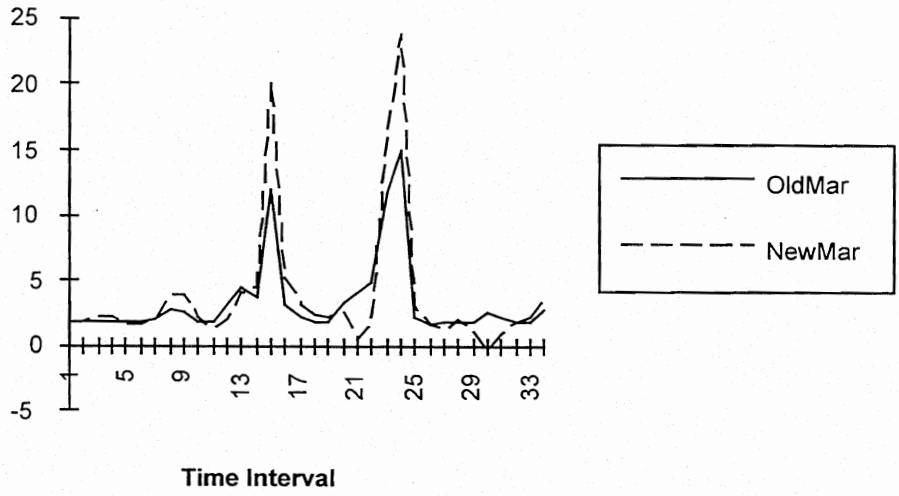


Figure 3.3 Marginal Trip Times for Left Turning Vehicles. Link 5-2. Destination Node 1.

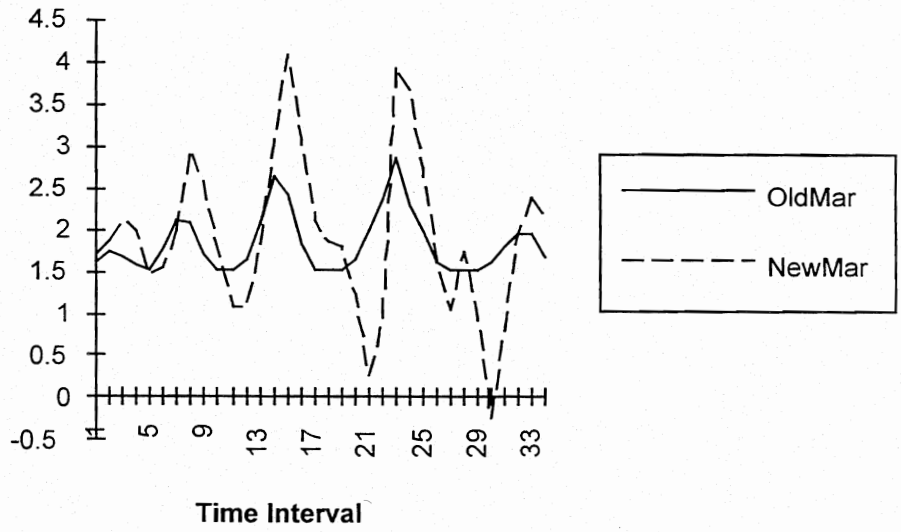


Figure 3.4 Marginal Trip Times for Straight Moving Vehicles. Link 5-2. Destination Node 1.

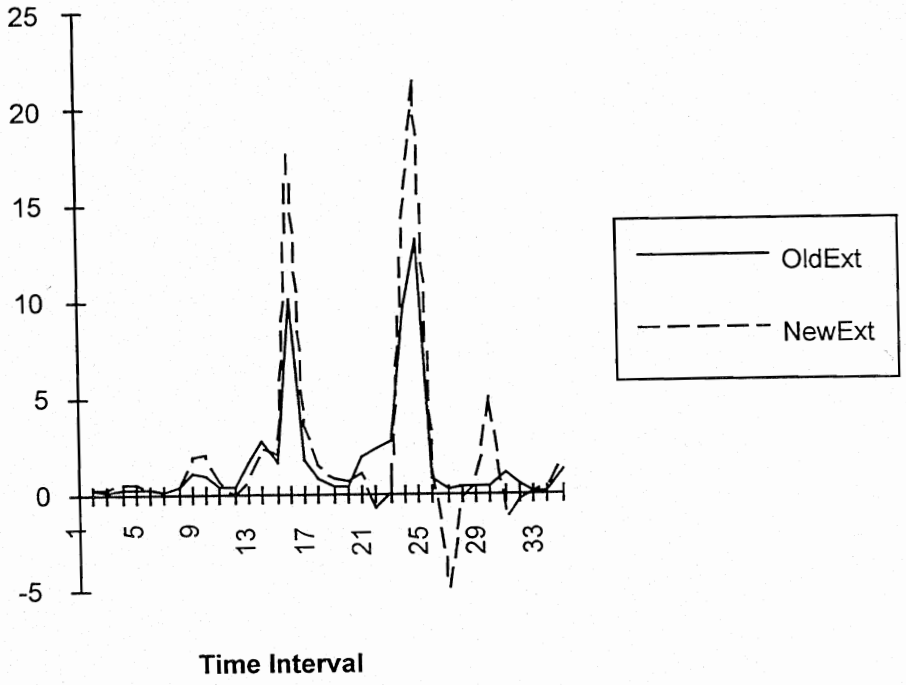


Figure 3.5 Externalities for Left Turning Vehicles. Link 5-2. Destination Node 1.

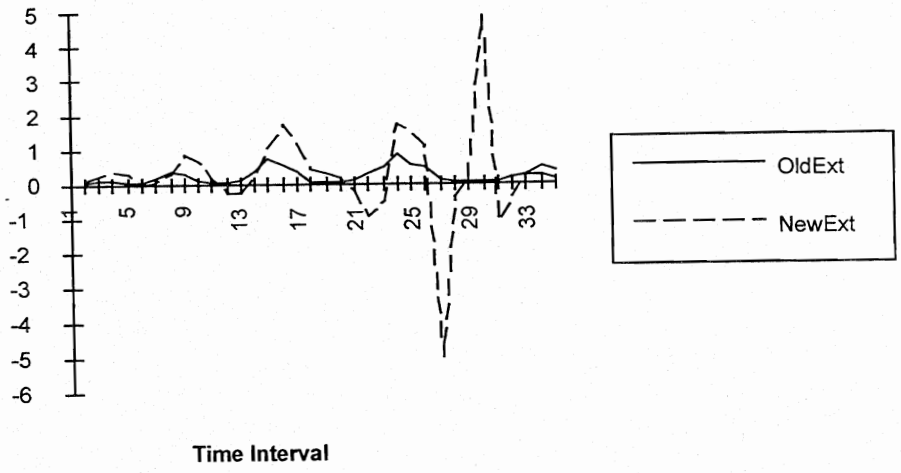


Figure 3.6 Externalities for Straight Moving Vehicles. Link 5-2. Destination Node 1.

CONCLUSION

New expressions for the calculation of the time-dependent externalities (optimal prices) were developed. Although they continue to be local in nature, they have partially incorporated both spatial and inter-temporal interactions. They are expected to provide a better approximation to the global marginals than previously implemented.

However, a benchmark remains to be calculated by running simulations with and without an additional vehicle entering the links under study at the correct times. This benchmark would provide the basis for a correct comparison of the approaches.

CHAPTER 4: CONSTANT AND VARIABLE PRICES IN DYNASMART

This chapter describes the incorporation of pricing in the dynamic traffic simulator used in the development of this research and is broken down into 6 sections. The first section of this chapter presents a justification for the use of a dynamic traffic simulator as a means to investigate the effect of pricing in a traffic network. The second section describes the DYNASMART simulation-assignment model as the main instrument used for the above purpose in this research. The third section describes the modifications made to DYNASMART to incorporate road pricing as a characteristic of a traffic network. The fourth section analyses the effects of constant prices in a single link of the network and in a restricted zone. Section five analyses the effects of variable prices applied in all the links of the network. Finally, section six presents the conclusions of this chapter.

INTRODUCTION

If monetary charges are applied for the use of some of the links or facilities in a traffic network, changes in its operation can be expected. Drivers tend to select travel routes that minimize their total travel cost. In their evaluation of that total, they will now include not only the travel times but also the out of pocket costs for the use of any link or facility. The possible selection of alternative routes may lead to a redistribution of the traffic flows, and consequently changes in the attributes that describe the operation of the network.

Chapter two of this report has discussed the objections to congestion pricing as a first-best option to manage congestion. Although second-best formulations avoid some of these objections in the static case, the non-static nature of congestion leads, in the time-dependent case, to complicated analytical formulations that are only approximations to the correct global marginals, as reviewed in chapter three. Alternative ways are needed to analyze the effect of pricing in a traffic network in a time-dependent framework. The problems with the analytical formulations make more attractive the use of a dynamic traffic simulator like DYNASMART as a tool for the analysis of the effects of pricing at the network level. Its flexibility and the information it provides give the necessary elements for the evaluation of those effects. The modifications completed as part of the present work allow DYNASMART to incorporate different pricing strategies, ranging from constant prices in a single network link to variable time-dependent pricing over the whole network. These modifications are described in the section on changes in DYNASMART to incorporate pricing.

This chapter also presents the experiments performed to investigate the effects that the application of constant and variable pricing has on several key network performance indicators, including total travel times, number of vehicles using the priced links, revenues and traveled

distances. The experiments were performed in a hypothetical test network used in several previous studies (Peeta, 1994; Hu, 1995). A given time-varying O-D demand matrix was loaded onto the network and the traffic assignment simulated. An initial simulation was performed with prices at zero level to provide a benchmark for comparison when prices are introduced. Then different pricing schemes with constant or variable tolls were considered.

Because of its importance for the development of this work, a brief description of the dynamic traffic simulator used is presented below. A complete documented description of DYNASMART and of its capabilities can be found in Jayakrishnan, Mahmassani and Hu (1994) or Hu (1995).

DYNASMART

DYNASMART is designed to model traffic patterns and evaluate overall network performance under real time information systems for a given network configuration (including traffic control system) and given time dependent Origin-Destination demand pattern. The modeling approach integrates traffic flow models, traffic control systems, network path processing, user behavior rules and information supply strategies. A principal feature is that vehicle paths are modeled explicitly as the outcome of individual path selection decisions at each node of the network. Traffic flow is represented using a hybrid approach where vehicles are tracked individually or as macro particles, and moved consistently with macroscopic traffic flow relations between speed and concentration on a roadway link (Chang, Mahmassani and Herman, 1985). Junction control and delay are explicitly modeled. Multiple user classes categorized by vehicle types, information availability and/or behavioral responses and/or traffic performance characteristics are also implemented in DYNASMART. Vehicles of different classes are routed in the network according to individual decisions made at decision points, under real time information availability. A version of DYNASMART that considers the passenger car mode only is used in this research. This version of DYNASMART is more efficient computationally and incorporates efficient data structures, and hence is better suited to incorporate road pricing.

The conceptual model structure for DYNASMART is presented in Figure 4.1. Additional details of the program components are shown in Fig. 4.2. The simulation model is deterministic and uses constant time increments. Vehicles are generated using a given time-dependent Origin-Destination zonal demand pattern and moved in the network according to macroscopic traffic flow relations ("link Pass 1" in Fig. 4.1). Supply information is provided to the drivers at the route decision points using a path database. The simulated driver response to the provided information and the nodal flow constraints (given by the traffic signal settings) gives the vehicles' distribution among the possible links to follow at the network nodes ("Node pass"). Vehicles are

then moved along the links they have just entered ("Link Pass 2" in Fig. 4.1), which defines the network traffic flow conditions at the end of the time step. The path database is updated ("Path processing" in Fig. 4.1), the time step is incremented and the simulation continues. Details of the major components of DYNASMART are described below.

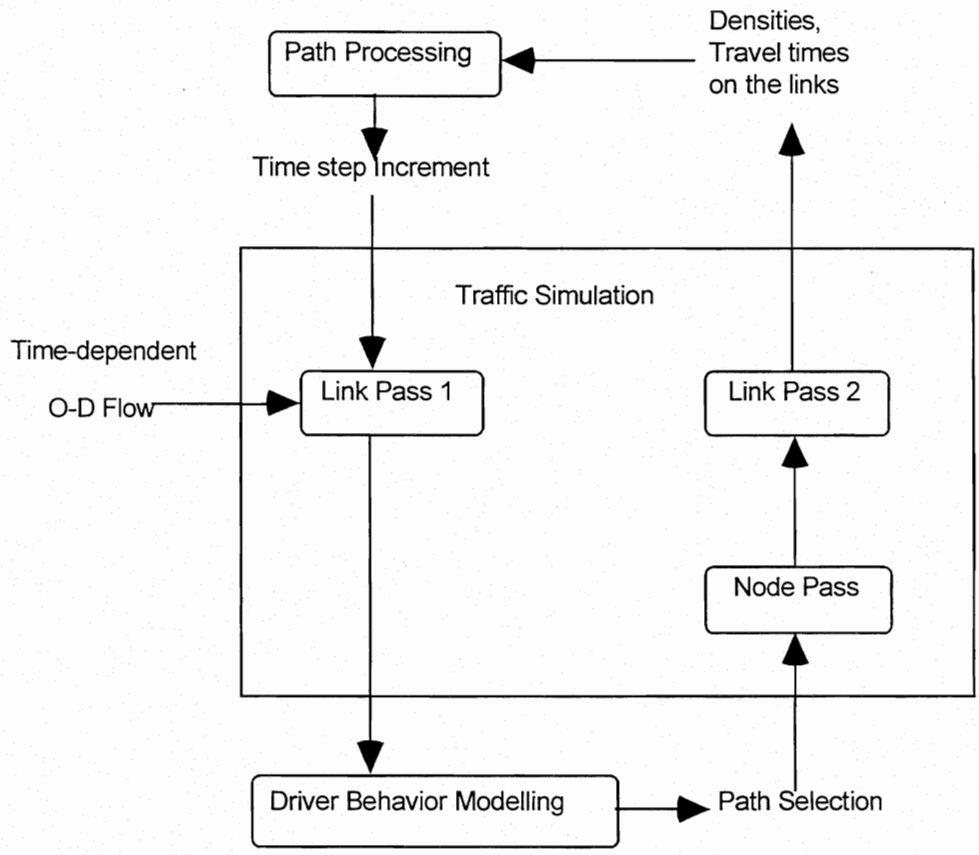


Figure 4.1 DYNASMART Model: Conceptual Structure (Jayakrishnan et al, 1994).

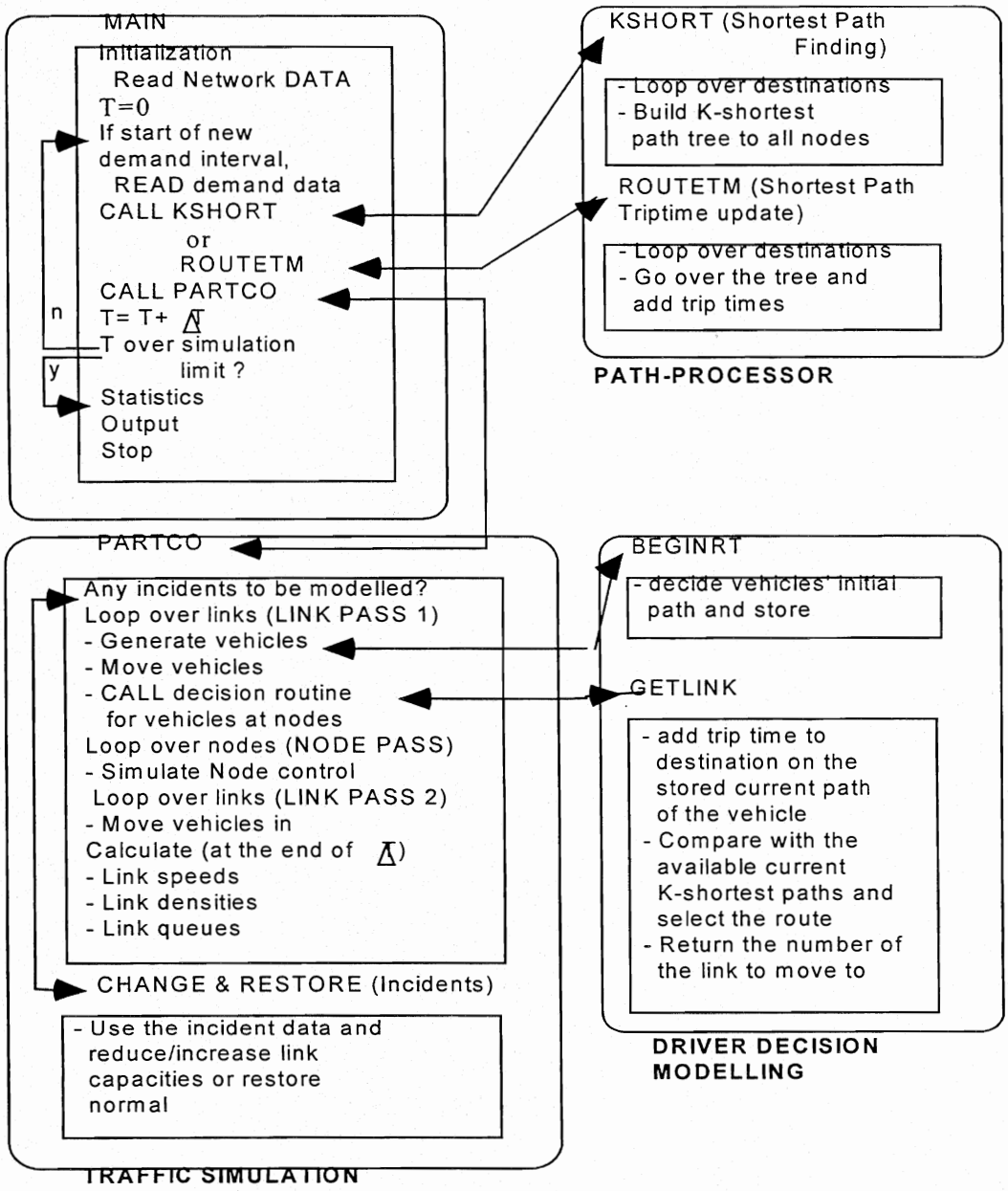


Figure 4.2 Program Flow and Communications Between Conceptual Modules (Jayakrishnan et al., 1994)

Traffic Simulation Component

Simulation of vehicle movement along a link takes place according to an extension of the macroparticle simulation model (MPSM) (Chang, Mahmassani, and Herman, 1985), initially developed as a special-purpose code for experimental studies of commuter behavior dynamics in traffic corridors. DYNASMART moves vehicles in discrete bunches or macroparticles, at the prevailing local speed determined by macroscopic traffic relations. In its current implementation, DYNASMART uses a macroparticle size of one vehicle, meaning that it effectively tracks the movement and location of individual vehicles (Mahmassani et al., 1993). Nevertheless, it does not consider the microscopic details or interactions such as car following, overtaking, etc. DYNASMART uses a fixed time increment simulation approach to move vehicles in the network. Two major aspects of the traffic simulation component are link movement and node transfer. Link movement is a process for moving vehicles on links during each scanning time interval in the simulation. The traffic flow model used is represented by the conservation equation,

$$\frac{\partial q}{\partial x}+\frac{\partial k}{\partial t}=g(x,t) \qquad (4.1)$$

where

q= flow (vehicles/hour),

k = density (vehicles/mile),

g(x,t) = net generation rate (vehicles/hour/mile), a function of

x = location, and

t = time,

and a modified version of the Greenshield's speed-density relationship as

$$v = v_0 + (v_f - v_0)(1-\frac{k}{k_0})^{\alpha} \qquad (4.2)$$

where

v = mean speed in the highway segment

v_0 = a user-specified minimum speed,

v_f = free-flow speed of the highway segment,

k = concentration in the highway segment

k_0 = jam concentration, and

α = a user-specified parameter to capture the sensitivity of speed to concentration.

DYNASMART moves vehicles according to the prevailing local speeds keeping track of their positions, consequently, the identity q = kv (where v is the average speed) is not used to solve

the conservation equation. The vehicle flux across link boundaries is based only on the number of vehicles reaching the link boundary during each time step, and the movement constraints at the link boundary.

All the vehicles in the link move at the same average speed associated with the concentration determined in the previous time step. The specified minimum speed ensures that the simulation does not stop due to zero speeds.

In DYNASMART no simulation of lane-changing maneuvers or car following is performed, and no platoon dispersion model to simulate the headway variations among vehicles is used. Besides the lack of definitive models for these processes, this microscopic level of detail is not necessary for the purpose of evaluating network level effects of pricing.

Traffic Generation and Initial Path Assignment. In DYNASMART, vehicles are generated according to the specified zone-to-zone demand during each demand subinterval. Vehicles enter the network at links identified as "generation links". The total specified generation from each zone during a subinterval is calculated from the O-D information. This defines the number of vehicles that will be generated during each time step in each zone. The generated vehicles are equally and randomly assigned to the generation links in each zone. The vehicles' destination zone is assigned according to destination probabilities computed from the O-D matrix. Vehicles are sent to specified destination nodes in each of the zones. When generated, each vehicle is randomly tagged as equipped or not equipped to receive information according to the user-specified fraction of equipped vehicles. Each vehicle is also assigned to an initial path. The path can be from the k-shortest stored paths or from outside files that prescribe an assignment.

Link Pass. Vehicles are moved along the links according to the prevailing speed calculated from the speed-density relation and based on the existing density at the beginning of the time step. The vehicles' positions are kept as the distance from the end of the link segment and updated as:

$$x_j^t = x_j^{t-1} - v_i^t \cdot \Delta t, \text{ if } \left(x_j^{t-1} / v_i^t\right) > \Delta t; \qquad (4.3)$$

$$x_j^t = 0, \text{ otherwise}$$

where

x_j^t = distance of vehicle j from the end of the link at the end of time step t,

Δt = simulation time step length,

v_i^t = speed in link i that the vehicle j is on, during time step t.

If the vehicle reaches the end of the link during the current time step, it may move to the next link (using the driver's decision module) and travel for the rest of the time step or it may join the queue at the end of the link if it cannot move to the desired link.

Node Pass. The node pass module moves the vehicles from link to link or from section to section. This module simulates the traffic control features at the end of the links and calculates the number of vehicles that can move to the downstream nodes. The driver's response is also simulated here to find the next link to follow. Output of this module includes the number of vehicles in the link-end queue at the end of the time step and the number of vehicles added to and subtracted from the links during the time step.

Travel Time and Queuing. Travel times are determined at the end of each time step and transferred to the path-processing module to find current path trip times. The travel time is calculated as:

$$T_i^t = Tm_i^t + Tq_i^t \tag{4.4}$$

where

T_i^t = Travel time for link i at time t,

Tm_i^t = moving time,

Tq_i^t = queue waiting time.

The moving time is based on the current link speed (v_i^t) and the available length of the link (length of the link (L_i) minus the length used by the queue), as:

$$Tm_i^t = \frac{L_i - Q_i^t \cdot l_c}{v_i^t} \tag{4.5}$$

where

Q_i^t = average queue length across the lane of the link, and

l_c = assumed vehicle length.

The queue waiting-time is calculated by dividing the queue length by a moving average of the discharge rate over a specified number of time steps.

Incidents. Incidents are modeled in DYNASMART by reducing the available number of lanes in computing the density of the affected links by specified fractions. Any number of incidents can be simulated in this form if the starting, end times and reduction factors (or severity) are specified for each incident.

Traffic Control Simulations

DYNASMART can model the control elements listed in Table 4.1, as described below.

TABLE 4.1. TRAFFIC CONTROL TYPES IN DYNASMART
(JAYAKRISHNAN ET AL., 1994)

Surface Street	Freeway system
No Control	Ramp metering
Stop signs	Pretimed
Signal control	Demand-responsive (ALINEA)
(green, red, amber time,	Variable message signs
cycle length, offsets, phases)	Route advisory signs
Pretimed	Route congestion warning signs
Pretimed coordinated	Speed control signs
Multidial pretimed	
Actuated (full)	

Link Outflow Constraints. These constraints limit the number of vehicles that are allowed to leave a link at an intersection approach. In DYNASMART the following equation is used:

$$VO_i = \text{Min}\{VQ_i, VS_i\} \quad (4.6)$$

where, for link i

VO_i = maximum number of vehicles that can enter the intersection during Δt.
(They may not enter if the receiving link's inflow constraints are not met),

VQ_i = available number of vehicles that can move out of the link during Δt,

VS_i = maximum number of vehicles that can enter the intersection during Δt,
based on the green time provided.

VS_i is equal to $G_i \cdot S_i$ where G_i is the effective green time during Δt and S_i is the saturation flow rate. Only certain movements receive green time during the given phase and hence the effective green time is calculated accordingly. The effective green time is also affected if the green phase ends during the current Δt.

Link Inflow Constraints. The inflow constraints limit the number of vehicles that can enter a link from all the approaches; they include the maximum number of vehicles from all the upstream links to link j, the available physical space on link j, and the section capacity constraint of link j. The equation used is stated as:

$$VI_j = \min\left\{\sum_{i \in U} VO_{ij}, VE_j, C_j \cdot \Delta_t\right\} \quad (4.7)$$

where, for link j,

VI_j = the maximum number of vehicles that can enter the link,

U = set of inbound links into link j,

VO_{ij} = number of vehicles ready to move from link i to link j,

VE_j = available space in the link, and

C_j = inflow capacity, which is normally 1800 by the number of lanes on link j, in vph.

Unsignalized Intersections. This represents the case of intersections with stop or yield controls. All the movements receive equivalent green times in the ratio of the critical volumes during each time step.

Signalized Intersections. Signal control can be pretimed signal control, pretimed coordinated control, multidial pretimed, and actuated signal control.

Freeway Control. Any link in the network can be specified as a freeway link. When several freeway links are connected in series, they are considered to have continuous green by the node pass module. Since the node saturation rate is set at a very high value, freeway traffic flow is governed by the continuum equation and the speed-density relationship, with no nodal constraints. DYNASMART can also model ramp entrance control and variable message signs.

Left Turn Movement. The left turn capacity is determined as follows:

(1) Calculate the left turn capacity for (a) protected left turn phase, based on the saturation flow rate; and (b) permissive phase from tables. (Lin et al., 1984).

(2) Calculate an average number of left-turn vehicles and reduce the saturation flow rate for straight and right-turn vehicles.

(3) Follow outflow-inflow constraints to move vehicles from link to link.

(4) Calculate the left-turn delay for path-processing.

Modeling of Network Path Dynamics

The path processing component of DYNASMART is essential to translate link-level travel time information (including queuing delays) from the simulation to the path-level attributes needed in the user decisions component.

For networks with ATMS/ATIS two different kinds of routes need to be modeled: (a) the routes provided by the central controller or ATIS, and (b) the routes that the drivers have in mind. As a consequence, the simulation model needs to store (a) the current alternatives from various

nodes to various destinations with their trip times, and (b) the current paths of the individual drivers. DYNASMART stores the routes at the network level using predecessor pointers. The routes in the mind of the individual drivers are stored as separate lists.

DYNASMART can also store and use externally specified paths. This feature is essential for the use of DYNASMART as a simulator for solving time-dependent assignment problems.

DYNASMART can find and store multiple paths rather than a single shortest path for each O-D pair. This helps to model decision mechanisms such as multiple paths of non-prescriptive guidance systems. A multiple user-class K-shortest path algorithm with left turn penalties is interfaced with the simulation model. However, to maintain computational performance, the K shortest paths from all the nodes to all the destinations are not recalculated for every simulation time step, but at pre-specified intervals. In the interim, travel times on the set of K current paths are updated using the prevailing link travel times at each simulation time step.

User Behavior Component

DYNASMART is designed to allow the incorporation of different user behavior rules in relation to different information supply strategies. It includes the behavioral rules governing travelers' route choice decisions. Basic information available to the drivers includes the actual or predicted travel times on alternative routes and in some cases the 'best' routes determined by the system. Since it is not expected that the drivers always follow the 'best' route provided, a boundedly-rational behavior rule, which has been supported by experimental evidence (Mahmassani and Stephan, 1988; Mahmassani and Liu, 1996), is incorporated in DYNASMART. The rule is stated as:

$$\delta_j(k) = 1 \text{ if } TTC_j(k) - TTB_j(k) > \max(TTC_j * \eta_j, \tau_j)$$
$$= 0 \text{ otherwise,} \qquad (4.8)$$

where, for driver j at node k,

$\delta_j(k) = 1$ means a change in route and 0 indicates no change.

$TTC_j(k)$ = trip time from node k to the destination on current path,

$TTB_j(k)$ = trip time on the best alternative path,

η_j = relative indifference threshold fraction, and

τ_j = minimum trip time reduction in order to switch routes.

The boundedly-rational rule implies that drivers will be looking for alternative routes only if the gains exceed a certain threshold. The driver's indifference threshold is a fraction (η_j) of the remaining trip time on the current path. The model also assumes that a minimum improvement (τ_j) will be needed in order to switch routes. The threshold value reflects particular

characteristics of the drivers such as perceptual factors, preferential indifference, or persistence and aversion to switching. The threshold value is treated as a random variable; at generation every user is assigned an independent random value for η_j. In the version of DYNASMART used, η_j is assumed to have a triangular distribution with mean η_j and range $\eta_j/2$. The value for the minimum improvement to switch τ_j is assumed to be the same for all the users and defined by the user. Ideally, both values should be estimated from field experiments.

If the values of the threshold level η_j and the minimum improvement τ_j are set to zero, the program replicates a myopic switching rule. In this situation, any driver will switch routes if alternative routes offer reduced travel times regardless of the magnitude of the improvement.

DYNASMART can also model route choice at a node according to a probabilistic discrete choice function such as one of the logit form.

CHANGES IN DYNASMART TO INCORPORATE PRICING

The version of DYNASMART available at the beginning of the present study did not consider pricing as an element of the operation of a network. However, only small modifications were needed to incorporate pricing. For this research, two kinds of user charges are considered: constant and variable tolls. Both kinds of charges can be used to simulate toll cordons or facility specific pricing. Constant prices will replicate the currently common application of constant tolls for entering a certain zone of a city or using a facility such as a bridge or a freeway. Constant prices are not related to the level of congestion. Sometimes they are applied only during the peak hours but are a flat rate regardless of traffic conditions. On the other hand, variable tolls can reflect prevailing congestion levels. They can be related to the number of vehicles in a certain link or zone, distance traveled in excess of minimum distances from one origin to a destination or time spent in the restricted zone or facility.

Constant pricing is the simplest form of road user charges. A fixed amount is charged every time that the vehicle passes through specific points of the road network. Charges depend only on the number of toll points crossed. It is the system most commonly used since it is the easiest to administer and drivers can get used to it without much difficulty.

Link characteristics in DYNASMART now include upstream node identification number, downstream node identification number, link length, number of lanes, maximum velocity, saturation flow rate and the link type (street, freeway, entrance or exit ramp). To incorporate constant pricing, the price is introduced as an additional characteristic for each of the links in the network. A monetary value is specified, which is then converted into its time equivalent as:

$$TToll_j = \frac{Toll_j}{VOT} \quad (4.9)$$

where

$TToll_j$ = time equivalent toll for link j,

$Toll_j$ = monetary value of the toll for link j and,

VOT = average value of travel time expressed in monetary units per unit time. This value is assumed constant in this research, but different values can be considered depending on the characteristics of the drivers.

Recently, there have been works that incorporate VOT as a continuously distributed characteristic of the drivers instead of the average value commonly used (Leurent, 1994; Dial, 1996). This could be considered in subsequent extensions of the present work

A generalized travel time function for each link was introduced in DYNASMART instead of the previously used travel time, and the program modified accordingly. This function is expressed as:

$$G_j = T_j + TToll_j = T_j + \frac{Toll_j}{VOT} \quad (4.10)$$

The time equivalent of the toll is therefore added to the travel time for each of the priced links. This will affect the determination of shortest travel routes at the beginning of the simulation and update times and the selection of the links to follow at the intersections since travel time along alternate routes may be supplied to the drivers.

Variable tolls are related to the prevailing level of congestion on the road network. These can be considered to be fairer than using constant tolls since charges are levied only when congestion occurs.

The incorporation of time dependent variable pricing into DYNASMART required additional work. The ratio of the number of vehicles to the maximum number of vehicles that can be present along the link defines the level of congestion. A threshold is set to reflect the level at which a toll will be charged. The same level of congestion will define the toll level. The system should be capable of identifying the prevailing congestion level, and charge for using the network at the time accordingly; the system should also be able to stop charging when congestion drops below a certain level. With this scheme, different tolls would be charged for different links at different times. The following expression is used to calculate the time equivalent of the toll at every time interval t that the toll is updated:

$$TToll_{tj} = \frac{\alpha X_{tj}(v_{fj} - v_0)(1 - \frac{k_{tj}}{k_0})^{\alpha - 1}}{k_0 v_{tj}^2} \quad (4.11)$$

where

$Ttoll_{tj}$ = time value of the toll for link j at time interval t,

α = user specified parameter to capture the sensitivity of speed to concentration,

X_{tj} = number of vehicles on link j at the beginning of time interval t,

v_{fj} = maximum value of the speed for link j,

v_0 = minimum value for the speed,

k_{tj} = concentration on link j at the beginning of time interval t,

k_0 = jam concentration, and

v_{tj} = average speed for link j at the beginning of time interval t.

The ratio k_{tj} to k_0 represents the prevailing congestion level.

Equation (4.11), found as the difference between the first derivative of the total travel time for the vehicles using link j at the beginning of time interval t with respect to the number of vehicles, and the average travel time for the same vehicles, is a local approximation to the global externalities. Its determination considers only the effect that an additional vehicle entering a link j at the beginning of time interval t has on the rest of the vehicles already using the link at the same time. It does not consider the effects on any other link at anytime or at any other time for the link under study. As such, Equation (4.11) considers only the second term of the right hand side of the global marginals expression (Equation (3.7)).

Variable tolls calculated with Equation (4.11) have the advantage of being calculated within the DYNASMART program. The modifications to DYNASMART to incorporate variable pricing are implemented in the PARTCO subroutine (Fig. 4.2) taking advantage of the fact that the variables needed for its calculations are already estimated in that subroutine. No expressions such as (3.12), (3.13), (3.60) or (3.61) (time-dependent externalities) were used since they calculate tolls outside DYNASMART, which complicates their determination. Future extensions of this work can consider those expressions.

Once the time equivalent of the toll is calculated, its monetary value can be found with equation (4.9) as:

$$Toll_{tj} = TToll_{tj} \cdot VOT$$

EFFECTS OF CONSTANT AND VARIABLE PRICING SCHEMES

The user's reaction to a pricing scheme will depend on the characteristics of the scheme used, such as whether the charges are constant or variable, if they are applied only at specific hours or all day long, if alternative routes to avoid tolls exist, and on the strategy followed to supply information about the charges to the drivers.

Of the multiple cases that could be considered for determining the price to charge in a road pricing scheme, three were examined in this research: (1) a fixed charge for using a link; (2) a fixed charge for crossing a boundary in a restricted zone of the network; and (3) a variable price based on the link's congestion level. The first two cases are considered in this section. The third case is considered in the upcoming section on variable pricing schemes.

Description of Experiments

The Test Network. This section describes the characteristics of the network used to simulate pricing. It also describes the demand pattern, demand levels, market penetration and the trip-maker switching behavior parameter values used in the simulation.

General Characteristics. The hypothetical test network (shown in Figure 4.3) used to simulate the application of pricing has the following characteristics: (1) it is formed by a freeway that runs in the middle of a street network; (2) it includes 168 links and 50 nodes; (3) all the nodes in the street network are origins. Only the extreme nodes in the freeway are origins and destinations. Nodes 2, 5, 13, 18, 25, 30, 35, and 36 in the street network are the centroids of their corresponding zones and the destination within that zone. As a consequence, the network has 38 origin nodes and 10 destination nodes; (4) all arcs in the network, but the entrance and exit ramps, are one-directional with two lanes in each direction; (5) entrance and exit ramps have only one lane; (6) the length for all the links is 0.5 miles; (7) free speeds are 55 mph for the freeway links and 30 mph for street links and entrance and exit ramps; (8) jam densities and maximum bumper to bumper densities are 160 vehicles per mile and 260 vehicles/mile respectively; (9) with respect to signal controls, 26 intersections are pre-timed, 8 have actuated controls, and the rest have no signal control. The pretimed signal controls, except for node 34, operate with a 60 second cycle length with only two phases, each phase has 25 seconds of green time and 5 seconds of yellow time. The pretimed signal control at node 34 operates with a 120 second cycle length with two phases. Each phase has 55 seconds of green time and 5 seconds of yellow time. Minimum green time for the actuated signals is 10 seconds. Maximum green time is 25 seconds with 5 seconds yellow time. Maximum cycle length is 60 seconds; and (10) except for nodes at the end of the freeway (nodes 37 and 44), origin-destination demand is evenly distributed both in terms of generation and attraction. Nodes 37 and 44 generate only about 25 percent of the vehicles generated by the rest of the origin-destination nodes.

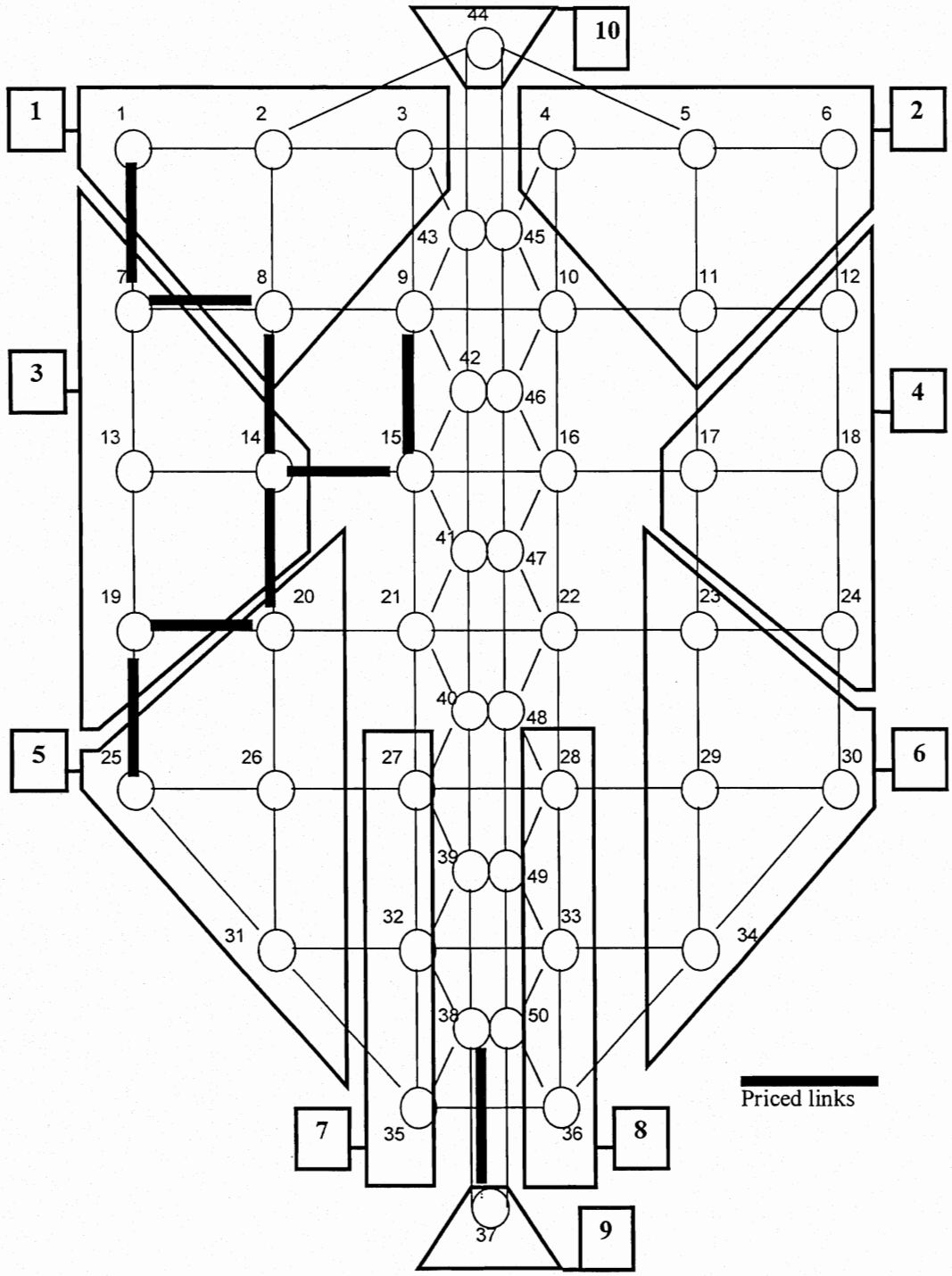


Figure 4.3 Test Network.

The value of travel time, according to Small (1982) that it should be approximately one half of the average salary, is assumed to be six dollars per hour.

Demand Pattern. Figure 4.4. Shows the loading profile used for the simulations. As in previous studies (Peeta, 1994), vehicles are generated over a 35-minute period. The first five minutes are considered to be a start-up time to have a loaded network. Statistics are collected for the vehicles generated after the five-minute warm-up period.

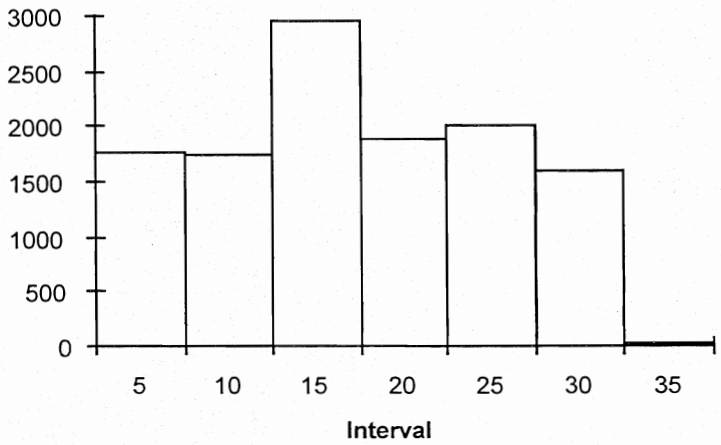


Figure 4.4 Loading Profile for Simulation Experiments; Base Demand Level.

For the base demand level, the network is loaded with 1760 vehicles during the first five-minute interval. Followed by 1740, 2960, 1880, 2020, 1600 and 20 vehicles in the second through seventh loading intervals respectively. This profile is intended to represent typical peak-period loading patterns.

Demand Levels. The loading factor (LF) is defined as the ratio of the total number of vehicles generated to the base value of 11980 vehicles; the values used in the simulations for this research were 1.00, 1.25, 1.50, 1.75, and 2.00. These different loading factors represent different levels of congestion for the network. The corresponding number of generated vehicles is given in Table 4.2

TABLE 4.2 NUMBER OF VEHICLES GENERATED OR THE DIFFERENT LOAD FACTORS.

Load Factor (LF)	Number of Vehicles
1.00	11980
1.25	14975
1.50	17970
1.75	20965
2.00	23960

Market Penetration. Vehicles can be equipped or non-equipped to receive information. Before departure, all drivers are provided with information about toll levels. Equipped vehicles receive real time descriptive information about travel times on alternative routes. With that information, drivers can make switching decisions according to the boundedly-rational behavioral rule presented in equation 4.8. Non-equipped vehicles follow the routes prescribed at the beginning of their journey without the possibility of switching routes.

The percentage of equipped drivers defines the market penetration. Two different levels of market penetration were considered in the simulations, 0 and 100 percent. No other information levels were considered since the effect of information has been studied elsewhere (Mahmassani and Jayakrishnan, 1991) and was not the purpose of this research.

Trip-Maker Switching Behavior. Informed drivers make switching decisions according to the boundedly-rational rule described in previous section on user behavior compenent. Two cases were considered for this research. The first case considers that drivers follow a myopic rule in their switching decisions, for which the values for the mean relative indifference band (η) and of the minimum threshold bound (τ) are set to zero, meaning that all the informed drivers will switch routes at decision points if following the new route represents a reduction in travel time.

In the second case, the value of the mean relative indifference band (η) was set to 0.2 and the minimum threshold bound (τ) was set to 1 minute for all users.

Initial Path Assignments. In this chapter, vehicles in the simulations are assigned to the current best path at the beginning of their journey. Other assignment rules are presented in chapters five and six of this report.

Constant Pricing Schemes Considered

Three constant pricing schemes were considered. The first consisted of pricing a pair of links (corresponding to the two opposite directions of the same highway section) for which alternative routes exist to any of the destinations. Both directions were priced at the same level. Drivers that are not willing to pay for the use of the tolled links can follow alternative paths with no extra out of pocket cost. The links considered are links 9-15 and 15-9 in the network (shown on Figure 4.3). The second scheme consisted in applying prices to a link that must be used by at least some of the vehicles to reach their respective destinations. The link considered was freeway link 38-37. Since node 37 is a destination (zone 9, in Figure 4.3), vehicles with that node as destination have no option but to pay for the use of the link. Other drivers desiring to enter the freeway at that link, but that do not have node 38 as destination, could pay the toll or follow alternative routes. The third consisted of a fixed charge for entering a restricted zone of thenetwork, defined here as zone 3 in Figure 4.3. Nodes 7, 13, 14, and 19 are within the restricted zone. Therefore, vehicles using arcs 1-7, 8-7, 8-14, 15-14, 20-14, 20-19, and 25-19 will be forced to pay a toll. Vehicles for which their destination is within zone 3 will have no alternative but to pay the toll. Vehicles with destinations other than zone 3 but desiring to use some of the above listed links could either pay the toll or use alternative routes.

Price Levels

Simulations were performed changing prices in twenty cents increments, from zero up to the level at which the number of vehicles using the priced arcs was constant. In the case of arcs 9-15 and 15-9, this number was zero. For the case of arc 38-37, the number was a function of the loading factor used, which gives the number of vehicles that have node 37 as destination. In the case of zone 3, as in the case of link 38-37, the number depended on the loading factor used.

Experiment Results

Experiment results are discussed for the constant pricing schemes described in previous section on constant pricing schemes considered. Network operation characteristics were extracted for each price level and presented in the following pages. These characteristics include: total overall trip time, which includes total travel time plus the total stopped time; total traveled distance; total number of vehicles using the priced links and; total revenue for the network.

Paired Links with Alternative Routes. The effects of constant prices on network operation when links 9-15 and 15-9 are tolled are presented in tables 4.3 to 4.17 and figures 4.5 to 4.16. Tables 4.3, 4.8 and 4.13 and the corresponding figures 4.5, 4.9 and 4.13 present the values of the overall travel times for the different loading factors and toll levels under the three market penetration scenarios considered (no information case, 100% percent informed vehicles under a myopic switching rule and 100% informed vehicles under a non-myopic switching rule).

In a similar way, tables 4.4, 4.9 and 4.14 present the percentage increase in overall travel time for the different loading factors and market penetration levels and switching rules; tables 4.5, 4.10 and 4.15 and the corresponding figures 4.6, 4.10 and 4.14 present the total trip distance summaries; tables 4.6, 4.11, and 4.16 along with figures 4.7, 4.11 and 4.15 present the information on number of vehicles using links 9-15 and 15-9 and; tables 4.7, 4.12 and 4.17 with figures 4.8, 4.12, and 4.16 present the information on network revenue.

In the base case, with no tolls, total overall times increase nearly exponentially with increased number of vehicles using the network. The provision of information appears to reduce significantly the total overall time especially under the higher loading factor, as described in Hu (1995). The incorporation of pricing does affect the overall travel times. Under no information, increases in travel times are significant for higher levels of demand and higher tolls. For a demand of 23,960 vehicles and a toll of 2 dollars, they are about 7 percent higher than in the base case of no toll for the same demand. At lower demand and lower tolling levels, total overall times do not appear to follow a systematic pattern, as they are sometimes smaller and at other times higher, but only in a few cases are they significantly different from the base case. For the cases of 100 percent informed vehicles, overall travel times increase for all the pricing levels, increases are smaller for lower levels of demand and prices and higher for high levels of demand and prices. For the myopic case, percentage increases in travel time are similar to the no information case for the highest levels of demand and prices considered. In the non-myopic case, although travel times increase with demand and prices, the increments are in general smaller than the myopic case.

Total trip distances increase for all but one of the loading factors and toll levels in the no information case. Highest values for trip distance occur at higher levels of demand and tolls.

However, in no case are the increments over 2 percent of the base case. For the 100 percent information cases, trip distances are higher for most of the toll levels and loading factors. In a few cases, trip distances are slightly smaller.

The number of vehicles entering the priced links exhibits a very similar behavior regardless of the information level. The number of vehicles using the priced links goes to zero as the toll increases. Steep reductions occur for low toll levels and this reduction is attenuated at higher levels of demand and tolls. The main difference among the no information, the 100 percent information case with myopic switching rule and the 100 percent information case with non-myopic switching rule is in the number of vehicles that use the links when no tolls are set. This number is much higher for the no information case than for the 100 percent information cases.

With respect to the revenues, they peak at low toll levels for low level of demand falling gradually to zero when toll levels are increased. For the highest level of demand, revenues peak at higher toll levels for the no information case and with the myopic switching rule, keeping the same behavior for the non-myopic switching rule case at the lower demand levels. With respect to the magnitude of the revenues, they are significantly small since the number of vehicles using link 15-9 and 9-15 is also small. Revenues are higher for the no information case than for the 100 percent information cases.

Results are as expected. When confronted with prices, drivers try to avoid payment when alternative routes exist. They do consider the benefits of lower out of pocket costs against increased travel times. Increased travel times occur when optimal routes under no price are replaced by routes with increased travel times but no tolls. Travel distances increase as a result of the same rerouting decisions. The number of vehicles using the priced links falls sharply since even small tolls outweigh travel times on alternate routes. Maximum revenues for this kind of pricing scheme are achieved with low toll levels. For congested networks under this pricing scheme, increased toll levels affect negatively the operation of the network and lead revenues to be minimal or zero. The use of information presents some advantages and disadvantages depending on the point of view. For the users, it improves the operation of the network by reducing overall travel times and trip distances but, from the toll operator standpoint, it reduces the revenue by reducing the number of vehicles using the priced links.

As stated in the previous section with the heading of general characteristics the value of time used in the experiments was constant and equal to six dollars per hour. Different values of travel time could have been used, but these values would only modify the scales of the figures presenting the different simulation results. The shape of the lines joining the results would remain equal. However, the relative desirability of a particular scheme in a given network would be affected by the value of time.

TABLE 4.3 OVERALL TRAVEL TIMES (HOURS) FOR DIFFERENT LOADING FACTOR AND TOLL LEVELS WHEN LINKS 9-15 AND 15-9 ARE TOLLED. NO EN-ROUTE INFORMATION CASE

	Toll Level										
Loading factor	0.00	0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.60	1.80	2.00
1.00	1540	1555									
1.25	2291	2367	2319	2369							
1.50	3827	3914	4006	3823	3798						
1.75	5978	5953	5927	5787	5865	5973	5771				
2.00	10333	10187	10328	9685	10034	10560	10981	11036	11050	11063	11064

TABLE 4.4 INCREASE IN OVERALL TRAVEL TIMES (%) FOR DIFFERENT LOADING FACTORS AND TOLL LEVELS WHEN LINKS 9-15 AND 15-9 ARE TOLLED. NO EN-ROUTE INFORMATION CASE

	Toll Level										
Loading factor	0.00	0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.60	1.80	2.00
1.00	0.00	0.99									
1.25	0.00	3.33	1.24	3.41							
1.50	0.00	2.27	4.68	-0.10	-0.76						
1.75	0.00	-0.43	-0.86	-3.19	-1.89	-0.09	-3.47				
2.00	0.00	-1.42	-0.05	-6.28	-2.90	2.20	6.27	6.80	6.94	7.07	7.07

TABLE 4.5 TOTAL TRIP DISTANCE (MILES) FOR DIFFERENT LOADING FACTORS AND TOLL LEVELS WHEN LINKS 9-15 AND 15-9 ARE TOLLED. NO EN-ROUTE INFORMATION CASE

	Toll Level										
Loading factor	0.00	0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.60	1.80	2.00
1.00	29901	2998									
1.25	38020	38249	38237	38151							
1.50	46942	47038	47498	47368	47351						
1.75	57116	57130	57512	57370	57523	57846	57405				
2.00	67458	67417	68682	68158	68078	68125	68733	68771	68792	68806	68804

TABLE 4.6 NUMBER OF VEHICLES USING THE LINK FOR DIFFERENT LOADING FACTORS AND TOLL LEVELS WHEN LINKS 9-15 AND 15-9 ARE TOLLED. NO EN-ROUTE INFORMATION CASE

	Toll Level										
Loading factor	0.00	0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.60	1.80	2.00
1.00	337	21									
1.25	577	109	20	0							
1.50	736	363	51	2	0						
1.75	1058	534	245	79	20	3	0				
2.00	1680	947	477	339	275	144	52	29	16	7	9

TABLE 4.7 NETWORK REVENUE (DOLLARS) FOR DIFFERENT LOADING FACTORS AND TOLL LEVELS WHEN LINKS 9-15 AND 15-9 ARE TOLLED. NO EN-ROUTE INFORMATION CASE

	Toll Level										
Loading factor	0.00	0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.60	1.80	2.00
1.00	0.00	4									
1.25	0.00	22	8	0							
1.50	0.00	73	20	1	0						
1.75	0.00	107	98	47	16	3	0				
2.00	0.00	189	191	203	220	144	62	41	26	13	18

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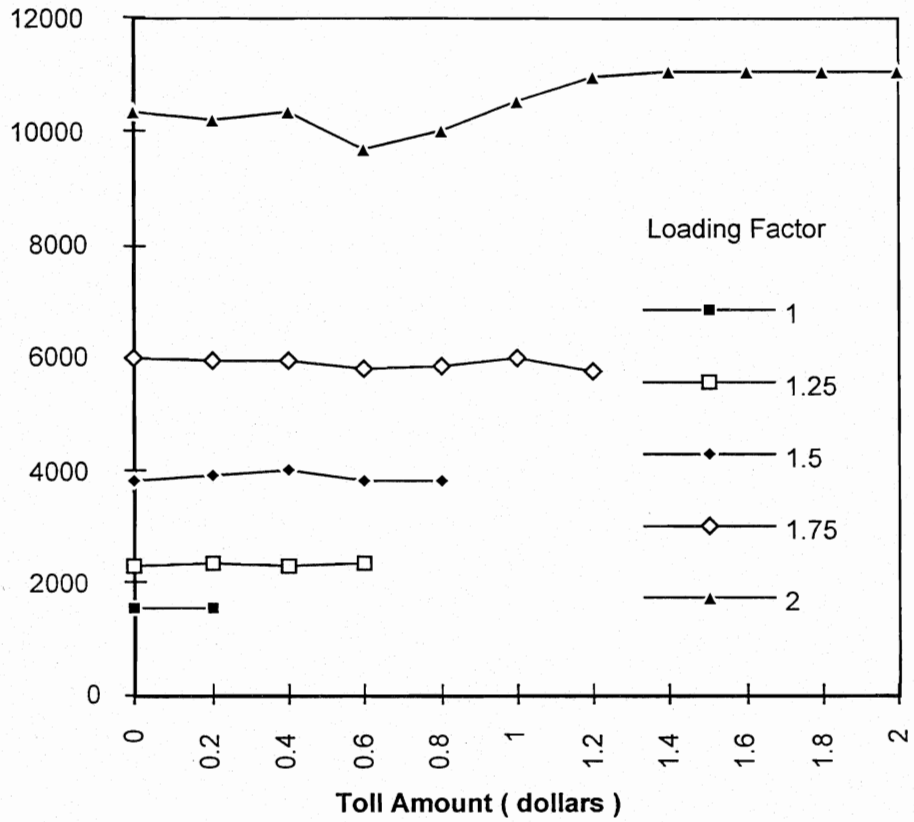

Figure 4.5 Variation of Network Total Overall Time for Different Loading Factors and Price Levels when Links 9-15 and 15-9 are Tolled. No en-route Information Case.

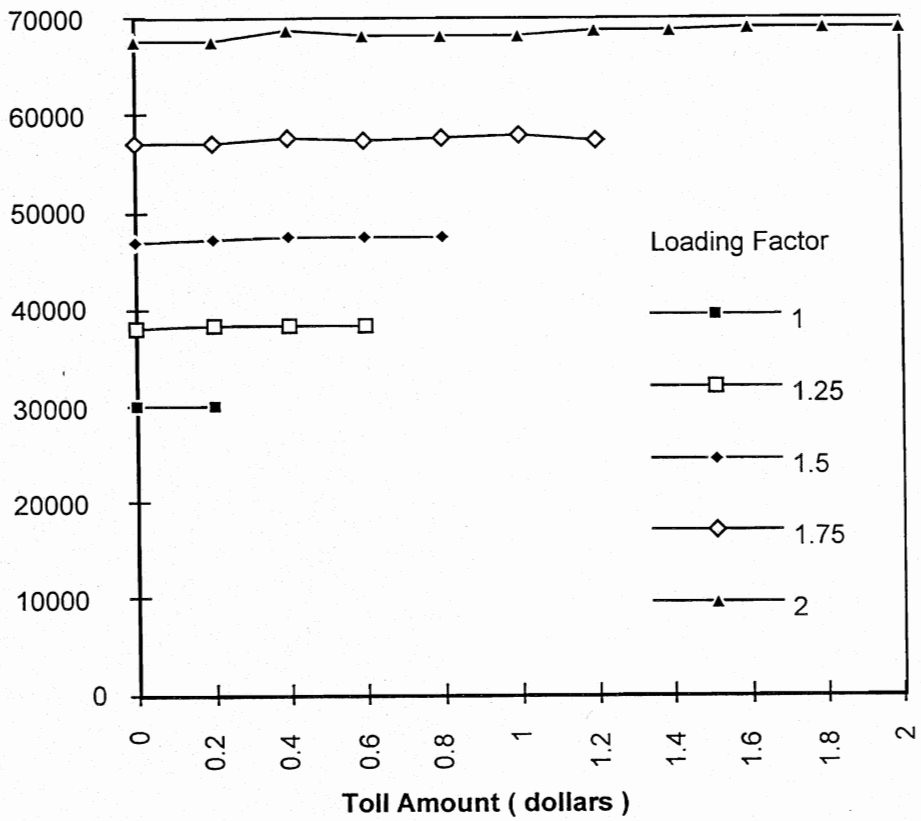

Figure 4.6 Variation of Total Trip Distance for Different Loading Factors and Price Levels when Links 9-15 and 15-9 are Tolled. No en-route Information Case.

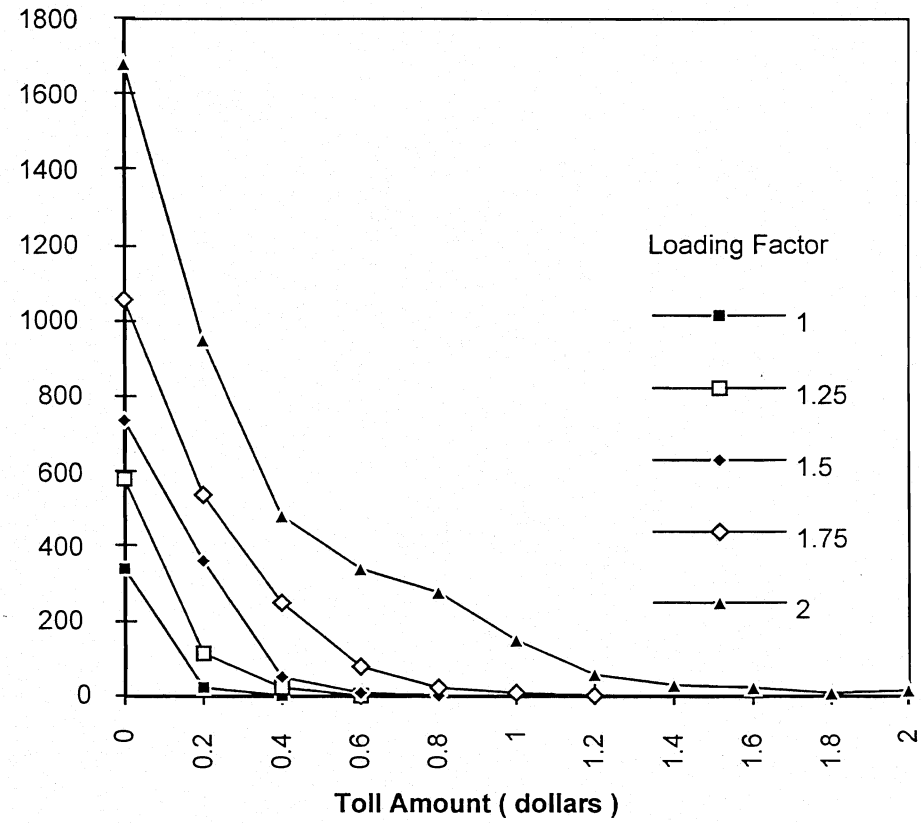

Figure 4.7 Variation in Number of Vehicles Using the Link for Different Loading Factors and Price Levels when Links 9-15 and 15-9 are Tolled. No en-route Information Case.

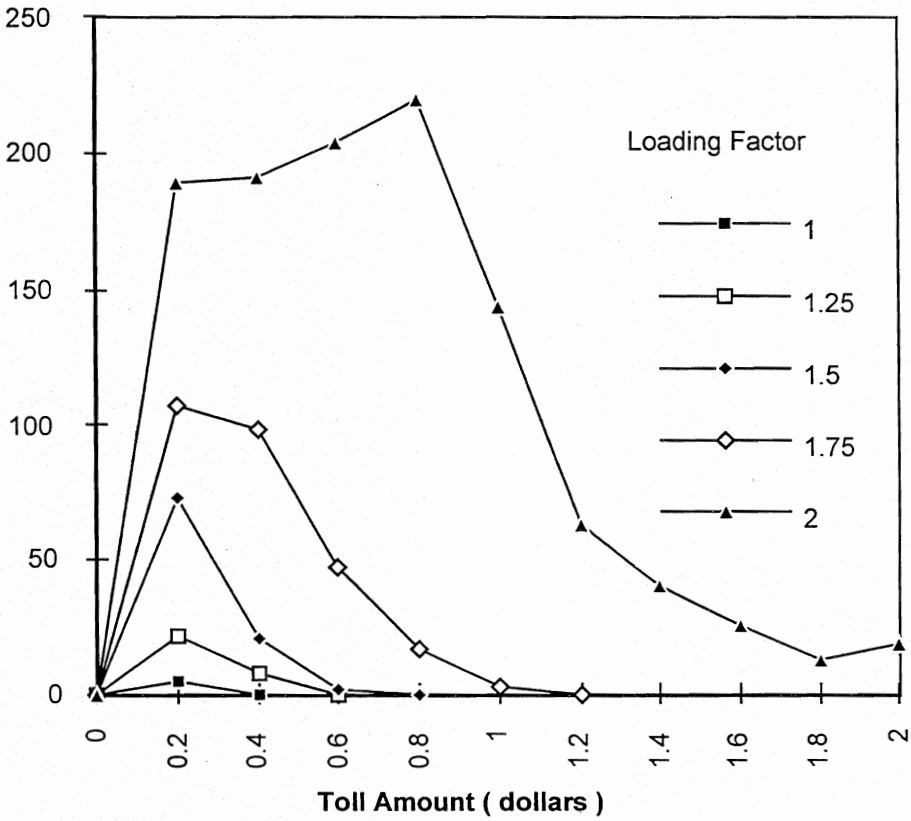

Figure 4.8 Variation in Network Revenue for Different Loading Factors and Price Levels when Links 9-15 and 15-9 are Tolled. No en-route Information Case.

TABLE 4.8 OVERALL TRAVEL TIMES (HOURS) FOR DIFFERENT LOADING FACTORS AND TOLL LEVELS WHEN LINKS 9-15 AND 15-9 ARE TOLLED. 100% INFORMED VEHICLES. MYOPIC CASE

	Toll Level									
Loading factor	0.00	0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.60	1.70
1.00	1354	1372								
1.25	1979	2003	1999							
1.50	2884	2898	2932	2910						
1.75	4147	4308	4243	4250	4227	3	0			
2.00	5871	5887	5922	5897	6018	5974	6022	5971	5979	5971

TABLE 4.9 INCREASE IN OVERALL TRAVEL TIMES (%) FOR DIFFERENT LOADING FACTORS AND TOLL LEVELS WHEN LINKS 9-15 AND 15-9 ARE TOLLED. 100% INFORMED VEHICLES. MYOPIC CASE

	Toll Level									
Loading factor	0.00	0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.60	1.70
1.00	0.00	1.38								
1.25	0.00	1.19	0.98							
1.50	0.00	0.49	1.67	0.93						
1.75	0.00	3.87	2.31	2.47	1.93					
2.00	0.00	0.27	0.86	0.43	6.94	7.07	7.07	7.07	7.07	7.07

TABLE 4.10 TOTAL TRIP DISTANCE (MILES) FOR DIFFERENT LOADING FACTORS AND TOLL LEVELS WHEN LINKS 9-15 AND 15-9 ARE TOLLED. 100% INFORMED VEHICLES. MYOPIC CASE

	Toll Level									
Loading factor	0.00	0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.60	1.70
1.00	26945	27082								
1.25	34294	34379	34427							
1.50	42250	42083	42462	42168						
1.75	50277	51009	50902	50438	50431	3	0			
2.00	59124	58713	58875	58497	58980	29400	59308	58908	59158	58887

TABLE 4.11 NUMBER OF VEHICLES USING THE LINK FOR DIFFERENT LOADING FACTORS AND TOLL LEVELS WHEN LINKS 9-15 AND 15-9 ARE TOLLED. 100% INFORMED VEHICLES. MYOPIC CASE

	Toll Level									
Loading factor	0.00	0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.60	1.70
1.00	245	2	0							
1.25	425	53	15	0						
1.50	649	196	18	3	0					
1.75	863	440	206	53	0					
2.00	1318	794	393	318	151	78	21	7	1	0

TABLE 4.12 NETWORK REVENUE (DOLLARS) FOR DIFFERENT LOADING FACTORS AND TOLL LEVELS WHEN LINKS 9-15 AND 15-9 ARE TOLLED. 100% INFORMED VEHICLES. MYOPIC CASE

	Toll Level									
Loading factor	0.00	0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.60	1.70
1.00	0	0.4	0							
1.25	0	10.6	6	0						
1.50	0	39.2	7.2	1.8	0					
1.75	0	88	82.4	31.8	0					
2.00	0	158.8	157.2	190.8	12	78	25.2	9.8	1.6	0

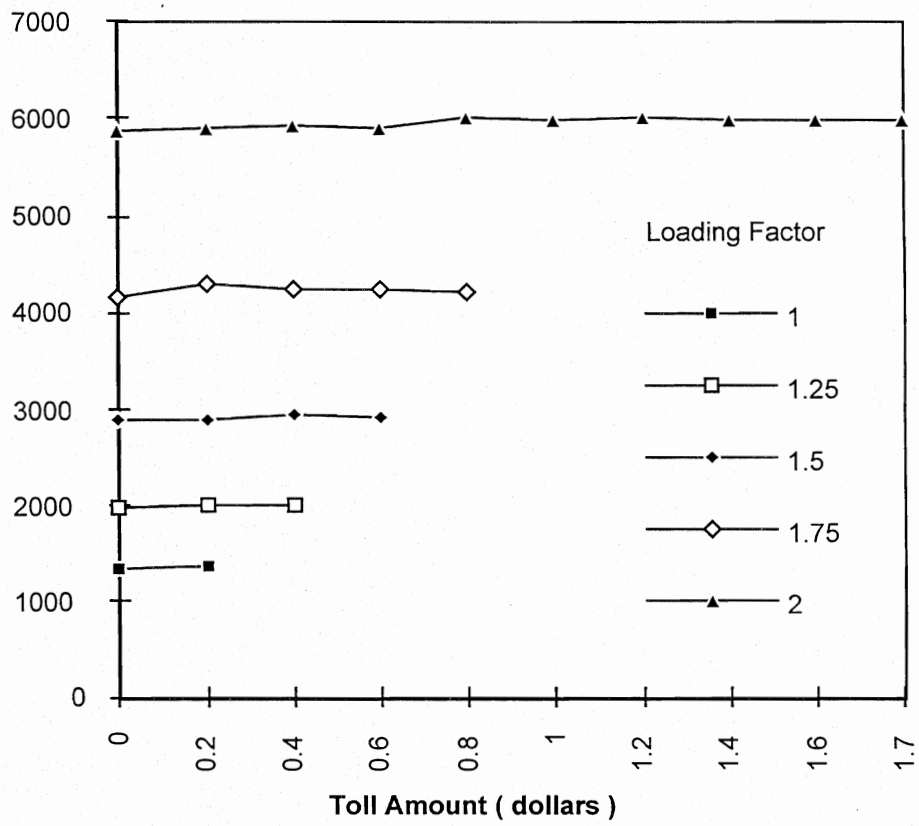

Figure 4.9 Variation of Network Total Overall Time for Different Loading Factors and Price Levels when Links 9-15 and 15-9 are Tolled. 100 % Informed Vehicles. Myopic Case.

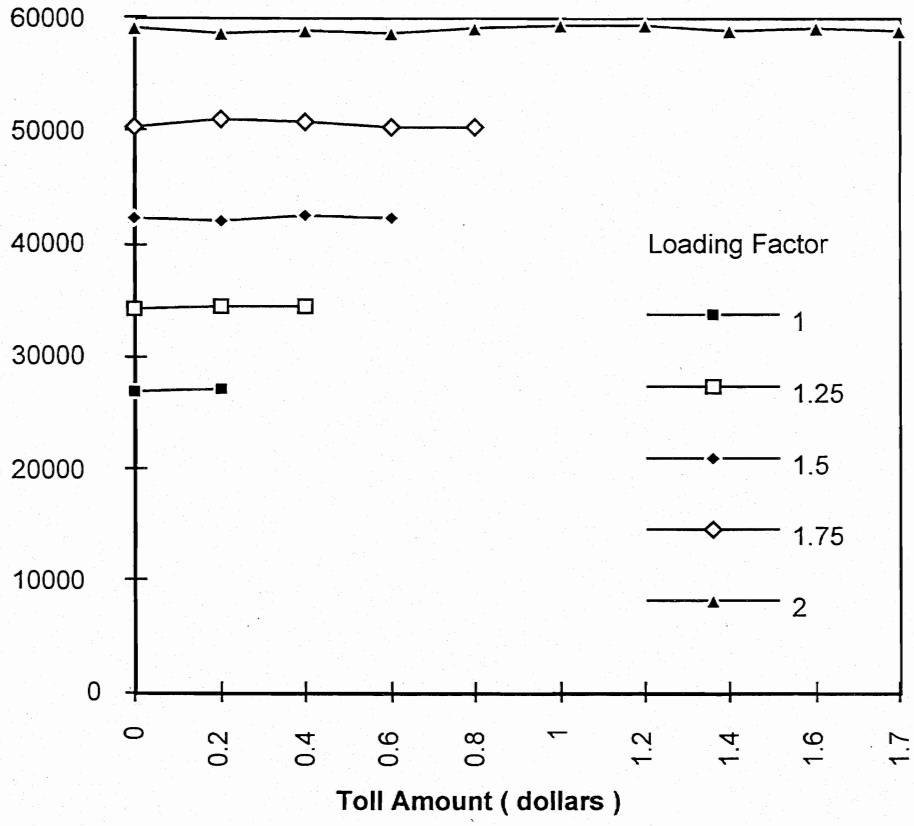

Figure 4.10 Variation of Total Trip Distance for Different Loading Factors and Price Levels when Links 9-15 and 15-9 are Tolled. 100% Informed Vehicles. Myopic Case.

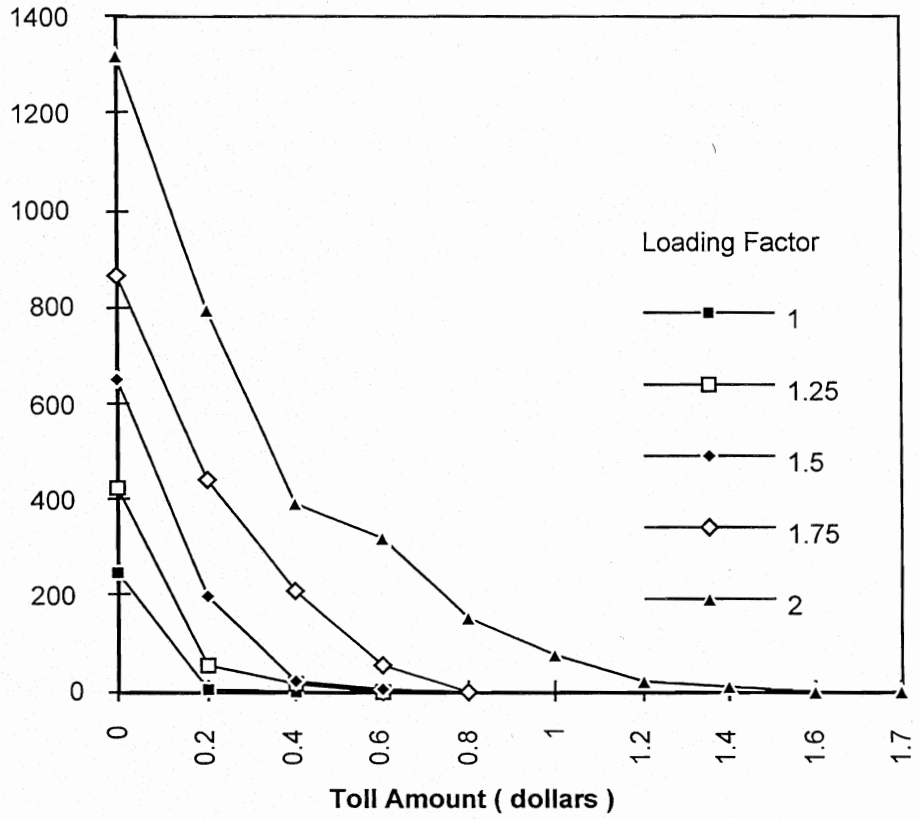

Figure 4.11 Variation in Number of Vehicles Using the Link for Different Loading Factors and Price Levels when Links 9-15 and 15-9 are Tolled. 100% Informed Vehicles. Myopic Case.

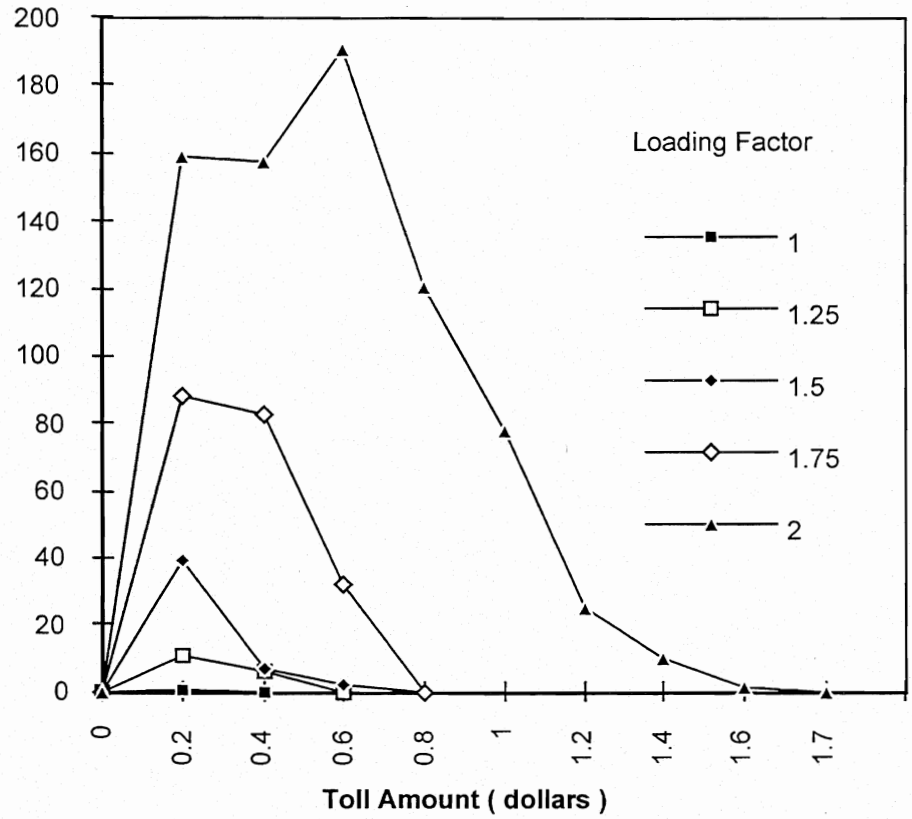

Figure 4.12 Variation in Network Revenue for Different Loading Factors and Price Levels when Links 9-15 and 15-9 are Tolled. 100% Informed Vehicles. Myopic Case.

TABLE 4.13 OVERALL TRAVEL TIMES (HOURS) FOR DIFFERENT LOADING FACTORS AND TOLL LEVELS WHEN LINKS 9-15 AND 15-9 ARE TOLLED. 100% INFORMED VEHICLES. MYOPIC CASE

	Toll Level								
Loading factor	0.00	0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.50
1.00	1351	1369							
1.25	1954	1982	1957						
1.50	2828	2864	2907						
1.75	4100	4128	4198	4198	4154	4154			
2.00	5753	5841	5866	5847	5948	6006	6026	6019	6036

TABLE 4.14 INCREASE INOVERALL TRAVEL TIMES (%) FOR DIFFERENT LOADING FACTORS AND TOLL LEVELS WHEN LINKS 9-15 AND 15-9 ARE TOLLED. 100% INFORMED VEHICLES. NON-MYOPIC CASE

	Toll Level								
Loading factor	0.00	0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.50
1.00	0.00	1.29							
1.25	0.00	1.43	0.17						
1.50	0.00	1.29	2.80						
1.75	0.00	0.68	2.39	2.35	2.14	1.32			
2.00	0.00	1.53	1.96	1.63	3.40	4.40	4.74	4.63	4.91

TABLE 4.15 TOTAL TRIP DISTANCE (MILES) FOR DIFFERENT LOADING FACTORS AND TOLL LEVELS WHEN LINKS 9-15 AND 15-9 ARE TOLLED. 100% INFORMED VEHICLES. NON-MYOPIC CASE

	Toll Level								
Loading factor	0.00	0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.50
1.00	27101	27246							
1.25	34369	34425	34401						
1.50	42132	42158	42339						
1.75	50083	50072	50355	50409	50172	50115			
2.00	58769	59156	58939	58862	59187	59345	59612	59638	59734

TABLE 4.16 NUMBER OF VEHICLES USING THE LINK FOR DIFFERENT LOADING FACTORS AND TOLL LEVELS WHEN LINKS 9-15 AND 15-9 ARE TOLLED. 100% INFORMED VEHICLES. NON-MYOPIC

	Toll Level								
Loading factor	0.00	0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.50
1.00	278	2							
1.25	459	42	0						
1.50	683	131	22						
1.75	860	356	83	21	3	0			
2.00	1258	849	371	245	76	21	8	4	0

TABLE 4.17 NETWORK REVENUE (DOLLARS) FOR DIFFERENT LOADING FACTORS AND TOLL LEVELS WHEN LINKS 9-15 AND 15-9 ARE TOLLED. 100% INFORMED VEHICLES. NON-MYOPIC CASE

	Toll Level								
Loading factor	0.00	0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.50
1.00	0	0							
1.25	0	8	0	0					
1.50	0	26	9	0	0				
1.75	0	71	33	13	2	0	0		
2.00	0	170	148	147	61	21	10	6	0

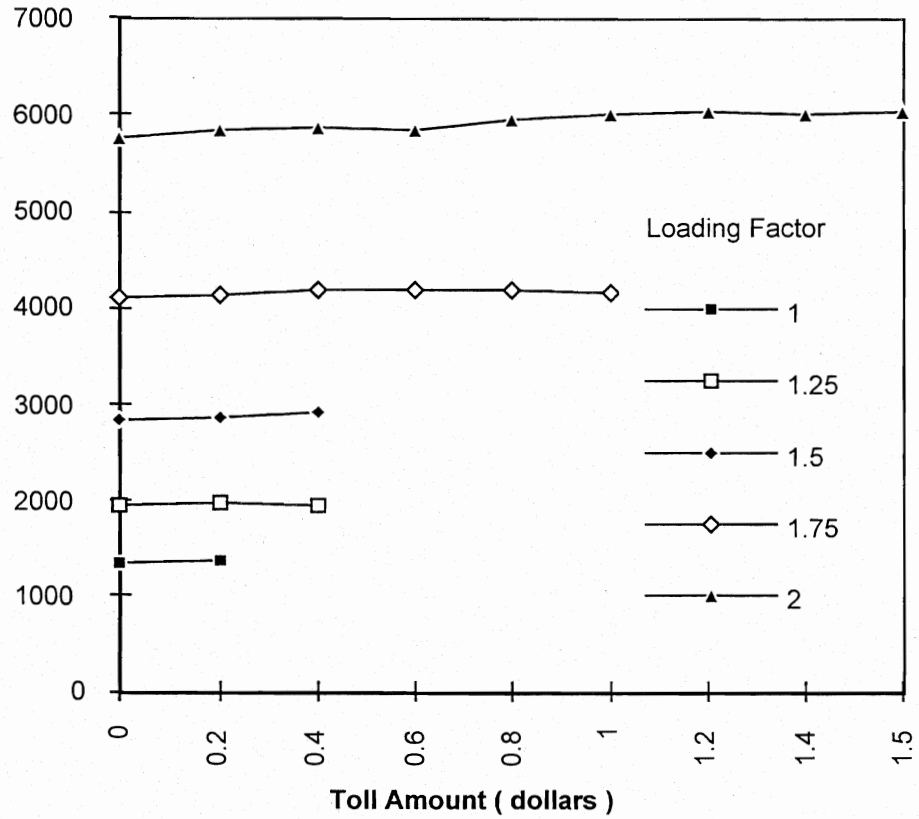

Figure 4.13 Variation of Network Total Overall Time for Different Loading Factors and Price Levels when Links 9-15 and 15-9 are Tolled. 100% Informed Vehicles. Non-Myopic Case.

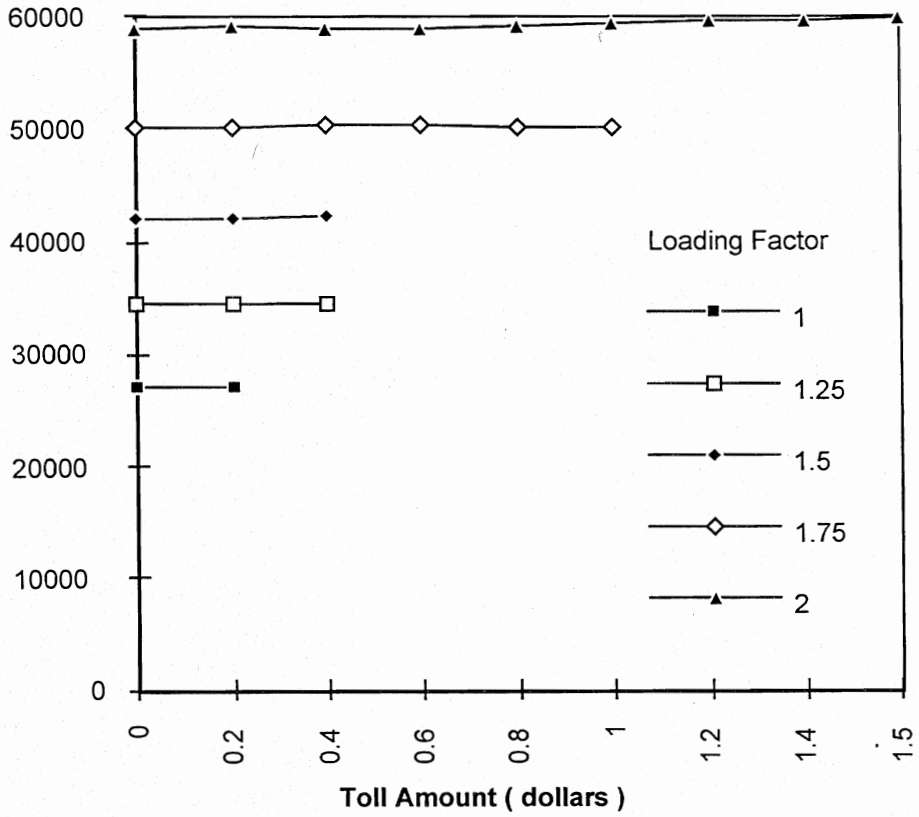

Figure 4.14 Variation of Total Trip Distance for Different Loading Factors and Price Levels when Links 9-15 and 15-9 are Tolled. 100% Informed Vehicles. Non-Myopic Case.

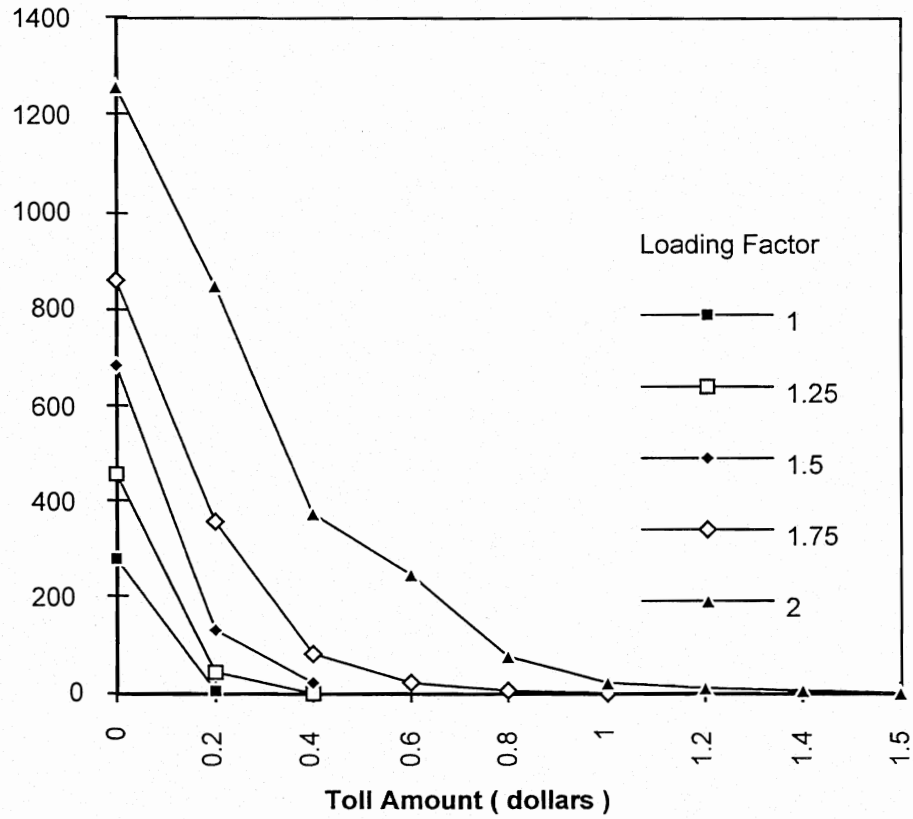

Figure 4.15 Variation in Number of Vehicles Using the Link for Different Loading Factors and Price Levels when Links 9-15 and 15-9 are Tolled. 100% Informed Vehicles. Non-Myopic Case.

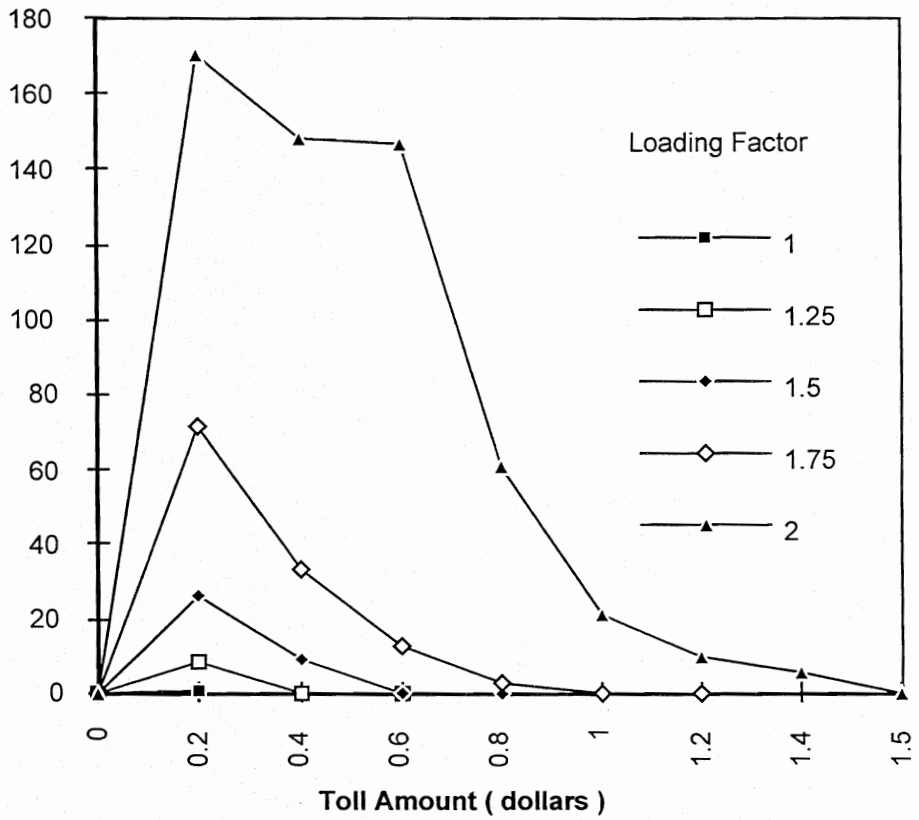

Figure 4.16 Variation in Network Revenue for Different Loading Factors and Price Levels when Links 9-15 and 15-9 are Tolled. 100% Informed Vehicles. Non-Myopic Case

A Single Link with no Alternative Routes for Some of the Vehicles. The effects of constant pricing for the use of link 38-37, which has no alternative for vehicles destined to node 37, on the operation of the network are presented in tables 4.18 to 4.32 and figures 4.17 to 4.28. Tables 4.18, 4.23 and 4.28 and the corresponding figures 4.17, 4.21 and 4.25 present the overall travel times for different loading factors and toll levels under the three market penetration scenarios considered (no information case, 100% percent informed vehicles under a myopic switching rule and 100% informed vehicles under a non-myopic switching rule).

In the same form, tables 4.19, 4.24 and 4.29 present the percentage increase in overall travel times for different loading factors, market penetration levels and switching rules; tables 4.20, 4.25 and 4.30 and their corresponding figures 4.18, 4.22 and 4.26 present the total trip distance; tables 4.21, 4.26, and 4.31 along with figures 4.19, 4.23 and 4.27 present the number of vehicles using link 38-37 and tables 4.22, 4.27 and 4.32 with figures 4.20, 4.24, and 4.28 present the network revenues.

As for the scheme reviewed on paired links with alternatives, the incorporation of constant prices does affect the overall travel times. However, unlike that same scheme, pricing one arc in this case reduces the overall travel time. For the no information case, travel times are significantly reduced especially at high levels of demand and tolls. Overall travel time is reduced by about 10 percent for a demand level of 23,960 vehicles and a toll of $1.30. The reduction in the number of vehicles using link 38-37, of about one third, significantly improves the operation of the network since vehicles using that link can achieve higher speeds and lower travel times. Due to its location, link 38-37 is used by more than 13 percent of the vehicles in the simulation when no price is considered. Any reduction in the number of vehicles using the link will have a significant impact on the operation of the network. With pricing, some vehicles are assigned to longer routes but that still improves the overall operation of the network.

For the cases with 100 percent information, overall travel times are not affected as much as in the no information case. Most of the gains are already achieved by the provision of information and only small reductions are achieved when pricing is incorporated.

Similarly to the overall travel times, total trip distances are smaller for most of the loading factors, toll levels and information characteristics. However, in no case are the reductions over 2.25 percent of the base case.

The number of vehicles entering the priced link exhibits a similar pattern regardless of the information level. Vehicles that do not have node 37 as destination are gradually shifted to routes that do not include link 38-37. The main reductions are at low toll levels, especially for the no information case. Then, they are smaller as the tolls increase. Unlike the scheme considered in

the section on paired links with alternative routes, reductions are not so sharp. Again, the main difference among the three information availability cases is in the number of vehicles that use the link when no tolls are considered. This number is much higher for the no information case than for the 100 percent information cases.

With respect to the revenues, these increase more sharply at higher demand levels. Here, there is a captive population that has no alternative but to use link 38-37. If there is no change in transportation mode, this segment will continue paying the higher tolls. The magnitude of the revenues is fairly similar for all the information cases considered.

TABLE 4.18 OVERALL TRAVEL TIMES (HOURS) FOR DIFFERENT LOADING FACTORS AND TOLL LEVELS WHEN LINK 38-37 IS TOLLED. NO EN-ROUTE INFORMATION CASE

	Toll Level							
Loading factor	0.00	0.20	0.40	0.60	0.80	1.00	1.20	1.30
1.00	1540	1555	1543					
1.25	2290	2320	2283					
1.50	3825	3709	3895	3678				
1.75	5978	6373	5936	6104	6035			
2.00	10296	9813	9850	9288	9617	9637	9454	9299

TABLE 4.19 INCREASE IN OVERALL TRAVEL TIMES (%) FOR DIFFERENT LOADING FACTORS AND TOLL LEVELS WHEN LINK 38-37 IS TOLLED. NO EN-ROUTE INFORMATION CASE

	Toll Level							
Loading factor	0.00	0.20	0.40	0.60	0.80	1.00	1.20	1.30
1.00	0.00	1.01	0.21					
1.25	0.00	1.28	-0.32					
1.50	0.00	-3.04	1.81	-3.84				
1.75	0.00	6.61	-0.70	2.12	0.96			
2.00	0.00	-4.69	-4.33	-9.79	-6.59	-6.39	-8.18	-9.68

TABLE 4.20 TOTAL TRIP DISTANCE (MILES) FOR DIFFERENT LOADING FACTORS AND TOLL LEVELS WHEN LINK 38-37 IS TOLLED. NO EN-ROUTE INFORMATION CASE

	Toll Level							
Loading factor	0.00	0.20	0.40	0.60	0.80	1.00	1.20	1.30
1.00	29901	29805	29757					
1.25	38020	37726	37706					
1.50	46942	46730	46614	46503				
1.75	57116	57063	56277	56028	56315			
2.00	67458	66909	66687	65936	66172	66291	66425	65991

TABLE 4.21 NUMBER OF VEHICLES USING THE LINK FOR DIFFERENT LOADING FACTORS AND TOLL LEVELS WHEN LINK 38-37 IS TOLLED. NO EN-ROUTE INFORMATION CASE

	Toll Level							
Loading factor	0.00	0.20	0.40	0.60	0.80	1.00	1.20	1.30
1.00	1176	1067	1065					
1.25	1550	1363	1349					
1.50	1880	1713	1603	1602				
1.75	2481	2348	2078	1854	1833			
2.00	3166	2650	2470	2245	2150	2113	2109	2109

TABLE 4.22 NETWORK REVENUE (DOLLARS) FOR DIFFERENT LOADING FACTORS AND TOLL LEVELS WHEN LINK 38-37 IS TOLLED. NO EN-ROUTE INFORMATION CASE

	Toll Level							
Loading factor	0.00	0.20	0.40	0.60	0.80	1.00	1.20	1.30
1.00	0	213						
1.25	0	273	540					
1.50	0	343	641	961				
1.75	0	470	831	1112	1466			
2.00	0	530	988	1347	1720	2113	2531	2742

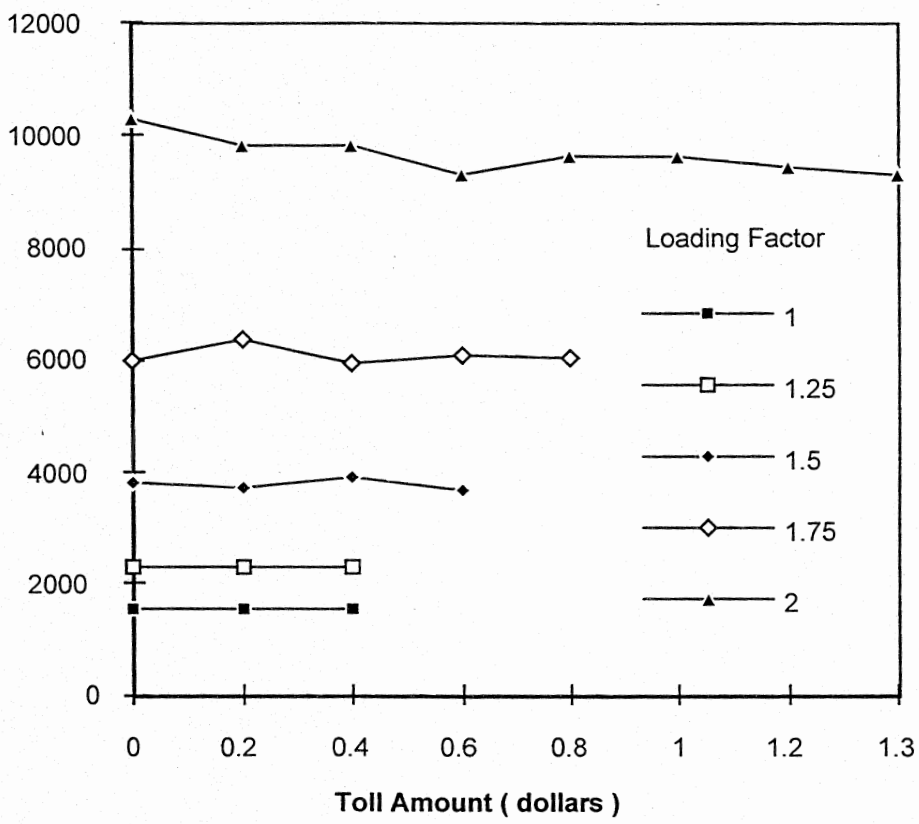

Figure 4.17 Variation of Network Total Overall Time for Different Loading Factors and Price Levels when Link 38-37 is Tolled. No en-route Information Case.

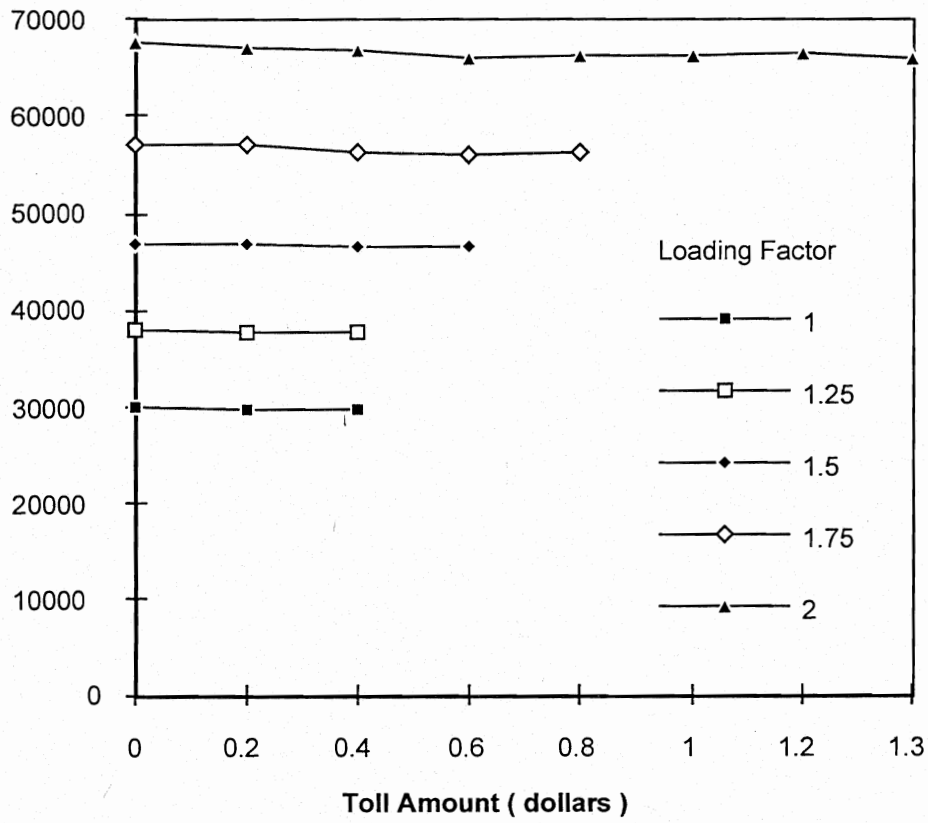

Figure 4.18 Variation of Total Trip Distance for Different Loading Factors and Price Levels when Link 38-37 is Tolled. No en-route Information Case.

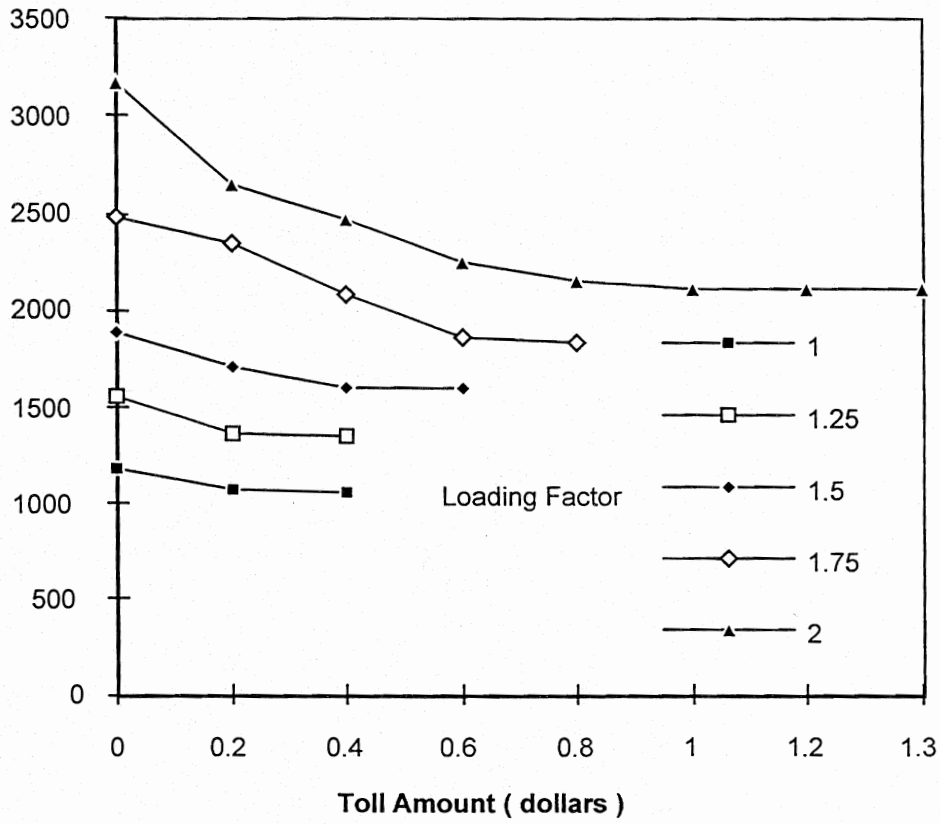

Figure 4.19 Variation in Number of Vehicles Using the Link for Different Loading Factors and Price Levels when Link 38-37 is Tolled. No en-route Information Case.

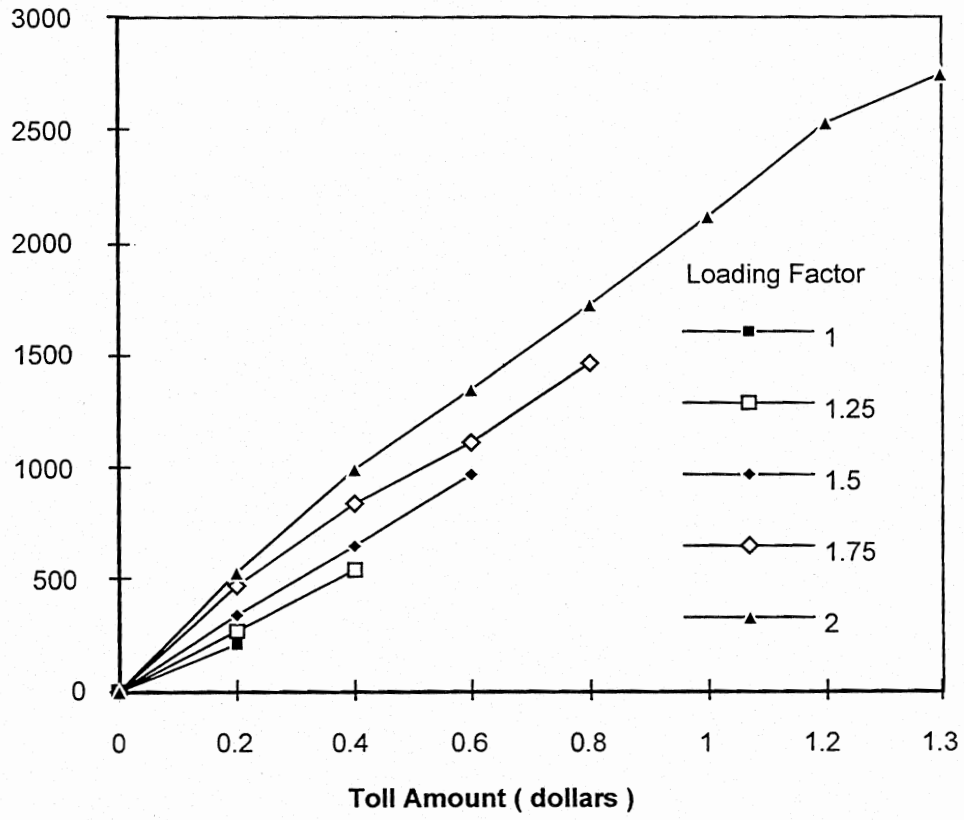

Figure 4.20 Variation in Network Revenue for Different Loading Factors and Price Levels when Link 38-37 is Tolled. No en-route Information Case

TABLE 4.23 OVERALL TRAVEL TIMES(HOURS) FOR DIFFERENT LOADING FACTORS AND TOLL LEVELS WHEN LINK 38-37 IS TOLLED. 100% INFORMED VEHICLES. MYOPIC CASE

	Toll Level										
Loading factor	0.00	0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.60	1.80	2.00
1.00	1354	1367	1371								
1.25	1979	2016	2026	1975							
1.50	2884	2895	2859	2868	2918	2861	2831				
1.75	4147	4102	4191	4088	4084	4121	4155	4121	4179	4084	
2.00	5871	5775	5872	5878	5893	5992	5873	5894	5767	5828	5927

TABLE 4.24 INCREASE IN OVERALL TRAVEL TIMES (%) FOR DIFFERENT LOADING FACTORS AND TOLL LEVELS WHEN LINK 38-37 IS TOLLED.100% INFORMED VEHICLES. MYOPIC CASE

	Toll Level										
Loading factor	0.00	0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.60	1.80	2.00
1.00	0.00	1.00	1.25								
1.25	0.00	1.86	2.34	-0.22							
1.50	0.00	0.41	-0.85	-0.53	1.20	-0.77	-1.84				
1.75	0.00	-1.08	1.06	-1.42	-1.51	-0.62	0.18	-0.64	0.76	-1.53	
2.00	0.00	-1.63	0.01	0.11	0.37	2.05	0.03	0.39	-1.78	-0.74	0.94

TABLE 4.25 TOTAL TRIP DISTANCE (MILES) FOR DIFFERENT LOADING FACTORS AND TOLL LEVELS WHEN LINK 38-37 IS TOLLED.100% INFORMED VEHICLES. MYOPIC CASE

	Toll Level										
Loading factor	0.00	0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.60	1.80	2.00
1.00	26945	26921	26820								
1.25	34294	34175	34361	34107							
1.50	42250	41928	41636	41580	41685	41778	41505				
1.75	50277	49867	50202	49996	49750	49811	50268	49855	50735	49829	
2.00	59124	58355	58575	58872	58661	59031	58737	58649	58271	58609	58546

TABLE 4.26 NUMBER OF VEHICLES USING THE LINK FOR DIFFERENT LOADING FACTORS AND TOLL LEVELS WHEN LINK 38-37 IS TOLLED.100% INFORMED VEHICLES. MYOPIC CASE

	Toll Level										
Loading factor	0.00	0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.60	1.80	2.00
1.00	1204	1092	1065								
1.25	1557	1426	1401	1349							
1.50	1841	1717	1688	1622	1635	1624	1602				
1.75	2171	2026	2041	1952	1871	1891	1885	1885	1890	1851	
2.00	2654	2614	2372	2414	2375	2262	2308	2182	2218	2194	2170

TABLE 4.27 NETWORK REVENUE (DOLLARS) FOR DIFFERENT LOADING FACTORS AND TOLL LEVELS WHEN LINK 38-37 IS TOLLED.100% INFORMED VEHICLES. MYOPIC CASE

	Toll Level										
Loading factor	0.00	0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.60	1.80	2.00
1.00	0	218									
1.25	0	285	560	809							
1.50	0	343	675	973	1308						
1.75	0	405	816	1171	1497	1891	2262				
2.00	0	523	949	1448	1900	2262	2770	3055	3549	3949	4340

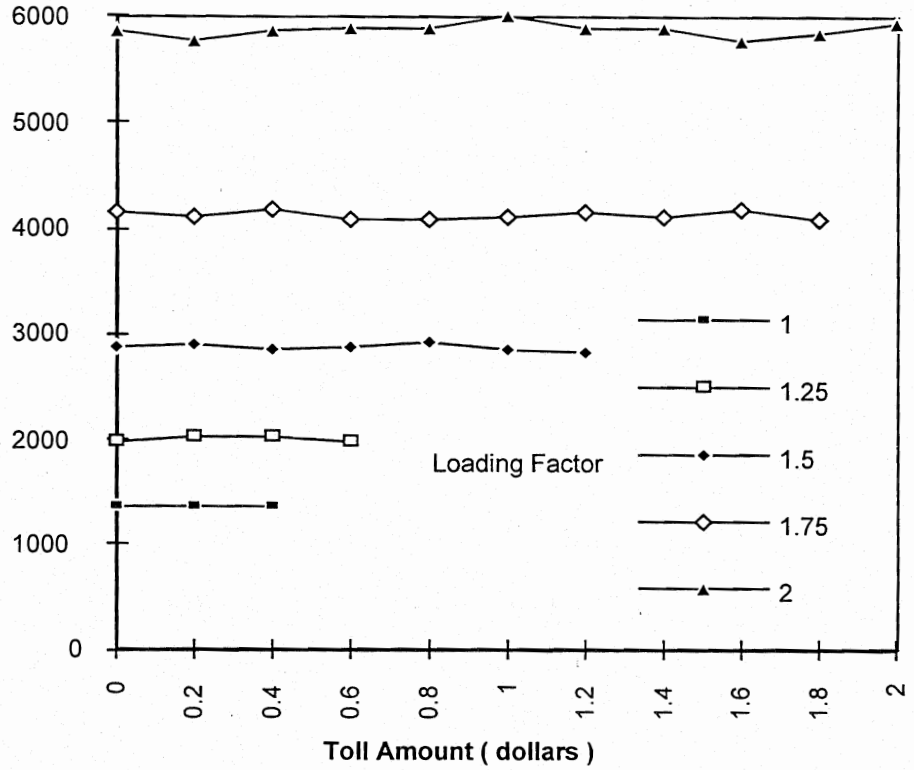

Figure 4.21 Variation of Network Total Overall Time for Different Loading Factors and Price Levels when Link 38-37 is Tolled. 100% Informed Vehicles. Myopic Case.

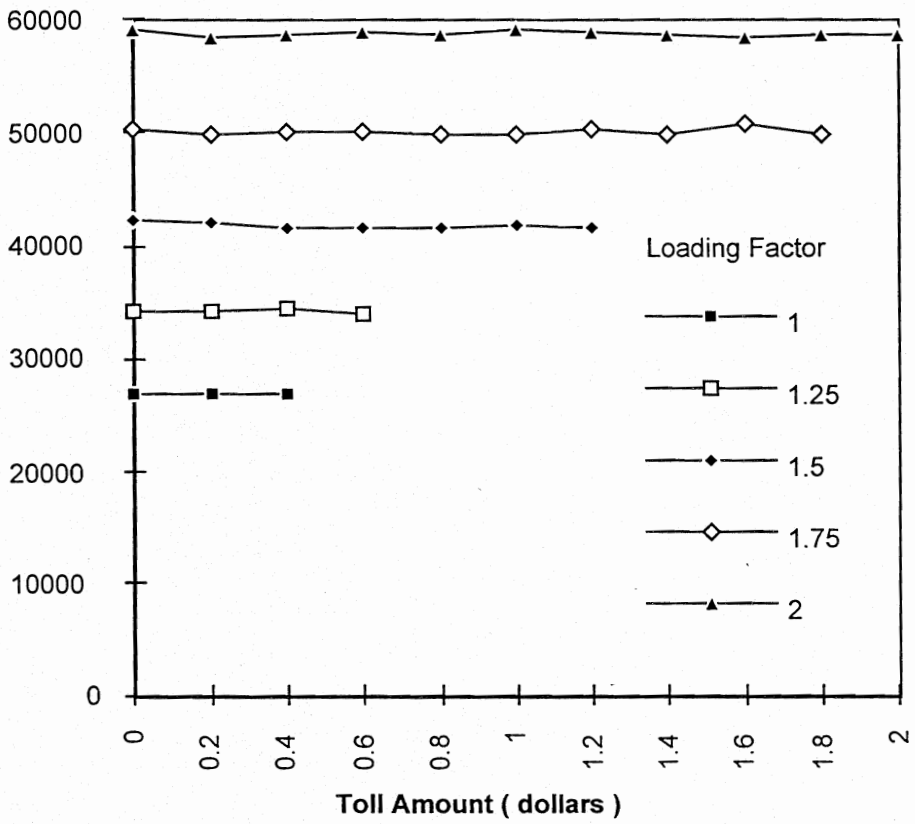

Figure 4.22 Variation of Total Trip Distance for Different Loading Factors and Price Levels when Link 38-37 is Tolled. 100% Informed Vehicles. Myopic Case.

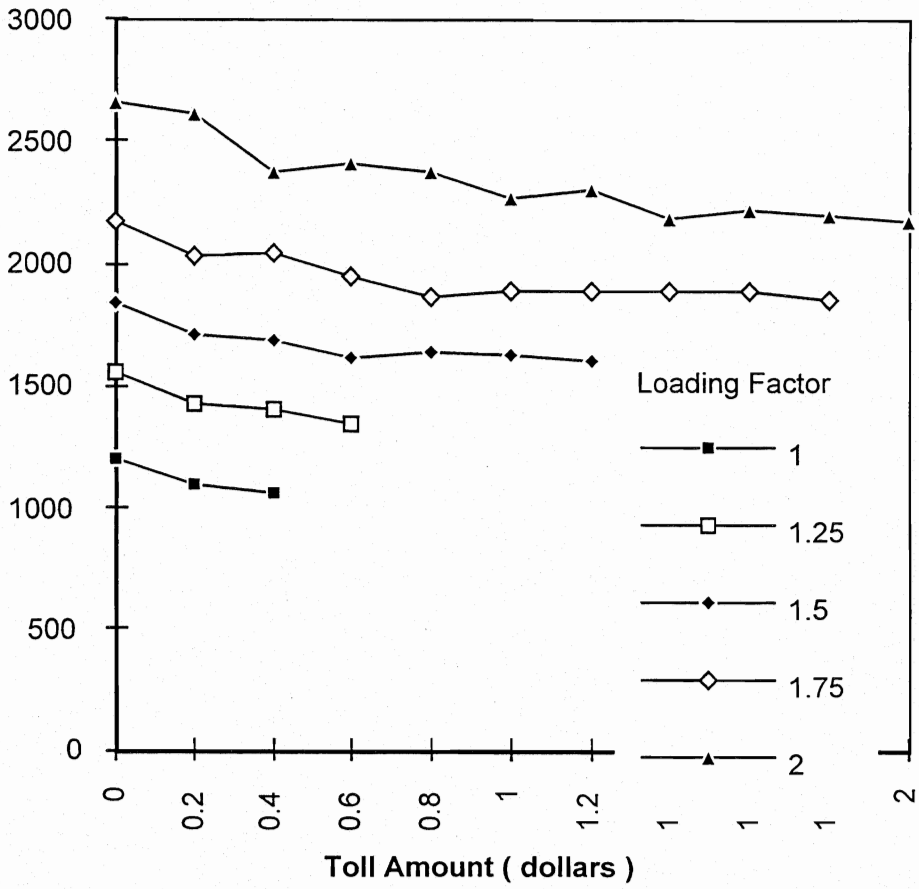

Figure 4.23 Variation in Number of Vehicles Using the Link for Different Loading Factors and Price Levels when Link 38-37 is Tolled. 100% Informed Vehicles. Myopic Case.

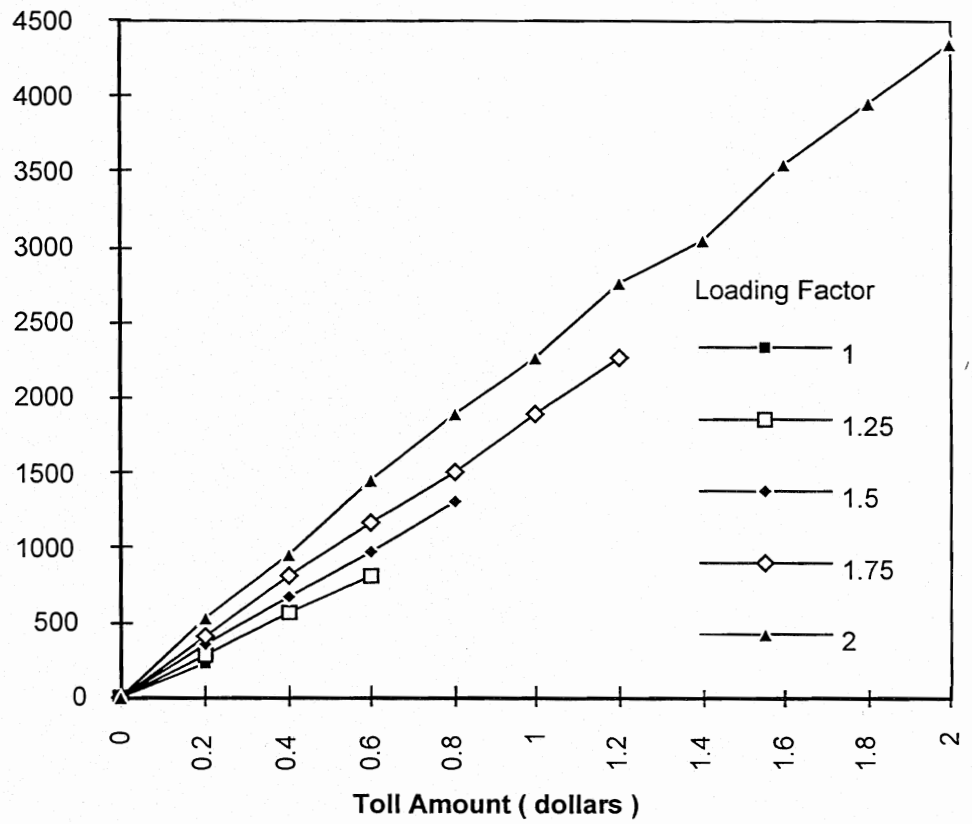

Figure 4.24 Variation in Network Revenue for Different Loading Factors and Price Levels when Link 38-37 is Tolled. 100% Informed Vehicles. Myopic Case.

TABLE 4.28 OVERALL TRAVEL TIMES (HOURS) FOR DIFFERENT LOADING FACTORS AND TOLL LEVELS WHEN LINK 38-37 IS TOLLED. 100% INFORMED VEHICLES. NON-MYOPIC CASE

	Toll Level										
Loading factor	0.00	0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.60	1.80	2.00
1.00	1351	1362									
1.25	1954	1962	1949								
1.50	2828	2826	2797	2787	2796	2835	2822				
1.75	4100	4121	4011	4052	3996	4054	4000	4047			
2.00	5753	5735	5700	5623	5579	5662	5660	5673	5706	5688	5696

TABLE 4.29 INCREASE IN OVERALL TRAVEL TIMES (%) FOR DIFFERENT LOADING FACTORS AND TOLL LEVELS WHEN LINK 38-37 IS TOLLED. 100% INFORMED VEHICLES. NON-MYOPIC CASE

	Toll Level										
Loading factor	0.00	0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.60	1.80	2.00
1.00	0.00	0.79									
1.25	0.00	0.44	-0.24								
1.50	0.00	-0.07	-1.10	-1.44	-1.11	0.25	-0.19				
1.75	0.00	0.51	-2.17	-1.16	-2.54	-1.11	-2.44	-1.28			
2.00	0.00	-0.32	-0.93	-2.27	-3.02	-1.59	-1.62	-1.40	-0.82	-1.13	-0.99

TABLE 4.30 TOTAL TRIP DISTANCE (MILES) FOR DIFFERENT LOADING FACTORS AND TOLL LEVELS WHEN LINK 38-37 IS TOLLED. 100% INFORMED VEHICLES. NON-MYOPIC CASE

	Toll Level										
Loading factor	0.00	0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.60	1.80	2.00
1.00	27101	27054									
1.25	34369	34201	34122								
1.50	42132	41936	41681	41760	41552	41838	41771				
1.75	50083	50181	49435	49932	49495	49713	49682	49731			
2.00	58769	58364	58162	57886	57588	58100	58099	57898	58490	58113	58261

TABLE 4.31 NUMBER OF VEHICLES USING THE LINK FOR DIFFERENT LOADING FACTORS AND TOLL LEVELS WHEN LINK 38-37 IS TOLLED. 100% INFORMED VEHICLES. NON-MYOPIC CASE

	Toll Level										
Loading factor	0.00	0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.60	1.80	2.00
1.00	1187	1091									
1.25	1635	1397	1353								
1.50	1837	1705	1674	1612	1607	1610	1611				
1.75	2179	2036	1948	1954	1887	1845	1841	1856			
2.00	2655	2559	2475	2401	2345	2362	2236	2182	2196	2156	2166

TABLE 4.32 NETWORK REVENUE (DOLLARS) FOR DIFFERENT LOADING FACTORS AND TOLL LEVELS WHEN LINK 38-37 IS TOLLED. 100% INFORMED VEHICLES. NON-MYOPIC CASE

	Toll Level										
Loading factor	0.00	0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.60	1.80	2.00
1.00	0	218									
1.25	0	279	541								
1.50	0	341	670	967	1286						
1.75	0	407	779	1172	1510	1845	2209				
2.00	0	512	990	1441	1876	2362	2683	3055	3514	3881	4332

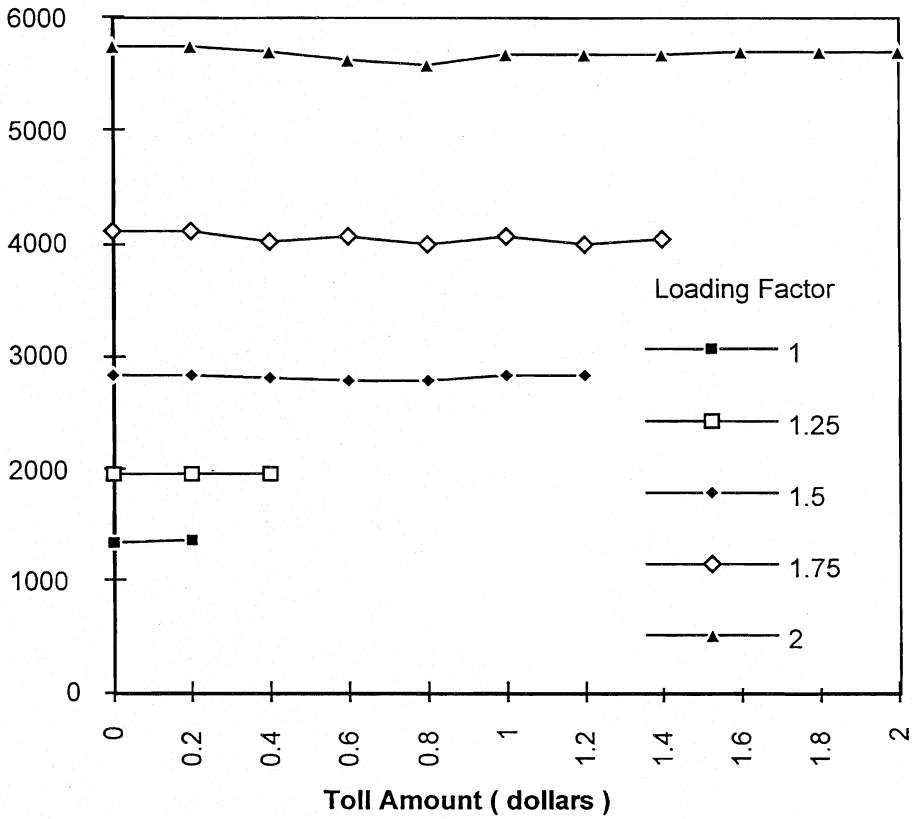

Figure 4.25 Variation of Network Total Overall Time for Different Loading Factors and Price Levels when Link 38-37 is Tolled. 100% Informed Vehicles. Non-Myopic Case.

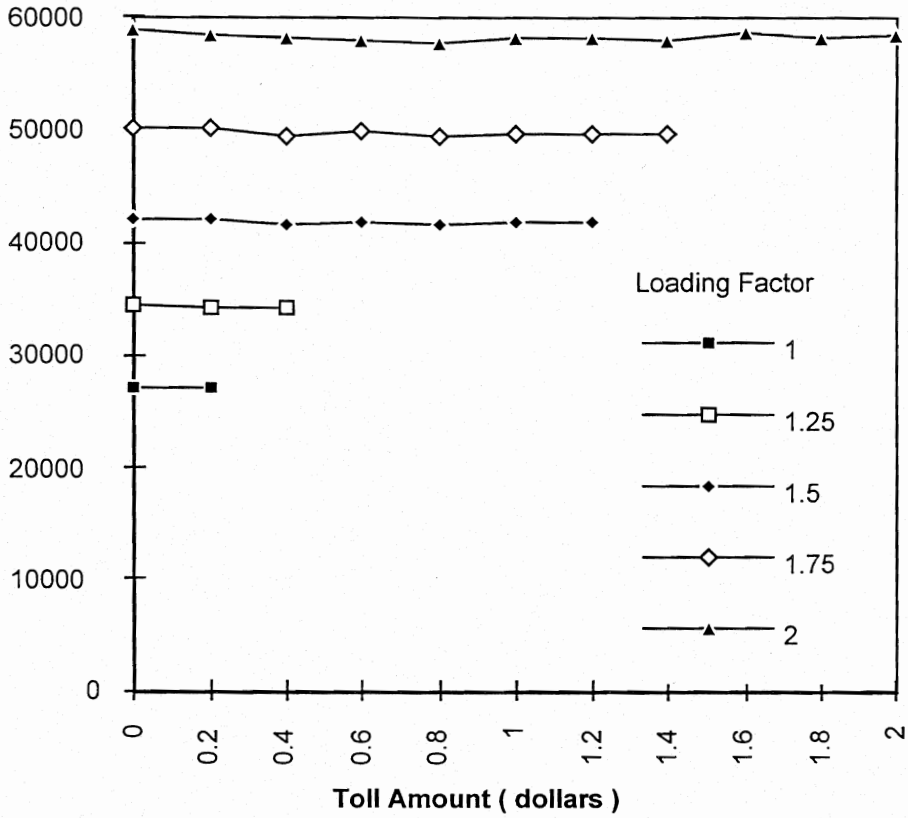

Figure 4.26 Variation of Total Trip Distance for Different Loading Factors and Price Levels when Link 38-37 is Tolled. 100% Informed Vehicles. Non-Myopic Case.

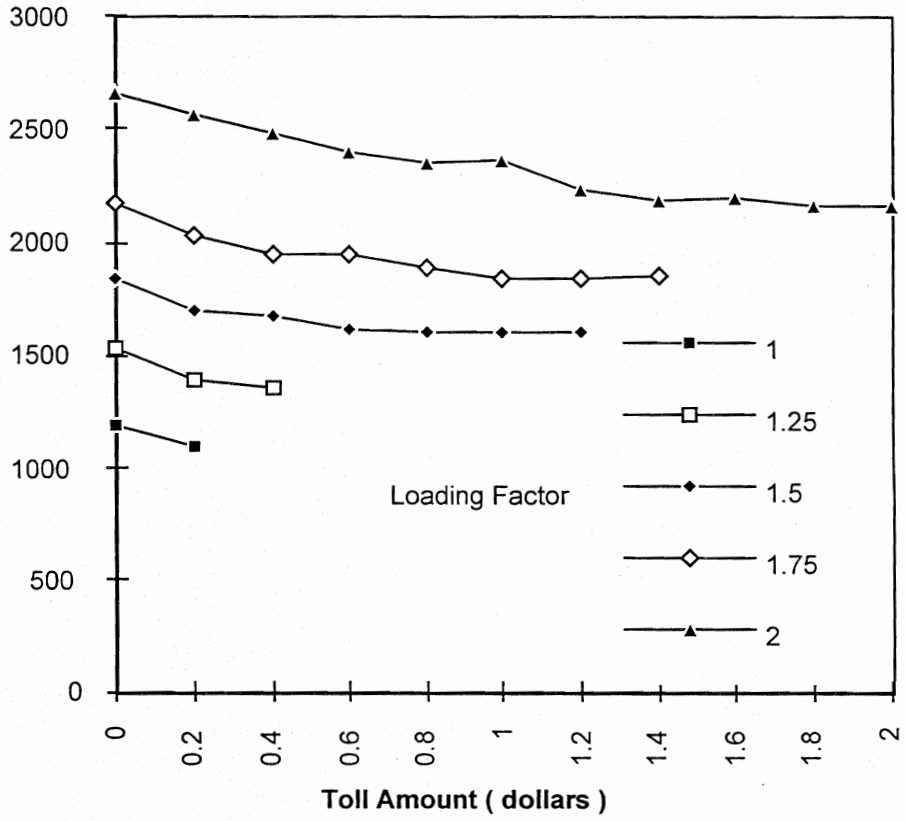

Figure 4.27 Variation in Number of Vehicles Using the Link for Different Loading Factors and Price Levels when Link 38-37 is Tolled. 100% Informed Vehicles. Non-Myopic Case.

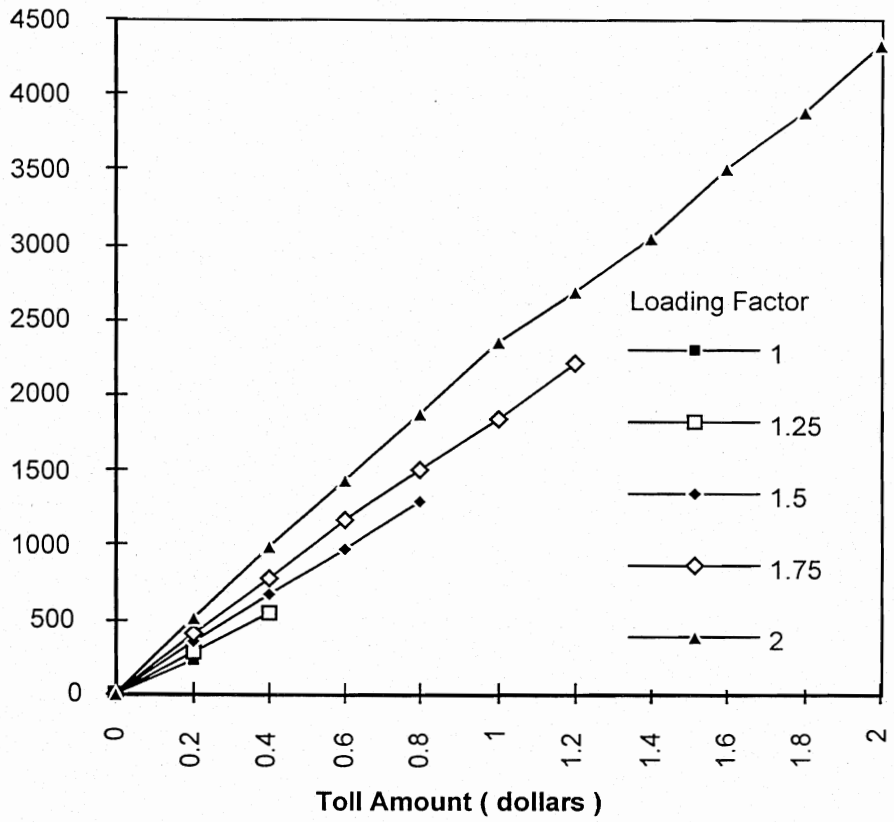

Figure 4.28 Variation in Network Revenue for Different Loading Factors and Price Levels when Link 38-37 is Tolled. 100% Informed Vehicles. Non-Myopic Case.

TABLE 4.33 OVERALL TRAVEL TIMES (HOURS) FOR DIFFERENT LOADING FACTORS AND TOLL LEVELS WHEN ACCESS TO ZONE 3 IS TOLLED. NO EN-ROUTE INFORMATION CASE

	Toll Level													
LF	0.00	0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.60	1.80	2.00	2.20	2.40	2.50
1.00	1540	1544	1545	1577	1583	1576								
1.25	2290	2387	2499	2489	2514	2468	2467							
1.50	3825	4018	3979	4187	4308	4164	4326	4395	4192	4082	4186			
1.75	5978	5965	6234	6409	6556	5887	6100	6986	6890	6330	6430	6556		
2.00	10296	9931	10320	10906	10692	11189	10639	11701	11222	12253	12111	11754	11579	12326

TABLE 4.34 INCREASE IN OVERALL TRAVEL TIMES (%) FOR DIFFERENT LOADING FACTORS AND TOLL LEVELS WHEN ACCESS TO ZONE 3 IS TOLLED. NO EN-ROUTE INFORMATION CASE

	Toll Level													
LF	0.00	0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.60	1.80	2.00	2.20	2.40	2.50
1.00	0.00	0.30	0.31	2.41	2.79	2.34								
1.25	0.00	4.21	9.10	8.68	9.76	7.78	7.72							
1.50	0.00	5.04	4.00	9.44	12.62	8.86	13.08	14.88	9.59	6.70	9.43			
1.75	0.00	-0.20	4.29	7.22	9.68	-1.51	2.04	16.88	15.26	5.89	7.57	9.67		
2.00	0.00	-3.54	0.24	5.93	3.85	8.67	3.34	13.65	9.00	19.01	17.64	14.16	12.47	19.73

TABLE 4.35 TOTAL TRIP DISTANCE (MILES) FOR DIFFERENT LOADING FACTORS AND TOLL LEVELS WHEN ACCESS TO ZONE 3 IS TOLLED. NO EN-ROUTE INFORMATION CASE

	Toll Level													
LF	0.00	0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.60	1.80	2.00	2.20	2.40	2.50
1.00	29901	30004	30128	30306	30379	30419								
1.25	38020	38002	38342	38422	38349	38464	38363							
1.50	46942	47126	47297	47579	47909	47719	47938	48316	48182	48081	47958			
1.75	57116	57167	57590	57375	57761	56721	57730	58414	58284	57489	57527	58144		
2.00	67458	67199	67378	68279	68487	68338	68369	68701	68723	69124	69103	69174	68927	68990

TABLE 4.36 NUMBER OF VEHICLES ENTERING ZONE 3 FOR DIFFERENT LOADING FACTORS AND TOLL LEVELS WHEN ACCESS IS TOLLED. NO EN-ROUTE INFORMATION CASE

	Toll Level													
LF	0.00	0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.60	1.80	2.00	2.20	2.40	2.50
1.00	2672	2083	1897	1858	1838	1835								
1.25	3410	2717	2398	2305	2274	2253	2253							
1.50	3822	3282	2900	2812	2810	2773	2703	2701	2697	2695	2689			
1.75	4725	3845	3645	3503	3399	3364	3253	3210	3214	3195	3185	3172		
2.00	5270	5386	4586	4437	4014	4151	3879	3783	3781	3791	3740	3748	3667	3654

TABLE 4.37 NETWORK REVENUE (DOLLARS) FOR DIFFERENT LOADING FACTORS AND TOLL LEVELS WHEN ACCESS TO ZONE 3 IS TOLLED. NO EN-ROUTE INFORMATION CASE

	Toll Level													
LF	0.00	0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.60	1.80	2.00	2.20	2.40	2.50
1.00	0	417	759	1115	1470	1835								
1.25	0	543	959	1383	1819	2253	2704							
1.50	0	656	1160	1687	2248	2773	3244	3781	4315	4851	5378			
1.75	0	769	1458	2102	2719	3364	3904	4494	5142	5751	6370	6978		
2.00	0	1077	1834	2662	3211	4151	4655	5296	6050	6824	7480	8246	8801	9135

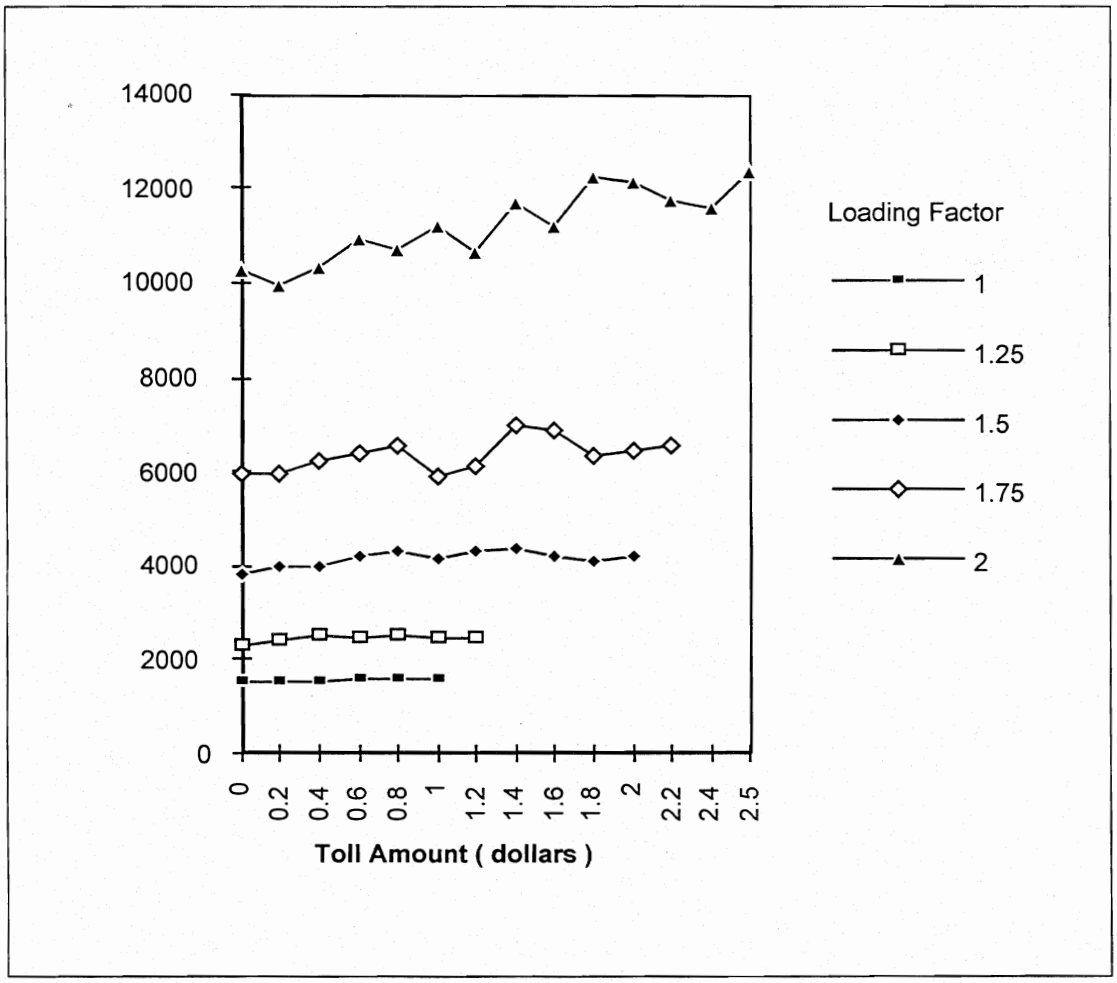

Figure 4.29 Variation of Network Total Overall Time for Different Loading Factors and Price Levels when Zone 3 is Tolled. No en-route Information Case.

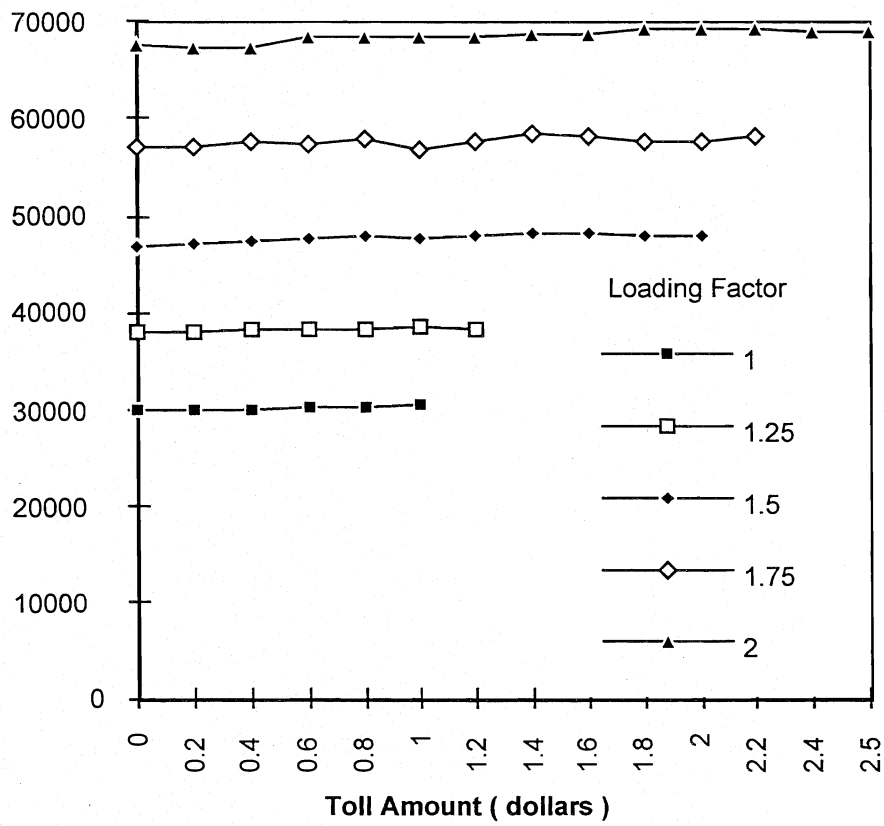

Figure 4.30 Variation of Total Trip Distance for Different Loading Factors and Price Levels when Zone 3 is Tolled. No en-route Information Case.

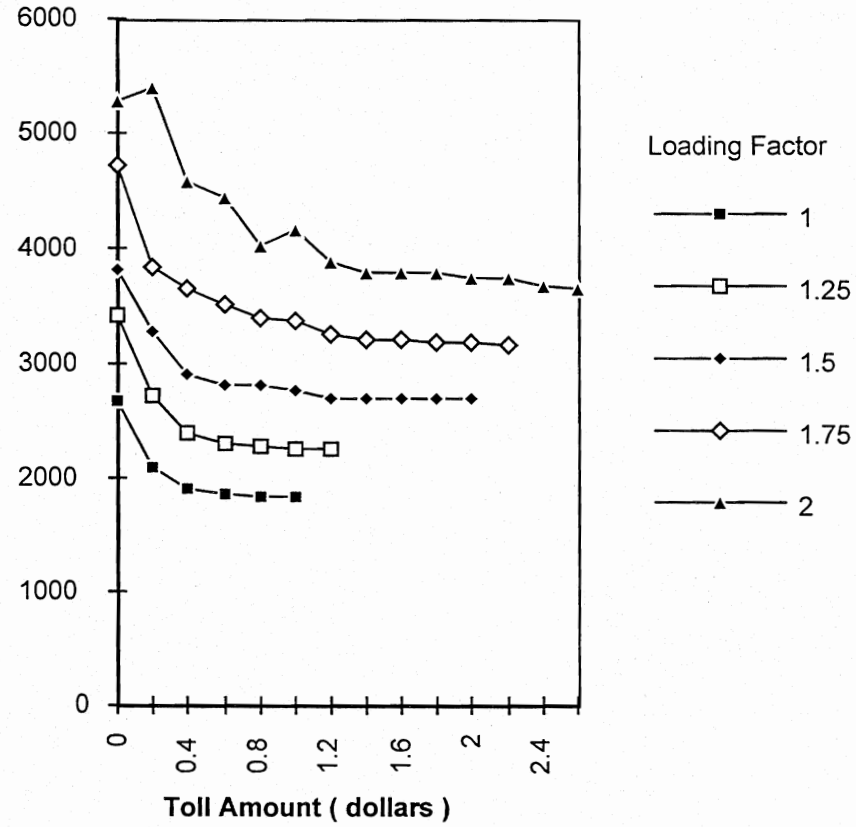

Figure 4.31 Variation in Number of Vehicles Using the Link for Different Loading Factors and Price Levels when Zone 3 is Tolled. No en-route Information Case.

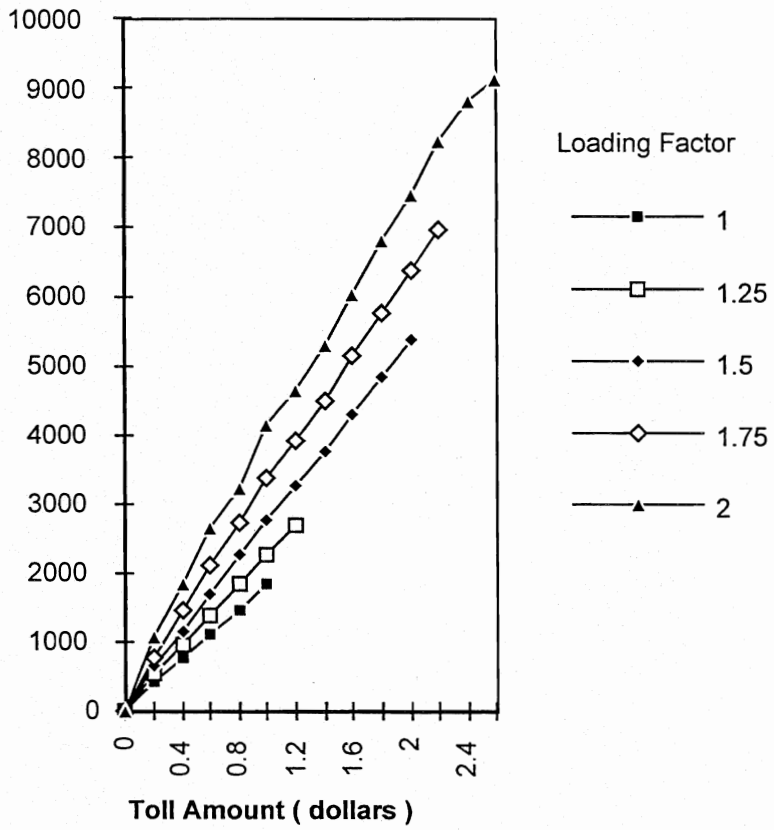

Figure 4.32 Variation in Network Revenue for Different Loading Factors and Price Levels when Zone 3 is Tolled. No en-route Information Case.

TABLE 4.38 OVERALL TRAVEL TIMES (HOURS) FOR DIFFERENT LOADING FACTORS AND TOLL LEVELS WHEN ACCESS TO ZONE 3 IS TOLLED. 100% INFORMED VEHICLES. MYOPIC CASE

	Toll Level													
LF	0.00	0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.60	1.80	2.00	2.20	2.40	2.50
1.00	1354	1356	1392	1384	1393	1388								
1.25	1979	1985	1966	2005	2016	2017	2048							
1.50	2884	2861	2925	2925	2936	2981	2971	3006	3035	2950	2959			
1.75	4147	4121	4171	4296	4253	4318	4300	4350	4335	4417	4345	4380		
2.00	5871	5819	5727	5812	6203	5994	6107	6130	6176	6201	6198	6131	6161	6187

TABLE 4.39 INCREASE IN OVERALL TRAVEL TIMES (%) FOR DIFFERENT LOADING FACTORS AND TOLL LEVELS WHEN ACCESS TO ZONE 3 IS TOLLED. 100% INFORMED VEHICLES. MYOPIC CASE

	Toll Level													
LF	0.00	0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.60	1.80	2.00	2.20	2.40	2.50
1.00	0.00	0.19	2.79	2.26	2.89	2.53								
1.25	0.00	0.30	-0.67	1.30	1.84	1.91	3.48							
1.50	0.00	-0.79	1.42	1.45	1.83	3.40	3.04	4.24	5.24	2.29	2.63			
1.75	0.00	-0.63	0.57	3.58	2.54	4.12	3.68	4.89	4.52	6.50	4.76	5.61		
2.00	0.00	-0.89	-2.47	-1.01	5.66	2.09	4.02	4.40	5.48	5.62	5.57	4.42	4.93	5.38

TABLE 4.40 TOTAL TRIP DISTANCE (MILES) FOR DIFFERENT LOADING FACTORS AND TOLL LEVELS WHEN ACCESS TO ZONE 3 IS TOLLED. 100% INFORMED VEHICLES. MYOPIC CASE

	Toll Level													
LF	0.00	0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.60	1.80	2.00	2.20	2.40	2.50
1.00	26945	27123	27409	27490	27583	27540								
1.25	34294	34419	34439	34782	34783	34856	35031							
1.50	42250	42038	42313	42586	42598	42758	42496	42908	43052	42625	42663			
1.75	50277	50361	50202	50463	50885	50485	50987	51244	51100	51294	51135	50866		
2.00	59124	59034	58312	58317	39757	58969	59236	59398	59877	59594	59520	59023	59523	59719

TABLE 4.41 NUMBER OF VEHICLES ENTERING ZONE 3 FOR DIFFERENT LOADING FACTORS AND TOLL LEVELS WHEN ACCESS IS TOLLED. 100% INFORMED VEHICLES. MYOPIC CASE.

	Toll Level													
LF	0.00	0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.60	1.80	2.00	2.20	2.40	2.50
1.00	2562	2106	1946	1869	1838	1835								
1.25	3140	2673	2490	2389	2312	2256	2257							
1.50	3835	3350	3036	2938	2874	2765	2759	2719	2708	2697	2691			
1.75	4495	4047	3712	3630	3415	3318	3270	3239	3197	3190	3182	3170		
2.00	5245	5020	4490	4313	4202	4059	4009	3873	3872	3813	3724	3689	3667	3672

TABLE 4.42 NETWORK REVENUE (DOLLARS) FOR DIFFERENT LOADING FACTORS AND TOLL LEVELS WHEN ACCESS TO ZONE 3 IS TOLLED. 100% INFORMED VEHICLES. MYOPIC CASE.

	Toll Level													
LF	0.00	0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.60	1.80	2.00	2.20	2.40	2.50
1.00	0	421.2	778.4	1121	1470	1835								
1.25	0	534.6	996	1433	1850	2256	2708							
1.50	0	670.0	1214	1763	2299	2765	3311	3807	4333	4855	5382			
1.75	0	809.4	1485	2178	2732	3318	3924	4535	5115	5742	6364	6974		
2.00	0	1004	1796	2588	3362	4059	4811	5422	6195	6863	7448	8116	8801	9180

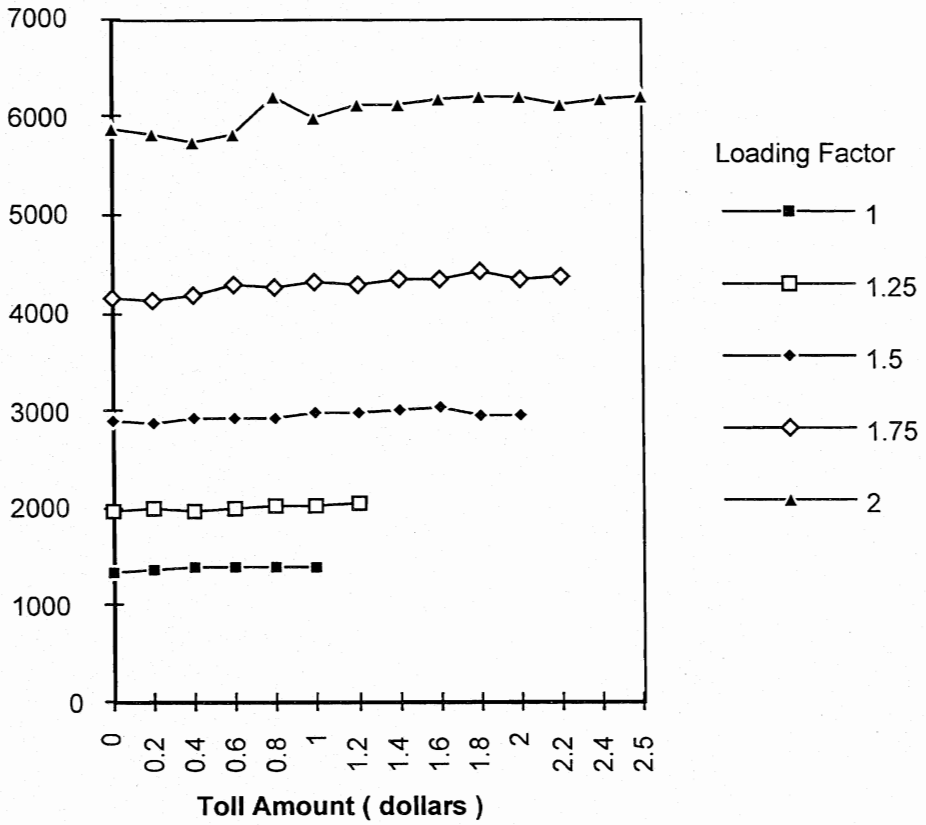

Figure 4.33 Variation of Network Total Overall Time for Different Loading Factors and Price Levels when Zone 3 is Tolled. 100% Informed Vehicles. Myopic Case.

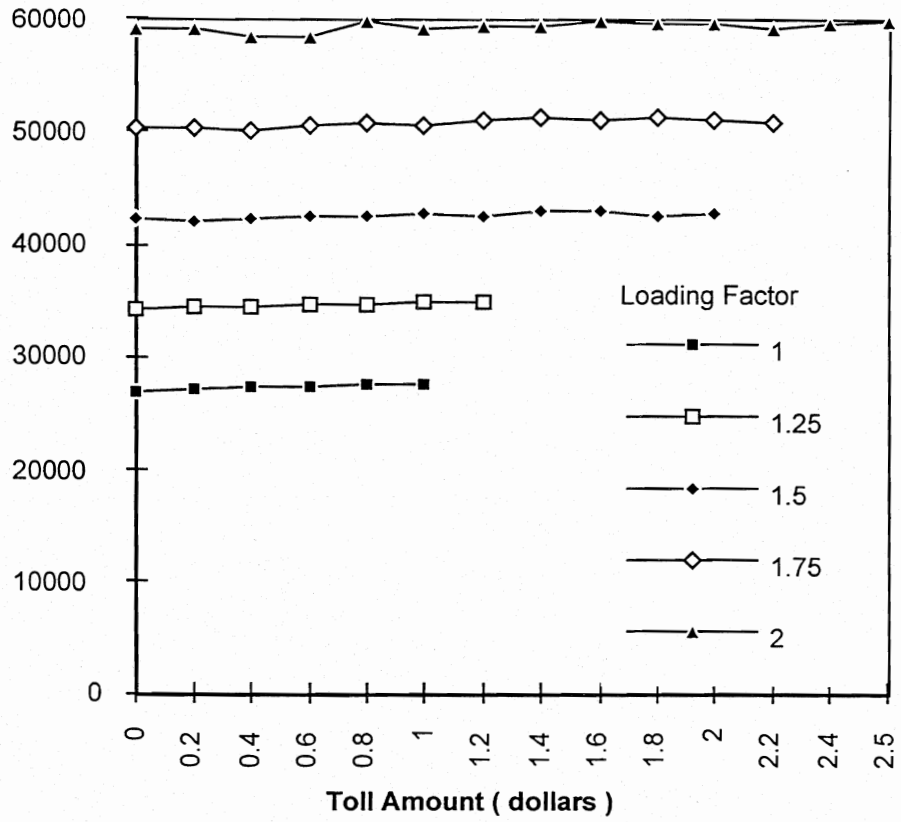

Figure 4.34 Variation of Total Trip Distance for Different Loading Factors and Price Levels when Zone 3 is Tolled. 100% Informed Vehicles. Myopic Case.

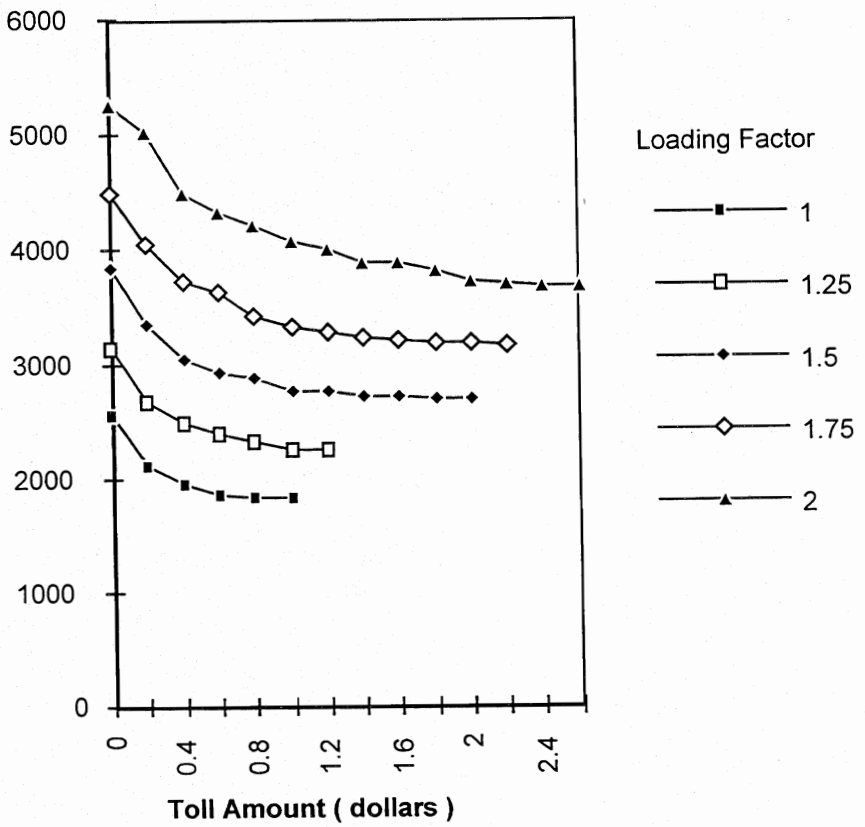

Figure 4.35 Variation in Number of Vehicles Using the Link for Different Loading Factors and Price Levels when Zone 3 is Tolled. 100% Informed Vehicles. Myopic Case.

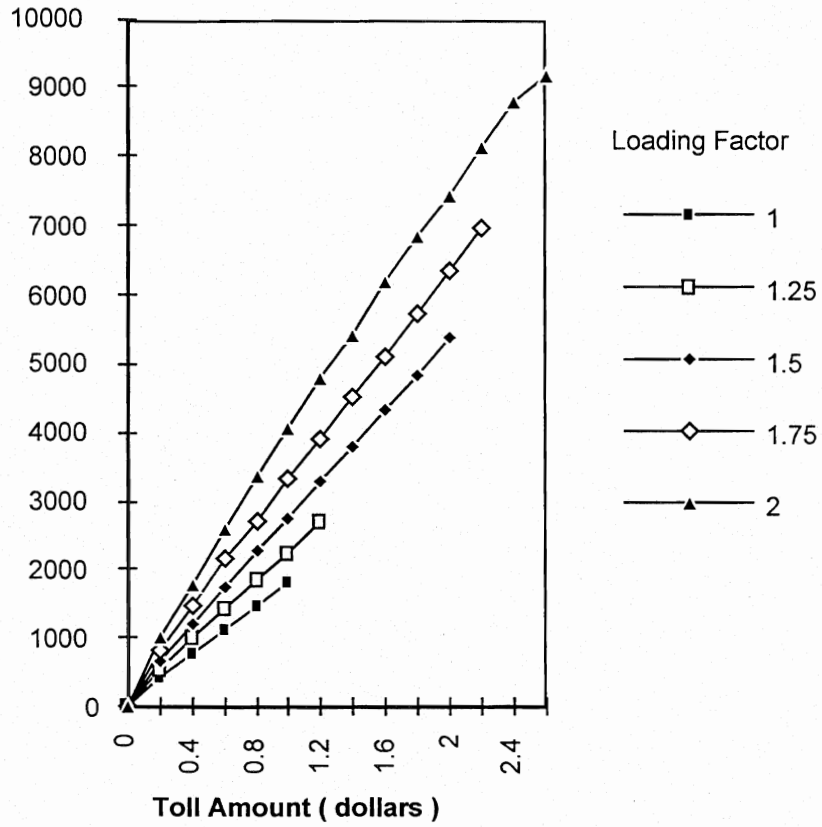

Figure 4.36 Variation in Network Revenue for Different Loading Factors and Price Levels when Zone 3 is Tolled.100% Informed Vehicles. Myopic Case.

TABLE 4.43 OVERALL TRAVEL TIMES (HOURS) FOR DIFFERENT LOADING FACTORS AND TOLL LEVELS WHEN ACCESS TO ZONE 3 IS TOLLED. 100% INFORMED VEHICLES. NON-MYOPIC CASE.

	Toll Level													
LF	0.00	0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.60	1.80	2.00	2.20	2.40	2.50
1.00	1351	1346	1360	1367	1368									
1.25	1954	1949	1963	1963	1997	1990	1977							
1.50	2828	2839	2822	2860	2934	2943	2948	2954	2954					
1.75	4100	4066	4185	4131	4133	4132	4212	4231	4262	4221	4236			
2.00	5753	5753	5813	5799	5793	5926	5940	6005	5922	5988	6040	6203	5909	6050

TABLE 4.44 INCREASE IN OVERALL TRAVEL TIMES (%) FOR DIFFERENT LOADING FACTORS AND TOLL LEVELS WHEN ACCESS TO ZONE 3 IS TOLLED. 100% INFORMED VEHICLES. NON-MYOPIC CASE.

	Toll Level													
LF	0.00	0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.60	1.80	2.00	2.20	2.40	2.50
1.00	0.00	-0.37	0.62	1.20	1.23									
1.25	0.00	-0.22	0.48	0.48	2.20	1.88	1.18							
1.50	0.00	0.39	-0.19	1.13	3.75	4.08	4.24	4.45	4.48					
1.75	0.00	-0.82	2.07	0.76	0.82	0.79	2.72	3.19	3.96	2.95	3.32			
2.00	0.00	0.00	1.04	0.79	0.69	3.01	3.25	4.39	2.93	4.08	5.00	7.83	2.70	5.17

TABLE 4.45 TOTAL TRIP DISTANCE (MILES) FOR DIFFERENT LOADING FACTORS AND TOLL LEVELS WHEN ACCESS TO ZONE 3 IS TOLLED. 100% INFORMED VEHICLES. NON-MYOPIC CASE.

	Toll Level													
LF	0.00	0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.60	1.80	2.00	2.20	2.40	2.50
1.00	27101	27354	27579	27638	27734									
1.25	34369	34618	34762	34960	35106	35134	34998							
1.50	42132	42046	42313	42528	42755	42831	42907	42976	42942					
1.75	50083	50155	50595	50544	50433	50331	50928	51164	50746	51120	51076			
2.00	58769	58893	59130	58883	58890	59006	59316	59300	59220	59272	59479	60224	58936	59783

TABLE 4.46 NUMBER OF VEHICLES ENTERING ZONE 3 FOR DIFFERENT LOADING FACTORS AND TOLL LEVELS WHEN ACCESS IS TOLLED. 100% INFORMED VEHICLES. NON-MYOPIC CASE.

	Toll Level													
LF	0.00	0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.60	1.80	2.00	2.20	2.40	2.50
1.00	2553	2044	1901	1853	1835									
1.25	3195	2652	2444	2333	2263	2255	2253							
1.50	3742	3268	2990	2873	2761	2739	2714	2697	2690					
1.75	4318	3970	3621	3494	3394	3350	3270	3264	3197	3184				
2.00	5224	4795	4519	4301	4154	3947	3978	3858	3813	3729	3705	3690	3692	3665

TABLE 4.47 NETWORK REVENUE (DOLLARS) FOR DIFFERENT LOADING FACTORS AND TOLL LEVELS WHEN ACCESS TO ZONE 3 IS TOLLED. 100% INFORMED VEHICLES. NON-MYOPIC CASE.

	Toll Level													
LF	0.00	0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.60	1.80	2.00	2.20	2.40	2.50
1.00	0	409	760	1112	1468									
1.25	0	530	978	1400	1810	2255	2704							
1.50	0	654	1196	1724	2209	2739	3257	3776	4304					
1.75	0	794	1448	2096	2715	3350	3924	4570	5115	5731				
2.00	0	959	1808	2581	3323	3947	4774	5401	6101	6712	7410	8118	8861	9163

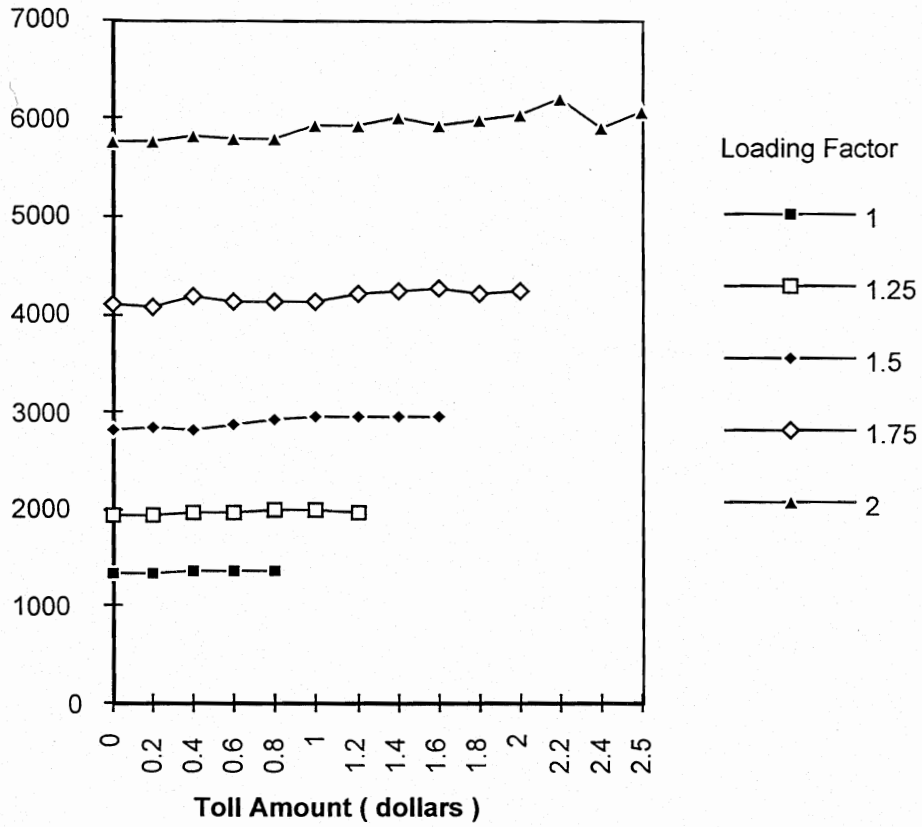

Figure 4.37 Variation of Network Total Overall Time for Different Loading Factors and Price Levels when Zone 3 is Tolled. 100% Informed Vehicles. Non-Myopic Case.

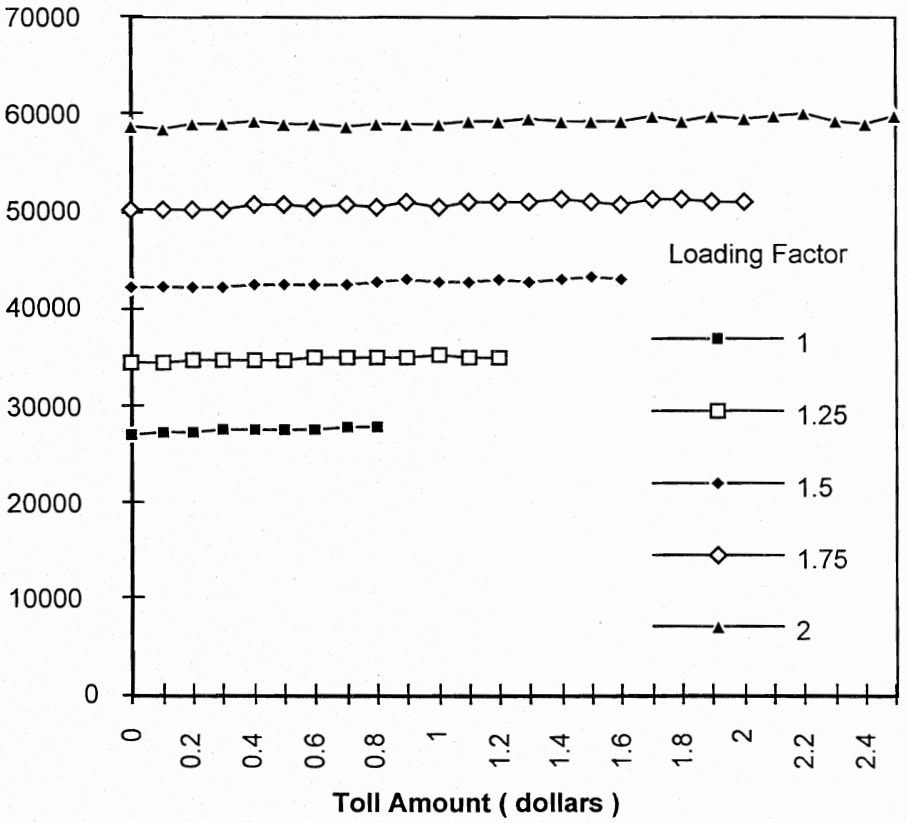

Figure 4.38 Variation of Total Trip Distance for Different Loading Factors and Price Levels when Zone 3 is Tolled. 100% Informed Vehicles. Non-Myopic Case.

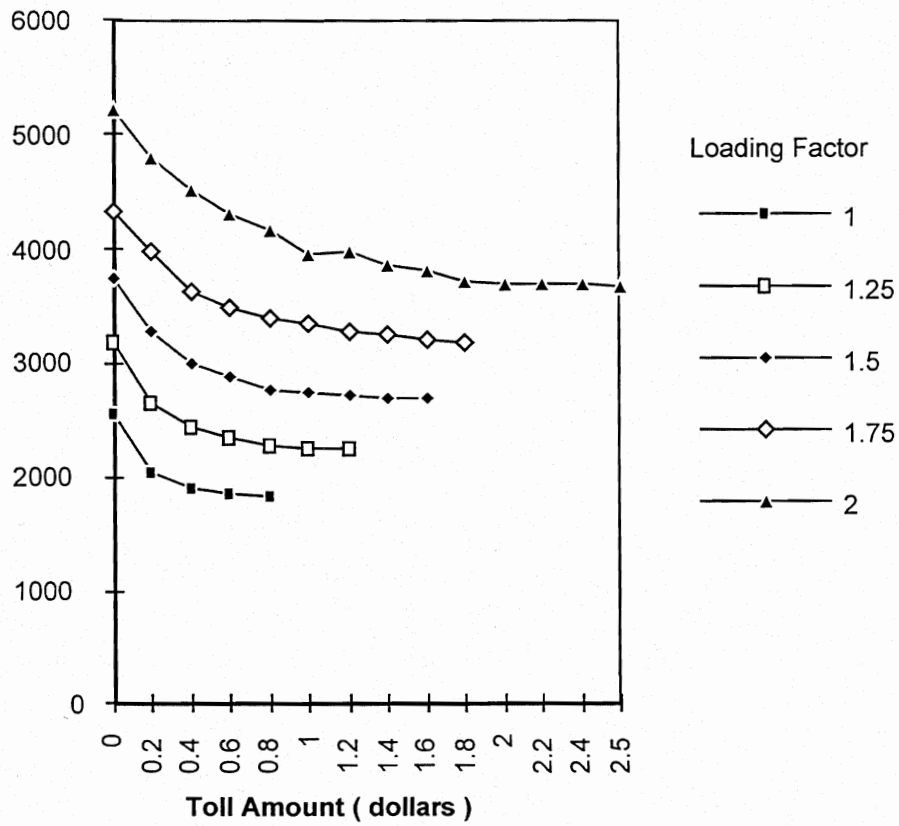

Figure 4.39 Variation in Number of Vehicles Using the Link for Different Loading Factors and Price Levels when Zone 3 is Tolled. 100% Informed Vehicles. Non-Myopic Case.

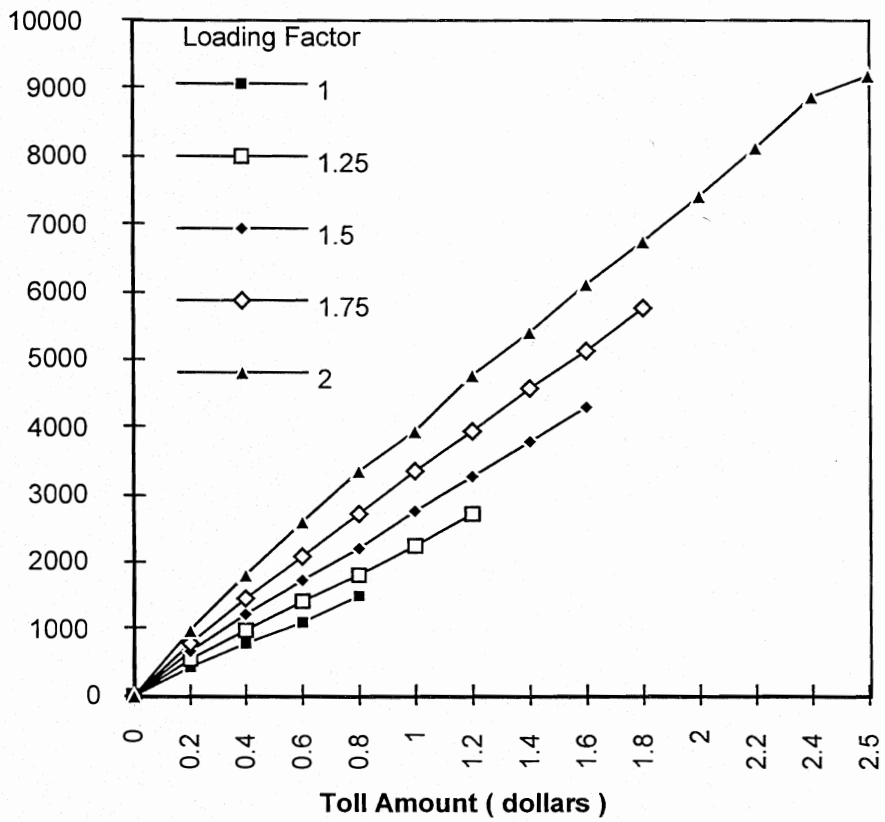

Figure 4.40 Variation in Network Revenue for Different Loading Factors and Price Levels when Zone 3 is Tolled. 100% Informed Vehicles. Non-Myopic Case.

VARIABLE PRICING SCHEMES

The general characteristics of the test network are the same as those used in preceding experiments, with constant tolls. The same demand pattern is used but only one demand level is reported in this document. The number of vehicles loaded onto the network was 23,960, which corresponds to very congested conditions. Two different market penetration levels were considered. 0 and 100 percent. The 100 percent information case was divided in two parts. The first part considers a myopic switching rule and the second a non myopic switching rule with the value of the mean relative indifference band set to 0.2 and the minimum threshold bound set to 1 minute for all the users. Initial assignment paths correspond to the current best path. The value of travel time is again assumed to be six dollars per hour. Different values of travel time could be used, but they will only change the revenues since the time equivalent tolls are calculated within DYNASMART.

Price Levels

Toll levels are calculated according to equation (4.11). Time-equivalent tolls are updated according to the user specified toll update interval and congestion level threshold. The toll update intervals considered were: 6 seconds, 30 seconds, 1 minute, 3 minutes, 5 minutes and 15 minutes. The congestion level thresholds considered were: 0, 20, 40, 60, 80 and 99 percent.

Pricing Schemes Considered

The only pricing scheme considered consisted of charging for the use of all the links of the traffic network. Once the congestion threshold is set, only those links that exceed that congestion level will be tolled.

Experiment Results

The effects of variable pricing on the operation of a traffic network, when all the links are priced, are presented in tables 4.48 to 4.62 and figures 4.41 to 4.49. Tables 4.48, 4.53 and 4.58 and the corresponding figures 4.41, 4.44 and 4.47 present the values of the overall travel times for different toll update intervals and density levels under the three market penetration scenarios considered (no information case, 100% percent informed vehicles under a myopic switching rule and 100% informed vehicles under a non-myopic switching rule).

In a similar way, tables 4.49, 4.54 and 4.59 present the percentage increase in overall travel times for different toll update intervals, density levels, market penetration level and switching rules; tables 4.50, 4.55 and 4.60 and the corresponding figures 4.42, 4.45 and 4.48 present the information on total trip distance; tables 4.51, 4.56, and 4.61 present the percentage increase in trip distance for different toll update intervals, density levels, market penetration level and switching rules and; tables 4.52, 4.57 and 4.62 with figures 4.43, 4.46, and 4.49 summarize the network revenue.

The clear effects on overall travel time identified for the constant toll schemes of the previous sections contrast with the effects of variable prices. Overall travel times behave in an irregular pattern. For the no information case, overall travel sometimes improve when tolls are updated at low density levels and shorter update times. A more than 6 percent reduction is observed for a threshold of 0 percent density and 6 seconds toll update interval, which would correspond to a continuously updated toll. When the density threshold is increased above 60 percent, the overall travel times also increase (less than 2 percent) with respect to the base case but variations are smaller than for the low density thresholds.

As in the constant toll cases, information provision reduces significantly the overall travel times. Regarding the effects of the variable tolls for the myopic case at 100 percent information level, overall travel times increase more than for any of the no information cases. The only identifiable tendency is that overall travel times increase less as the congestion threshold is increased. For the non myopic and 100 percent information case similar increases to the overall travel time as in the myopic case occur. The only difference is that the decreasing marginal increase in the travel time as the congestion level threshold increases is more clear.

Trip distances are not significantly affected by the variable tolls in the no information case. They increase for all the toll updates intervals and congestion thresholds, but they show a tendency to smaller increments as the threshold for congestion is increased. For the 100 percent information cases, the effects on trip distance are more significant. In some cases trip distances increase more than 5 percent with respect to the base case of no toll and the same information level. No clear pattern is established in the myopic case. The non myopic case exhibits a decreasing marginal increase in trip distance as the density threshold is increased.

Revenues behave in a more predictable way. As the density threshold is increased, the revenues collected decrease for all the cases. With the respect to the toll update interval, for the no information case, revenues do not present a clear tendency, sometimes shorter update intervals produce higher revenues and some other times longer update intervals produce higher revenues. The level of revenues is significantly high and in some cases more than $ 100,000. Here, an increased number of drivers will pay tolls since any link in the network can be potentially tolled. For the 100 percent information cases, except for the 30 seconds update interval, as the toll update interval is increased, the revenues collected decrease. Revenues behave as should be expected. If the density threshold is increased, a reduced number of links will exceed it and smaller number of vehicles will be paying the fees. Revenues for these cases are about 60 percent smaller than for the no information case. Revenues decrease as information improves the operation of the network, reducing congestion levels and the magnitude of the tolls.

The use of variable tolls has the potential of charging drivers for the true value of the congestion they caused when driving. However, the practical implementation of variable tolls is still uncertain. Although a continuously updated toll has been shown to improve the overall travel time for the no information case, to update tolls continuously may lead to erratic drivers' decisions in their search for cheaper routes. Besides, it is hard to think of a congestion scheme that charges drivers even though congestion levels are minimal or nonexistent.

TABLE 4.48 OVERALL TRAVEL TIMES (HOURS) FOR DIFFERENT TOLL UPDATE INTERVALS AND DENSITY LEVELS WHEN VARIABLE PRICING IS APPLIED TO ALL THE LINKS IN THE NETWORK. NO EN-ROUTE INFORMATION CASE.

	Density Level					
Interval for Toll Update	0.00	0.20	0.40	0.60	0.80	0.99
6 seconds	9618	10558	10104	10443	10370	10311
30 seconds	10261		9810	10305	10276	10345
1 minute			9878	10402	10333	10297
3 minutes	10540		10238	10222	10393	10300
5 minutes	9816		10454	10118	10324	10296
15 minutes	10434	10666	10582	10362	10314	10295

TABLE 4.49 INCREASE IN OVERALL TRAVEL TIMES (%) FOR DIFFERENT TOLL UPDATE INTERVALS AND DENSITY LEVELS WHEN VARIABLE PRICING IS APPLIED TO ALL THE LINKS IN THE NETWORK. NO EN-ROUTE INFORMATION CASE.

	Density Level					
Interval for Toll Update	0.00	0.20	0.40	0.60	0.80	0.99
6 seconds	-6.58	2.55	-1.86	1.43	0.72	0.15
30 seconds	-0.33		-4.72	0.09	-0.19	0.48
1 minute			-4.05	1.03	0.36	0.01
3 minutes	2.37		-0.56	-0.71	0.94	0.04
5 minutes	-4.66		1.54	-1.72	0.28	0.01
15 minutes	1.35	3.60	2.78	0.65	0.18	-0.01

TABLE 4.50 TOTAL TRIP DISTANCE (MILES) FOR DIFFERENT TOLL UPDATE INTERVALS AND DENSITY LEVELS WHEN VARIABLE PRICING IS APPLIED TO ALL THE LINKS IN THE NETWORK. NO EN-ROUTE INFORMATION CASE.

	Density Level					
Interval for Toll Update	0.00	0.20	0.40	0.60	0.80	0.99
6 seconds	67659	68540	67901	67834	67542	67408
30 seconds	69301		67636	67660	67482	67503
1 minute			67622	67642	67508	67479
3 minutes	68271		68106	67537	67522	67478
5 minutes	68050		67972	67573	67491	67478
15 minutes	67488	67932	68110	67720	67513	67458

TABLE 4.51 INCREASE IN TOTAL TRIP DISTANCE (%) FOR DIFFERENT TOLL UPDATE INTERVALS AND DENSITY LEVELS WHEN VARIABLE PRICING IS APPLIED TO ALL THE LINKS IN THE NETWORK. NO EN-ROUTE INFORMATION CASE.

	Density Level					
Interval for Toll Update	0.00	0.20	0.40	0.60	0.80	0.99
6 seconds	0.30	1.60	0.66	0.56	0.13	0.03
30 seconds	2.73		0.26	0.30	0.04	0.07
1 minute			0.24	0.27	0.07	0.03
3 minutes	1.21		0.96	0.12	0.09	0.03
5 minutes	0.88		0.76	0.17	0.05	0.03
15 minutes	0.04	0.70	0.97	0.39	0.08	0.00

TABLE 4.52 NETWORK REVENUE (DOLLARS) FOR DIFFERENT TOLL UPDATE INTERVALS AND DENSITY LEVELS WHEN VARIABLE PRICING IS APPLIED TO ALL THE LINKS IN THE NETWORK. NO EN-ROUTE INFORMATION CASE.

	Density Level					
Interval for Toll Update	0.00	0.20	0.40	0.60	0.80	0.99
6 seconds	72693	91524	67509	78642	73750	32198
30 seconds	110731		75396	95821	91759	74015
1 minute			62319	82645	78616	49201
3 minutes	98282		82842	82625	79067	55260
5 minutes	94355		69521	80801	77336	53571
15 minutes	94160	93136	102423	81688	75519	52237

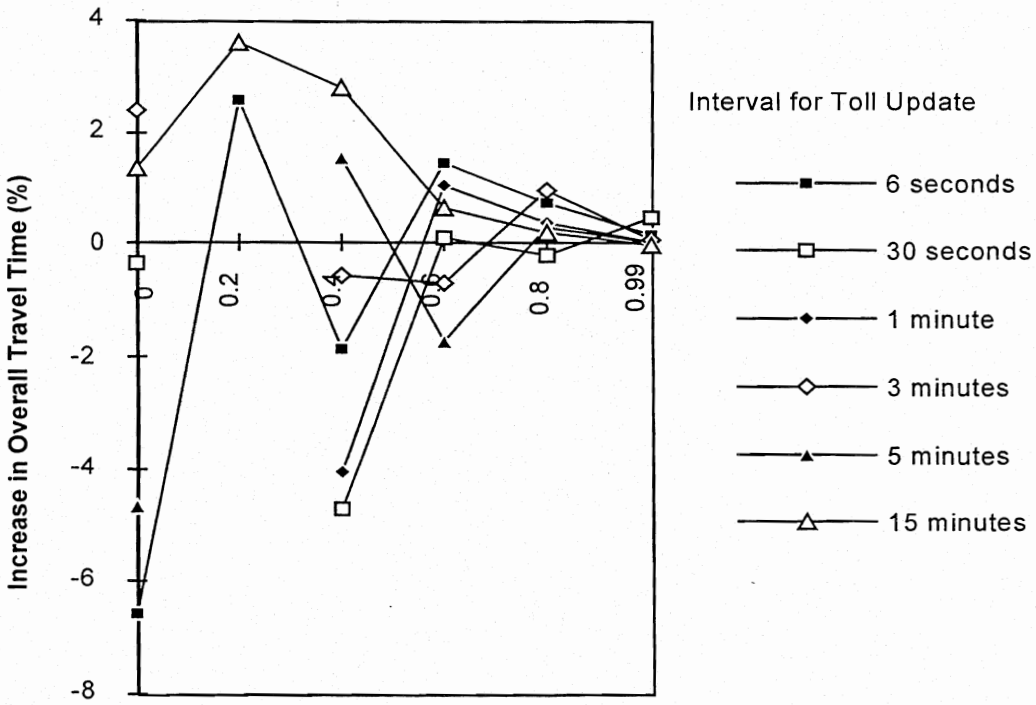

Figure 4.41 Variation of Network Total Overall Time for Different Toll Update Intervals and Density Levels When Variable Pricing is Applied To all the Links in the Network. No en-route Information Case.

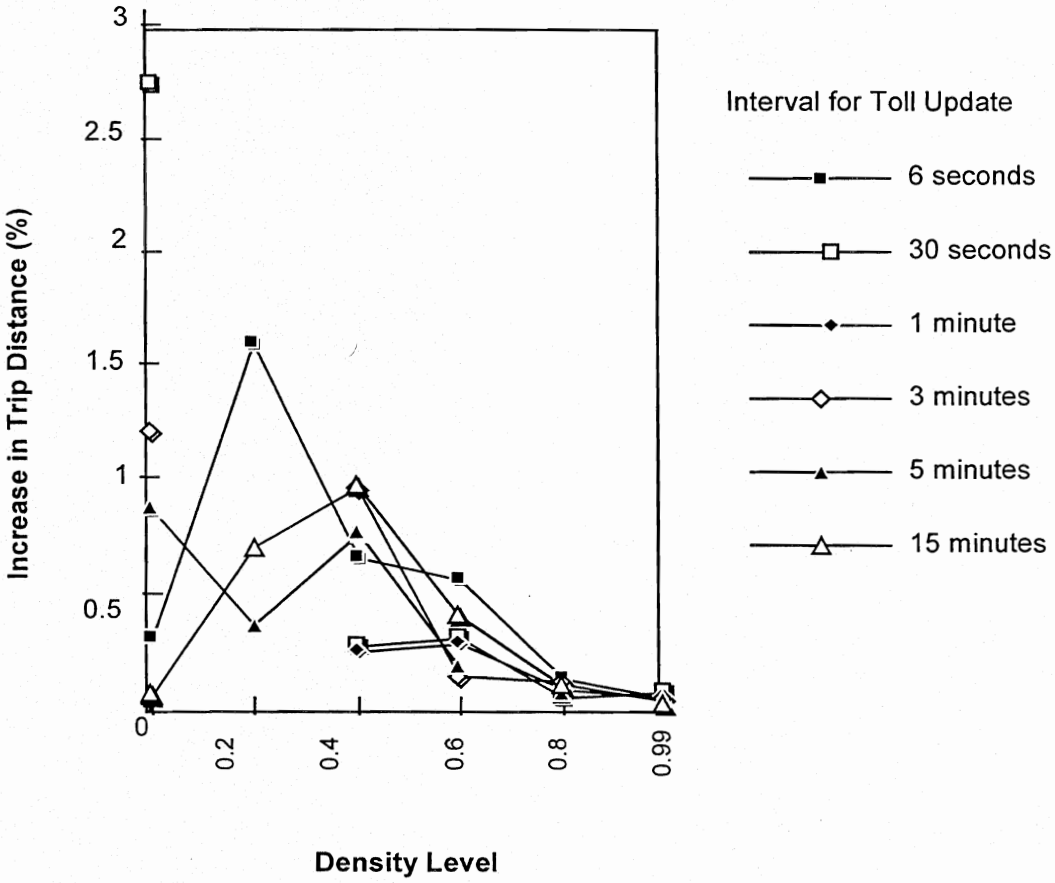

Figure 4.42 Variation of Total Trip Distance for Different Toll Update Intervals and Density Levels When Variable Pricing is Applied To all the Links in the Network. No en-route Information Case.

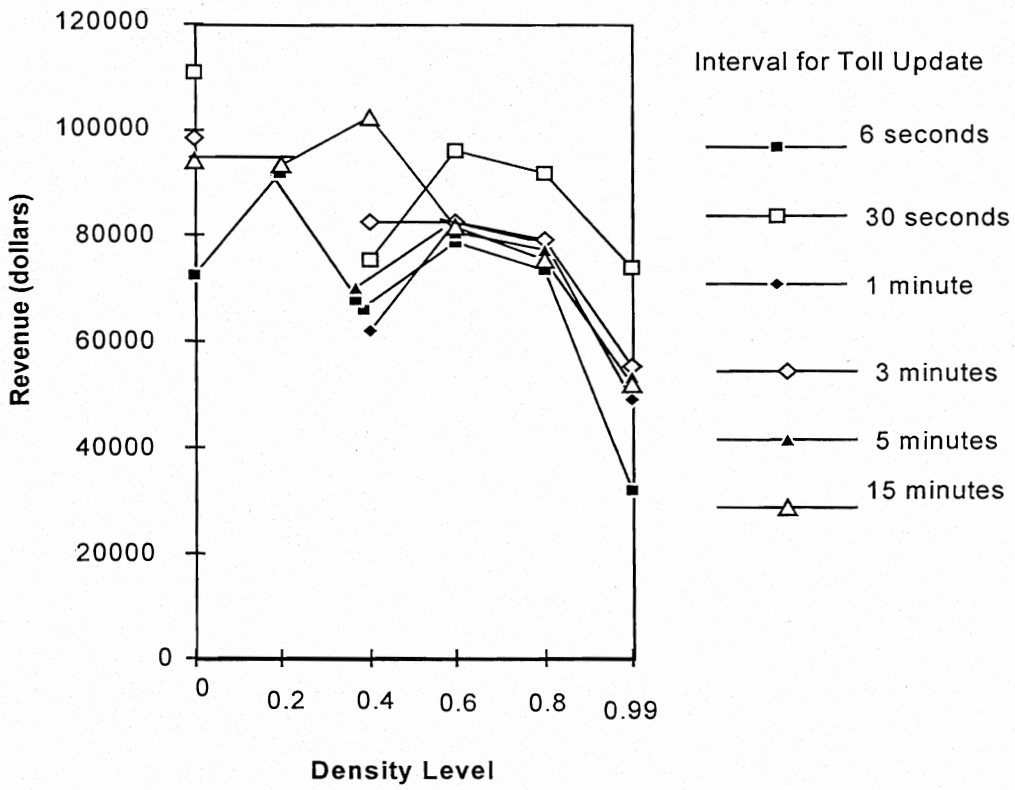

Figure 4.43 Variation in Network Revenue for Different Toll Update Intervals and Density Levels When Variable Pricing is Applied To all the Links in the Network. No en-route Information Case

TABLE 4.53 OVERALL TRAVEL TIMES (HOURS) FOR DIFFERENT TOLL UPDATE INTERVALS AND DENSITY LEVELS WHEN VARIABLE PRICING IS APPLIED TO ALL THE LINKS IN THE NETWORK. 100 % INFORMED VEHICLES. MYOPIC CASE.

	Density Level					
Interval for Toll Update	0.00	0.20	0.40	0.60	0.80	0.99
6 seconds	5820	6077	6038	5995	6028	5946
30 seconds	5932	6037	5962	5902	5973	5941
1 minute	5912	6136	5988	5816	6065	5872
3 minutes	6129	6317	6134	6008	6163	5856
5 minutes	6086	6189	6009		5944	5865
15 minutes	6017	6074	6059	6099	6025	5885

TABLE 4.54 INCREASE IN OVERALL TRAVEL TIMES (%) FOR DIFFERENT TOLL UPDATE INTERVALS AND DENSITY LEVELS WHEN VARIABLE PRICING IS APPLIED TO ALL THE LINKS IN THE NETWORK. 100 % INFORMED VEHICLES. MYOPIC CASE.

	Density Level					
Interval for Toll Update	0.00	0.20	0.40	0.60	0.80	0.99
6 seconds	-0.87	3.50	2.83	2.11	2.66	1.27
30 seconds	1.03	2.82	1.55	0.51	1.73	1.18
1 minute	0.70	4.51	1.99	-0.93	3.29	0.01
3 minutes	4.40	7.60	4.48	2.33	4.96	-0.27
5 minutes	3.66	5.42	2.35		1.23	-0.12
15 minutes	2.49	3.46	3.19	3.88	2.62	0.24

TABLE 4.55 TOTAL TRIP DISTANCE (MILES) FOR DIFFERENT TOLL UPDATE INTERVALS AND DENSITY LEVELS WHEN VARIABLE PRICING IS APPLIED TO ALL THE LINKS IN THE NETWORK. 100 % INFORMED VEHICLES. MYOPIC CASE.

	Density Level					
Interval for Toll Update	0.00	0.20	0.40	0.60	0.80	0.99
6 seconds	59208	61088	60023	60021	60689	59986
30 seconds	59521	60472	60295	60227	59709	60015
1 minute	59629	60741	61029	58992	60720	59804
3 minutes	60402	62156	61351	60842	61624	59396
5 minutes	60577	61481	60370		60048	59255
15 minutes	58801	60127	60354	61327	61356	59119

TABLE 4.56 INCREASE IN TOTAL TRIP DISTANCE (%) FOR DIFFERENT TOLL UPDATE INTERVALS AND DENSITY LEVELS WHEN VARIABLE PRICING IS APPLIED TO ALL THE LINKS IN THE NETWORK. 100 % INFORMED VEHICLES. MYOPIC CASE.

	Density Level					
Interval for Toll Update	0.00	0.20	0.40	0.60	0.80	0.99
6 seconds	0.14	3.32	1.52	1.52	2.65	1.46
30 seconds	0.67	2.28	1.98	1.87	0.99	1.51
1 minute	0.85	2.73	3.22	-0.22	2.70	1.15
3 minutes	2.16	5.13	3.77	2.91	4.23	0.46
5 minutes	2.46	3.99	2.11		1.56	0.22
15 minutes	-0.55	1.70	2.08	3.73	3.78	-0.01

TABLE 4.57 NETWORK REVENUE (DOLLARS) FOR DIFFERENT TOLL UPDATE INTERVALS AND DENSITY LEVELS WHEN VARIABLE PRICING IS APPLIED TO ALL THE LINKS IN THE NETWORK. 100 % INFORMED VEHICLES. MYOPIC CASE.

	Density Level					
Interval for Toll Update	0.00	0.20	0.40	0.60	0.80	0.99
6 seconds	31814	31848	27113	25785	28669	3760
30 seconds	41559	37308	33626	32014	30396	29010
1 minute	31557	29261	24783	25418	22472	11026
3 minutes	30687	29973	23404	24722	21623	7024
5 minutes	29727	25641	24330		21352	9302
15 minutes	24756	20995	17986	18728	18194	8119

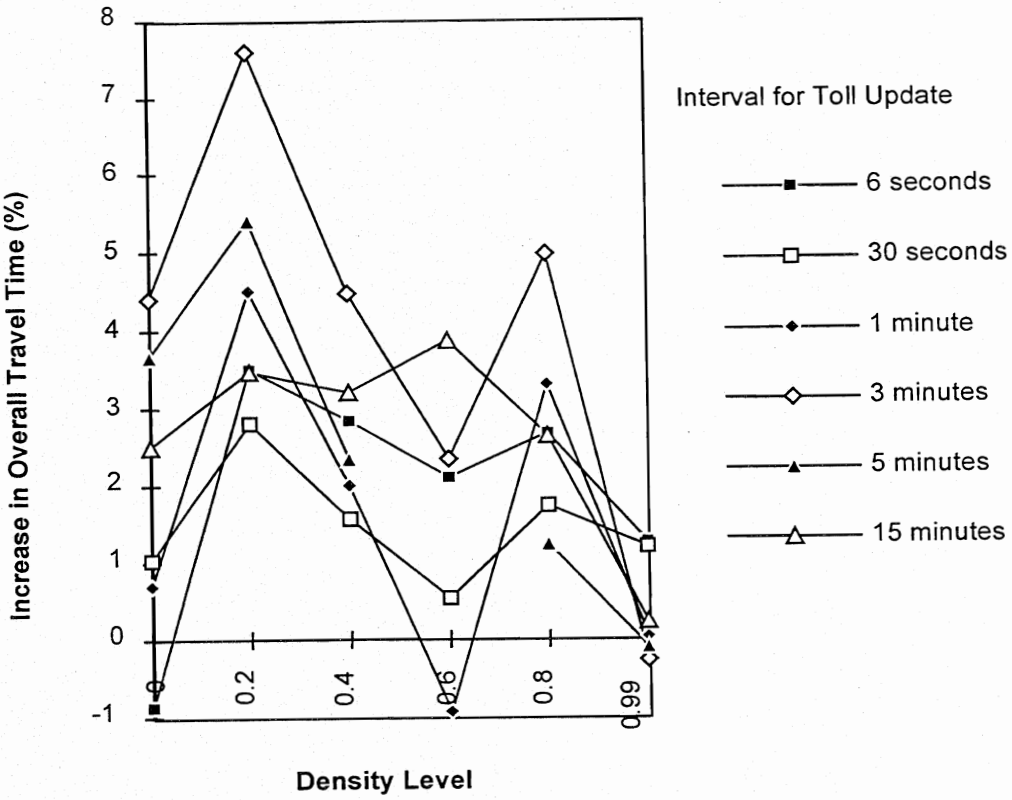

Figure 4.44 Variation of Network Total Overall Time for Different Toll Update Intervals and Density Levels When Variable Pricing is Applied To all the Links in the Network. 100% Informed Vehicles. Myopic Case.

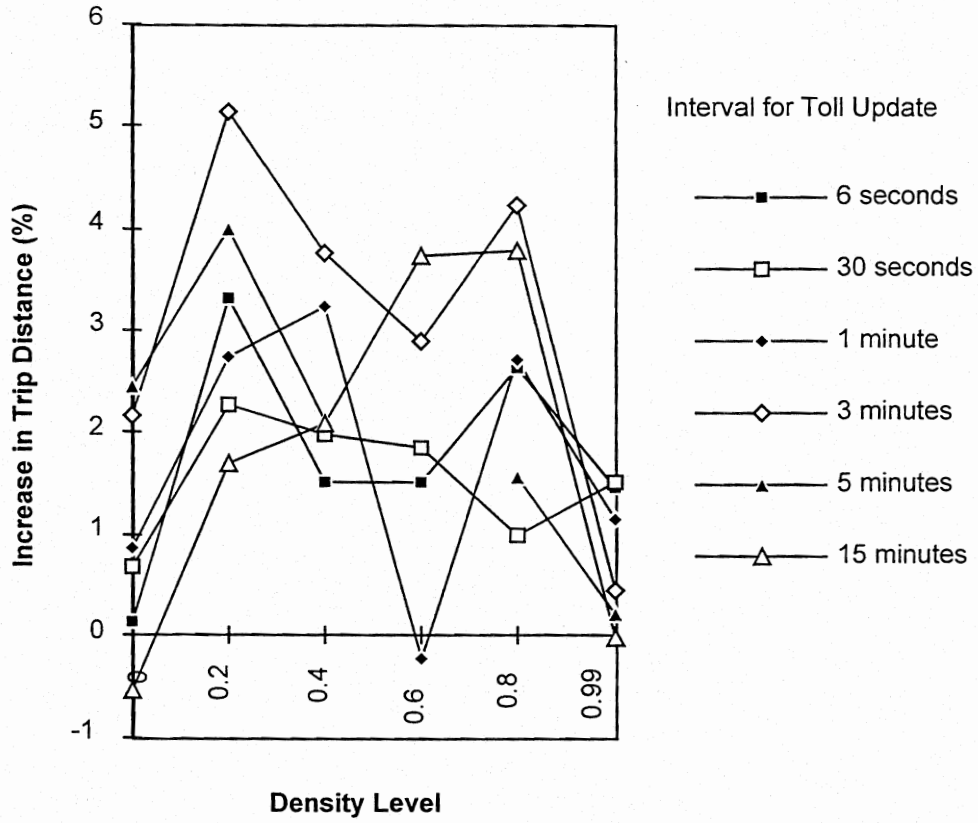

Figure 4.45 Variation of Total Trip Distance for Different Toll Update Intervals and Density Levels When Variable Pricing is Applied To all the Links in the Network. 100% Informed Vehicles. Myopic Case.

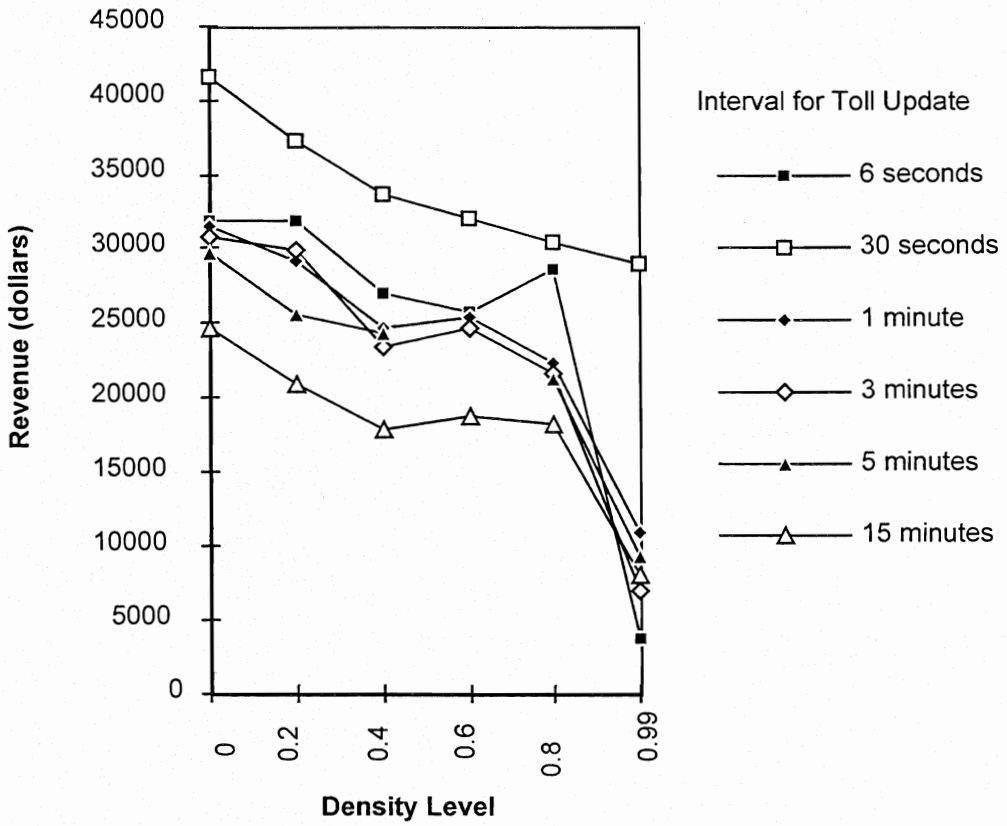

Figure 4.46 Variation in Network Revenue for Different Toll Update Intervals and Density Levels When Variable Pricing is Applied To all the Links in the Network. 100% Informed Vehicles. Myopic Case.

TABLE 4.58 OVERALL TRAVEL TIMES (HOURS) FOR DIFFERENT TOLL UPDATE INTERVALS AND DENSITY LEVELS WHEN VARIABLE PRICING IS APPLIED TO ALL THE LINKS IN THE NETWORK. 100 % INFORMED VEHICLES. NON-MYOPIC CASE.

	Density Level					
Interval for Toll Update	0.00	0.20	0.40	0.60	0.80	0.99
6 seconds	5851	5881	5926	5734	5762	5783
30 seconds	5950	6012	5805	5857	5823	5785
1 minute	5988	5906	5840	5868	5816	5758
3 minutes	5966	6094	6085	5830	5833	5807
5 minutes	5941	6137	5958	5893	5830	5784
15 minutes	5885	6024	5944	5903	5779	5776

TABLE 4.59 INCREASE IN OVERALL TRAVEL TIMES (%) FOR DIFFERENT TOLL UPDATE INTERVALS AND DENSITY LEVELS WHEN VARIABLE PRICING IS APPLIED TO ALL THE LINKS IN THE NETWORK. 100 % INFORMED VEHICLES. NON-MYOPIC CASE.

	Density Level					
Interval for Toll Update	0.00	0.20	0.40	0.60	0.80	0.99
6 seconds	1.70	2.23	3.00	-0.34	0.15	0.53
30 seconds	3.42	4.50	0.90	1.80	1.22	0.56
1 minute	4.08	2.66	1.51	2.00	1.10	0.08
3 minutes	3.69	5.92	5.77	1.34	1.39	0.94
5 minutes	3.28	6.67	3.56	2.43	1.34	0.54
15 minutes	2.30	4.71	3.33	2.60	0.44	0.39

TABLE 4.60 TOTAL TRIP DISTANCE (MILES) FOR DIFFERENT TOLL UPDATE INTERVALS AND DENSITY LEVELS WHEN VARIABLE PRICING IS APPLIED TO ALL THE LINKS IN THE NETWORK.
100 % INFORMED VEHICLES. NON-MYOPIC CASE.

	Density Level					
Interval for Toll Update	0.00	0.20	0.40	0.60	0.80	0.99
6 seconds	59672	60144	60352	59099	59121	59340
30 seconds	59612	60410	59365	59497	59640	59673
1 minute	60359	59914	59573	59611	59662	58893
3 minutes	60223	61625	62149	59983	59930	59363
5 minutes	60084	61418	60569	60442	59945	59045
15 minutes	59701	60203	59562	59636	58819	58761

TABLE 4.61 INCREASE IN TOTAL TRIP DISTANCE (%) FOR DIFFERENT TOLL UPDATE INTERVALS AND DENSITY LEVELS WHEN VARIABLE PRICING IS APPLIED TO ALL THE LINKS IN THE NETWORK.
100 % INFORMED VEHICLES. NON-MYOPIC CASE.

	Density Level					
Interval for Toll Update	0.00	0.20	0.40	0.60	0.80	0.99
6 seconds	1.54	2.34	2.69	0.56	0.60	0.97
30 seconds	1.44	2.79	1.01	1.24	1.48	1.54
1 minute	2.71	1.95	1.37	1.43	1.52	0.21
3 minutes	2.47	4.86	5.75	2.07	1.98	1.01
5 minutes	2.24	4.51	3.06	2.85	2.00	0.47
15 minutes	1.59	2.44	1.35	1.48	0.09	-0.01

TABLE 4.62 NETWORK REVENUE (DOLLARS) FOR DIFFERENT TOLL UPDATE INTERVALS AND DENSITY LEVELS WHEN VARIABLE PRICING IS APPLIED TO ALL THE LINKS IN THE NETWORK.
100 % INFORMED VEHICLES. NON-MYOPIC CASE.

	Density Level					
Interval for Toll Update	0.00	0.20	0.40	0.60	0.80	0.99
6 seconds	33958	31370	28190	24023	22743	2202
30 seconds	41667	39593	32543	31635	29157	25875
1 minute	36319	30508	24915	23645	21640	4100
3 minutes	30238	30838	24824	22650	21034	3461
5 minutes	28827	26812	23845	24067	21616	5450
15 minutes	22608	27281	21141	18512	16093	3311

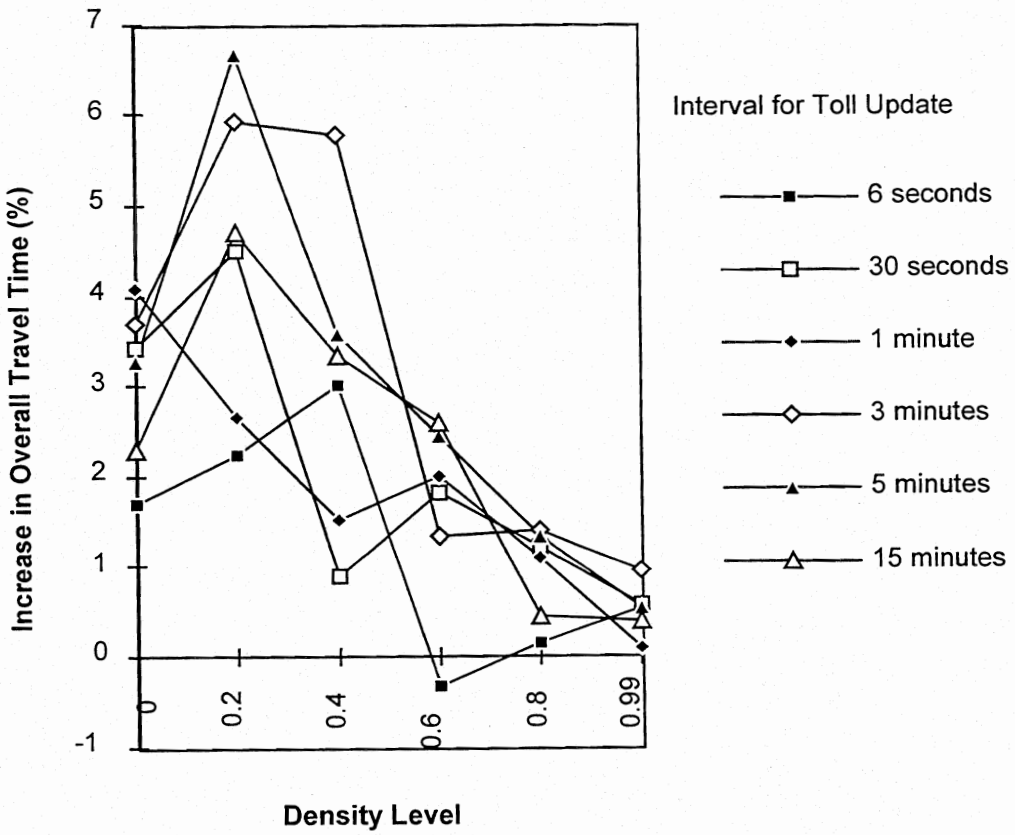

Figure 4.47 Variation of Network Total Overall Time for Different Toll Update Intervals and Density Levels When Variable Pricing is Applied To all the Links in the Network. 100% Informed Vehicles. Non Myopic Case.

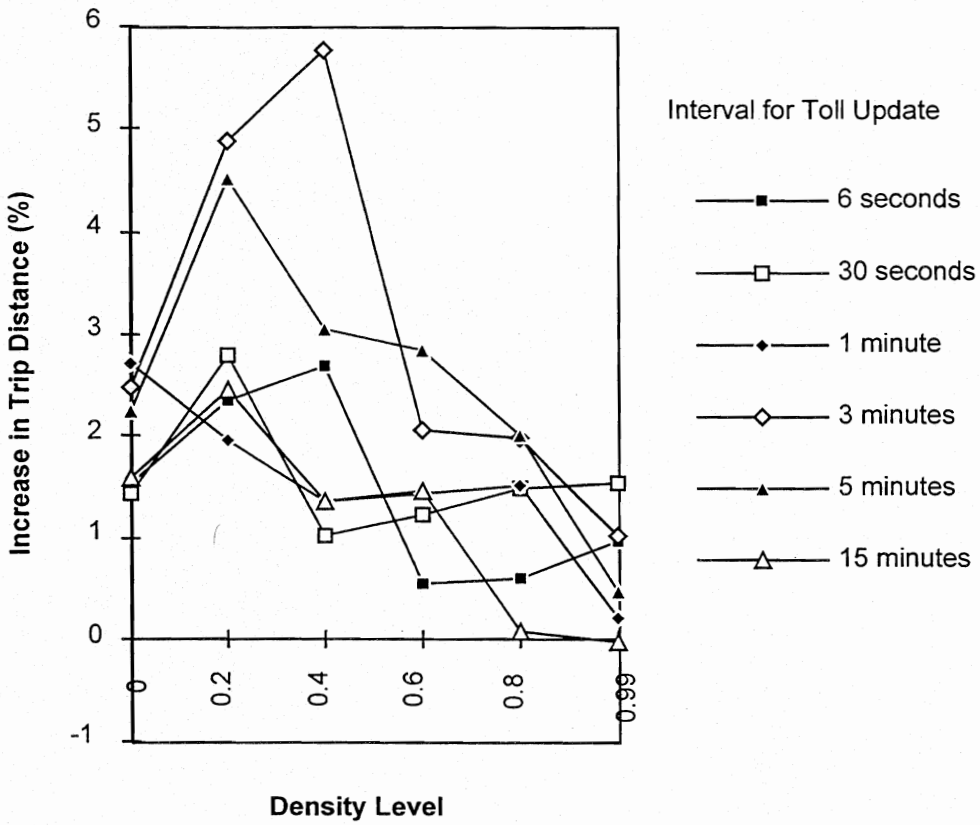

Figure 4.48 Variation of Total Trip Distance for Different Toll Update Intervals and Density Levels When Variable Pricing is Applied To all the Links in the Network. 100% Informed Vehicles. Non Myopic Case.

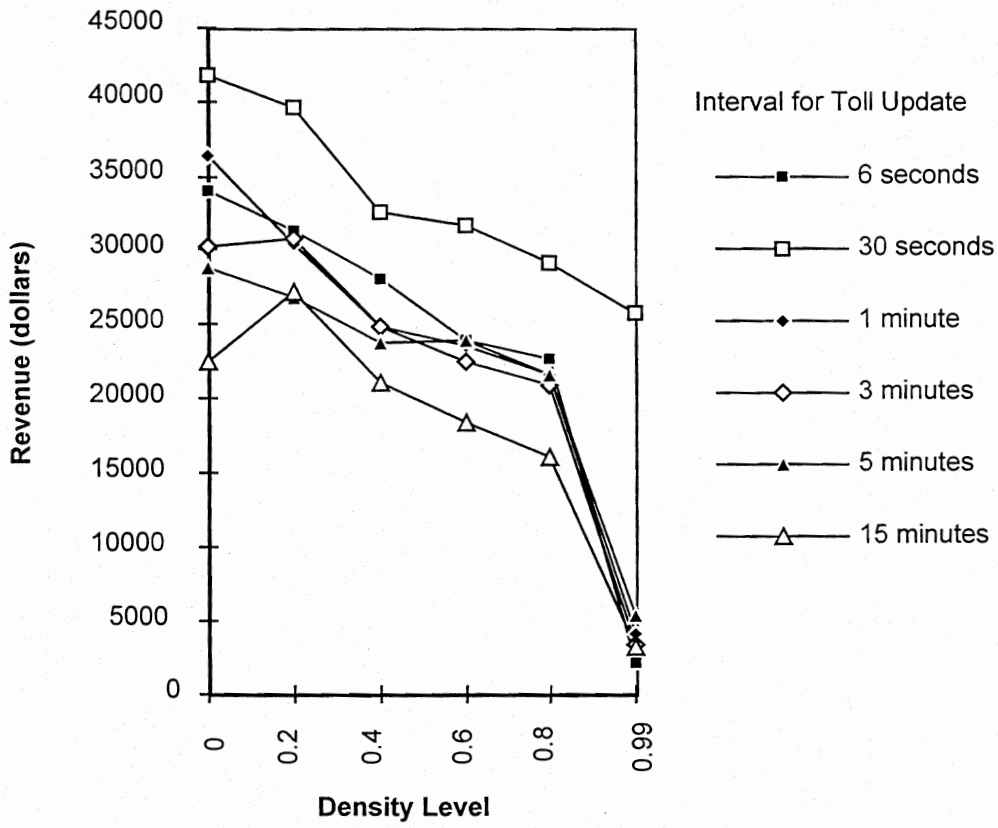

Figure 4.49 Variation in Network Revenue for Different Toll Update Intervals and Density Levels When Variable Pricing is Applied To all the Links in the Network. 100% Informed Vehicles. Non Myopic Case.

CONCLUSION

This chapter has reviewed different pricing schemes for (constant and variable) prices. While the experiments performed in the chapter did not cover all the possible cases, solutions for different schemes should no be drastically different from the ones found here.

The effects of constant tolls, as anticipated, are easier to predict. Drivers are more comfortable with this kind of pricing and can take decisions based on easier to understand information. They wont be surprised by sudden changes in the toll levels, as in the case of variable tolls, that are not only their responsibility. Constant tolls will affect the operation of the network in a more predictable way. Constant tolls, however, do not relate to the level of congestion and penalized those using the priced links or entering the restricted zones without

regard to their contribution to the level of congestion. In some cases the penalty can be much higher than their contribution and in others can be too small.

Variable tolls can better reflect the true costs of congestion. However, its practical implementation is still uncertain. Technical considerations are no longer an issue, but drivers will not be enthusiastic with continuously changing tolls. Unpredicted and in most cases negative effects make variable tolls less attractive. The only consistently attractive feature of variable tolls is the high revenues that can be obtained. This feature can be particularly attractive to operating agencies but, not to drivers.

The methodology developed and illustrated in this chapter allows careful investigation of the network level implications of different pricing schemes, both spatially and temporally.

CHAPTER 5: PRICING UNDER A USER EQUILIBRIUM ASSIGNMENT RULE

Chapter four has presented the analysis of the effects of constant and variable pricing when vehicles are initially assigned to the prevailing best route and follow during their journey a boundedly rational behavioral rule. However, that kind of assignment is only one of the assignment rules that could be followed to characterize the distribution of traffic in a network. A more common assumption in the analysis of traffic networks is that each driver tries to minimize his/her travel time when going from an origin to a destination. As stated by Wardrop's first principle, by following their individual desires, drivers lead the system to a stable condition where *no traveler can improve his travel time by unilaterally changing routes.* This equilibrium state is known as the *user-equilibrium* (UE) condition.

By incorporating pricing in the time dependent user equilibrium traffic assignment algorithm developed by Peeta (1994), this chapter presents the analysis of the effects of constant and variable pricing under a user equilibrium assignment rule. The first section reviews the concepts of user equilibrium in the time-dependent case. It describes the algorithm used for the solution of the user equilibrium problem and the changes made to incorporate pricing. The second section describes the experiments with constant prices. The third section presents the experiments with variable prices. The fourth section presents the conclusions of this chapter.

USER EQUILIBRIUM

It is commonly assumed that the user equilibrium state is attained under conditions where each driver has full information about the travel times along available routes, and that they behave identically and rationally in selecting the "best" route. When these conditions are relaxed, and perceived travel times are considered instead of the actual times, another equilibrium is defined. This equilibrium is known as the stochastic user equilibrium (SUE), and is characterized as a state where *no traveler* believes *that his travel time can be improved by unilaterally changing routes* (Sheffi, 1985; Daganzo and Sheffi, 1977). When perceived travel times are equal to actual travel times, both definitions of equilibrium will lead to the same distribution of flows.

The analysis of the user equilibrium starts by considering steady-sate conditions. Beckmann et al. (1956) provided the first formulation of the static user equilibrium problem with fixed demand as a mathematical program. They also proved, for their formulation, the existence and uniqueness of the solution.

Sheffi (1985) provides a comprehensive review of the static UE problem. Although extensively researched, the static UE analysis is useful only if the flows and travel times can be assumed time-invariant during the period under study. As such, static conditions fail to consider

the dynamics of congestion and are not adequate for real-time applications in which traffic changes continuously. These limitations have motivated the development of time-dependent assignment models, where time is explicitly included as a new dimension in the formulation of the problem. Peeta (1994) presents a thorough review of contributions to the UE problem for the time-dependent case. Here, only recent developments are presented.

Wie et al. (1995) present a discrete time formulation of the dynamic user equilibrium problem in which route and departure time choices are simultaneous. Their analysis uses the so called link exit flow functions to calculate unit path costs, this being the main limitation of their formulation.

Chen and Hsueh (1998), like Wie et al. (1995), followed a variational inequality approach to formulate a discrete time, link-based, dynamic user equilibrium problem. Their concern is only the determination of the optimal route choices. The main limitation of their approach continues to be the use of exit flow functions and restrictive assumptions regarding the traffic network.

Although a number of researchers have been trying to find an analytical formulation and solution approach for dynamic traffic assignment, and in particular for the user equilibrium time-dependent traffic assignment (UETDTA) problem, the reasons cited by Peeta (1994) for the use of a simulation-based approach continue to be valid. Analytical formulations still require restrictive assumptions or simplifications that make them impractical for realistic large traffic networks.

Formulation of the UETDTA Problem with Pricing

The objective of the UETDTA problem with pricing is to extend Wardrop´s first principle to the time-dependent case, i.e., to find a stable condition where no traveler can reduce his/her travel cost by unilaterally changing routes, where cost is defined as the (value of the) total travel time for each driver plus any toll paid. The conditions are stated as:

(1) All paths $k^* \in K_{ij}$, connecting an O-D pair (i-j), that are assigned vehicles in any time interval τ, have the same experienced path travel cost $C^t_{ijk^*}$ (equal to q^{*t}_{ij}) .

(2) All paths connecting a given O-D pair, and that are not assigned vehicles in a given time interval, have experienced travel costs greater than or equal to θ^{*t}_{ij} .

Mathematically, these conditions can be expressed as:

$$r^t_{ijk} \, (C^t_{ijk} - \theta^{*t}_{ij}) = 0, \quad \forall \; i, j, k, \tau \qquad (5.1)$$

$$(C^t_{ijk} - \theta^{*t}_{ij}) \geq 0, \forall \; i, j, k, \tau \qquad (5.2)$$

where

r^{t}_{ijk} = number of vehicles that depart along path k = 1,..., K_{ij} between i and j at time τ

Problem Statement . As in chapter three, assume that a matrix of known time-dependent O-D origin-destination demands expressed as the number of vehicle trips r^{t}_{ij} leaving node i for node j in departure time interval τ, $\forall$ i $\in$ I, j $\in$ J and τ = 1,....., T, is loaded onto a traffic network, represented by a directed graph G = {N, A}, where N is the set of nodes N={1,2,...,q,...}, and A represents the set of directed arcs joining the nodes. A node can be a trip origin and/or a destination and/or a junction of physical links. A network with multiple origins i$\in$I and destinations j$\in$J is considered for generality. The overall planning period is divided into T small equal time intervals t=1,..., T. Find the time-dependent assignment of vehicles to network paths and corresponding arcs in such a way that each vehicle uses the least cost route. Then, the objective is to find the number of vehicles r^{t}_{ijk} that depart along path k = 1,..., K_{ij} between i and j at time τ, $\forall$ i $\in$ I, j $\in$ J and τ = 1,....., T, as well as the associated numbers of vehicles $x^{\tau ta}_{ijk}$ on each arc a $\in$ A in each time interval t of the duration of interest. τ and t are used to differentiate between the departure time of a vehicle and the current time. $\tau \leq t$ in the definition of $x^{\tau ta}_{ijk}$.

Find

r^{τ}_{ijk}, $\forall$ i, j and τ = 1,....., T

Such that

$$r^{\tau}_{ijk}\,(C^{\tau}_{ijk} - \theta^{*\tau}_{ij}) = 0, \qquad \forall\ i, j, k, \tau$$

$$(C^{\tau}_{ijk} - \theta^{*\tau}_{ij}) \geq 0, \qquad \forall\ i, j, k, \tau$$

conditions are fulfilled.

Subject to:

$$r^{\tau}_{ij} = \sum_{k} r^{\tau}_{ijk}, \qquad \forall\ i, j, \tau \qquad (5.3)$$

$$\sum_{b} d^{tb} = \sum_{c} m^{tc} + I_n^t - O_n^t, \quad \forall\, t, n, b \in B(n), c \in C(n) \qquad (5.4)$$

$$x^{ta} = x^{t-1a} + d^{t-1a} - m^{t-1a}, \quad \forall\, t, a \qquad (5.5)$$

$$x^{ta} = \sum_{k}\sum_{\tau}\sum_{i}\sum_{j} (r_{ijk}^{\tau} \cdot \delta_{ijk}^{\tau ta}), \quad \forall\, t, a \qquad (5.6)$$

$$T_{ijk}^{\tau} = \sum_{t}\sum_{a} [\delta_{ijk}^{\tau ta} \cdot \Delta], \quad \forall\, i, j, k, \tau \qquad (5.7)$$

$$\delta_{ijk}^{\tau ta} = F[(r_{ijk}^{\tau}), \forall\, i, j, k, \tau] \quad \forall\, i, j, k, \tau, t, a \qquad (5.8)$$

$$d^{ta} = \sum_{k}\sum_{\tau}\sum_{i}\sum_{j} d_{ijk}^{\tau ta}, \quad \forall\, t, a \qquad (5.9)$$

$$m^{ta} = \sum_{k}\sum_{\tau}\sum_{i}\sum_{j} m_{ijk}^{\tau ta}, \quad \forall\, t, a \qquad (5.10)$$

$$I_n^t = \sum_{j} r_{nj}^{t}, \quad \forall\, t, n \in I \qquad (5.11)$$

$$O_n^t = \sum_{k}\sum_{\tau}\sum_{i}\sum_{c} m_{ink}^{\tau tc}, \quad \forall\, t, n \in J, c \in C(n) \qquad (5.12)$$

$$\tau \le t \qquad (5.13)$$

$$\delta_{ijk}^{\tau ta} = 0 \text{ or } 1, \quad \forall\, i, j, k, \tau, t, a \qquad (5.14)$$

$$\text{All variables} \ge 0 \qquad (5.15)$$

where

r_{ijk}^{τ} = number of vehicles who wish to depart from origin i to destination j in period τ assigned to path k

C_{ijk}^{τ} = experienced path travel cost

$\theta_{ij}^{*\tau}$ = experienced path travel cost

r_{ij}^{τ} = number of vehicles who wish to depart from origin i to destination j in period τ

d^{ta} = total number of vehicles which enter link a in period t

m^{ta} = total number of vehicles which exit link a in period t

I_n^t = number of vehicles generated at node n in period t

O_n^t = number of vehicles exiting the network through node n in period t

x^{ta} = total number of vehicles on link a at the beginning of period t

$\delta_{ijk}^{\tau ta}$ = time-dependent link-path incidence indicator, equal to 1 if vehicles going from i to j assigned to path k at time τ are on link a in period t, i.e.,

$$\delta_{ijk}^{\tau ta} = \begin{cases} 1, \text{if } r_{ijk}^{\tau} \text{ is on arc a during period t} \\ 0, \text{if arc a does not belong to path k} \\ 0, \text{if } \tau > t \\ 0, \text{if } r_{ijk}^{\tau} \text{ is not on arc a during period t} \end{cases}$$

T_{ijk}^{τ} = experienced path travel time for vehicles going from i to j that are assigned to path k at time τ

T' = total duration (peak period) for which assignments are to be made

Δ = length of a time interval (equal to T'/T)

d^{ta} = total number of vehicles which enter link a in period t

$d_{ijk}^{\tau ta}$ = number of vehicles going from origin i to destination j assigned to path k in period τ which enter arc a in period t

m^{ta} = total number of vehicles which exit link a in period t

$m_{ijk}^{\tau ta}$ = number of vehicles going from origin i to destination j assigned to path k in period τ which exit link a in period t

C(n) = set of links incident to node n

B(n) = set of links incident from node n

This formulation is not amenable to an analytic solution. Equation (5.8) does not have a known explicit expression that considers all the complex time-dependent interactions among vehicles. Most existing time-dependent formulations make unacceptable simplifications regarding this equation in order to have a tractable problem. Peeta's solution algorithm uses a simulation model that captures the essential dynamic phenomena and circumvents its analytical intractability.

Constraints (5.3) ensure that all the vehicles desiring to depart at the origin nodes $i \in I$ will be assigned to a path. Constraints (5.4) denote the conservation of vehicles at nodes. Vehicles cannot be stored at nodes, and at any time t on a node n, the number of vehicles entering all links incident to the node should be equal to the sum of the number of vehicles exiting from all links incident to that node plus the net generation. Constraints (5.5) express the conservation of vehicles along links, and state that the number of vehicles on an arc in one period equals the number of vehicles at the beginning of the previous period, minus the outflow plus the inflow.

Constraints (5.6), (5.7) and (5.8) use the time-dependent link-path incidence variables $\delta_{ijk}^{\tau ta}$ to characterize dynamic assignment problems. Constraints (5.11) are time-dependent incidence relationships. They express the number of vehicles on a link, x^{ta}, in terms of path vehicle assignments r_{ijk}^{τ}. They relate link-based constraints (5.4) and (5.5) to the path-based decision variables. They are nonlinear due to the non-linear nature of (5.8). Constraints (5.7) show the calculation of the path travel times using the link-path incidence variables. The number of time steps in which $\delta_{ijk}^{\tau ta}$ takes a value of 1 implies the number of discrete time steps that the corresponding "packet" of vehicles r_{ijk}^{τ} spend in the system, and multiplying with Δ gives the actual travel time for that packet. By using constraints (5.7) and the time-dependent link-path incidence variables $\delta_{ijk}^{\tau ta}$, it is possible to compute the actual travel time of vehicles. This avoids the need for analytical link performance functions.

Constraint (5.9) states that the number of vehicles entering a link at time t is the total of all the vehicles following path k from origin i to destination j and departing at time τ entering the link a at time t. Constraint (5.10) states that the number of vehicles leaving a link at time t is the total of all the vehicles following path k from origin i to destination j and departing at time τ leaving the link a at time t. In the same form constraints (5.11) and (5.12) define conditions for vehicles entering or exiting the network through link n at time t. Constraint (5.13) restricts vehicles to leave at time τ before or at most at the current interval. Constraint (5.14) is the time-dependent incidence variable. Constraint (5.15) restricts the variables to be all positive or zero to have a physically meaningful problem.

Solution Algorithm for the UETDTA Problem with Pricing. The solution algorithm follows closely the solution methods for the static UE assignment problem. The Solution algorithm framework for the UETDTA with pricing is presented in Figure 5.1. It consists of an iterative procedure in which DYNASMART is used to simulate the traffic.

The least cost paths algorithm requires the time-dependent link average travel times $T^{ta}(.)$ and constant or variable tolls as input data ($CT^{a}(.)$ or $VT^{ta}(.)$). Paths are calculated using the time-dependent least cost path algorithm proposed by Ziliaskopoulos and Mahmassani (1992a, 1992b).

The steps of the UETDTA with pricing algorithm are:

(1) Set the iteration counter $i = 0$. Assign the given O-D desires r_{ij}^{τ} $\forall$ i, j and τ = 1,....., T, to a time-dependent initial set of feasible paths K(I). Hence, the initial solution is given by the assignment $r_{ijk}^{\tau,0}$, $\forall$ i, j, τ = 1,....., T, and k $\in$ K(I).

(2) Simulate, using DYNASMART, the traffic network under the set of path assignments $r_{ijk}^{\tau,i}$ for the entire duration of interest and evaluate the function F(.) in equation (5.13) of the formulation. A number of time-dependent link level performance measures are obtained as simulation output including the link travel times T^{ta}, constant or variable tolls CT^{a} or VT^{ta} and the number of vehicles on links x^{ta}, $\forall$ t, a. Aggregate network level performance measures like the total system travel time $\mathbf{T}(\mathbf{r})$ are also obtained.

(3) Compute the time-dependent least cost paths k^{*}, $\forall$ i, j and τ. Hence, k^{*} represents a path on which $C_{ijk^{*}}^{t} \leq C_{ijk}^{t}$, $\forall$ k $\in$ K_{ij}, where K_{ij} represents the set of feasible paths from i to j. The path $k^{*} \in K_{ij}$; however, unless k^{*} is an already existing path for a given i, j and τ, it does not belong to the set K_{ij}^{t}

(4) Perform an all-or-nothing assignment by assigning all O-D desires r_{ij}^{t} for a given i, j and τ to the corresponding least cost path k^{*}. This gives the auxiliary number of vehicles on paths, $y_{ijk}^{\tau,i}$ $\forall$ i, j and τ.

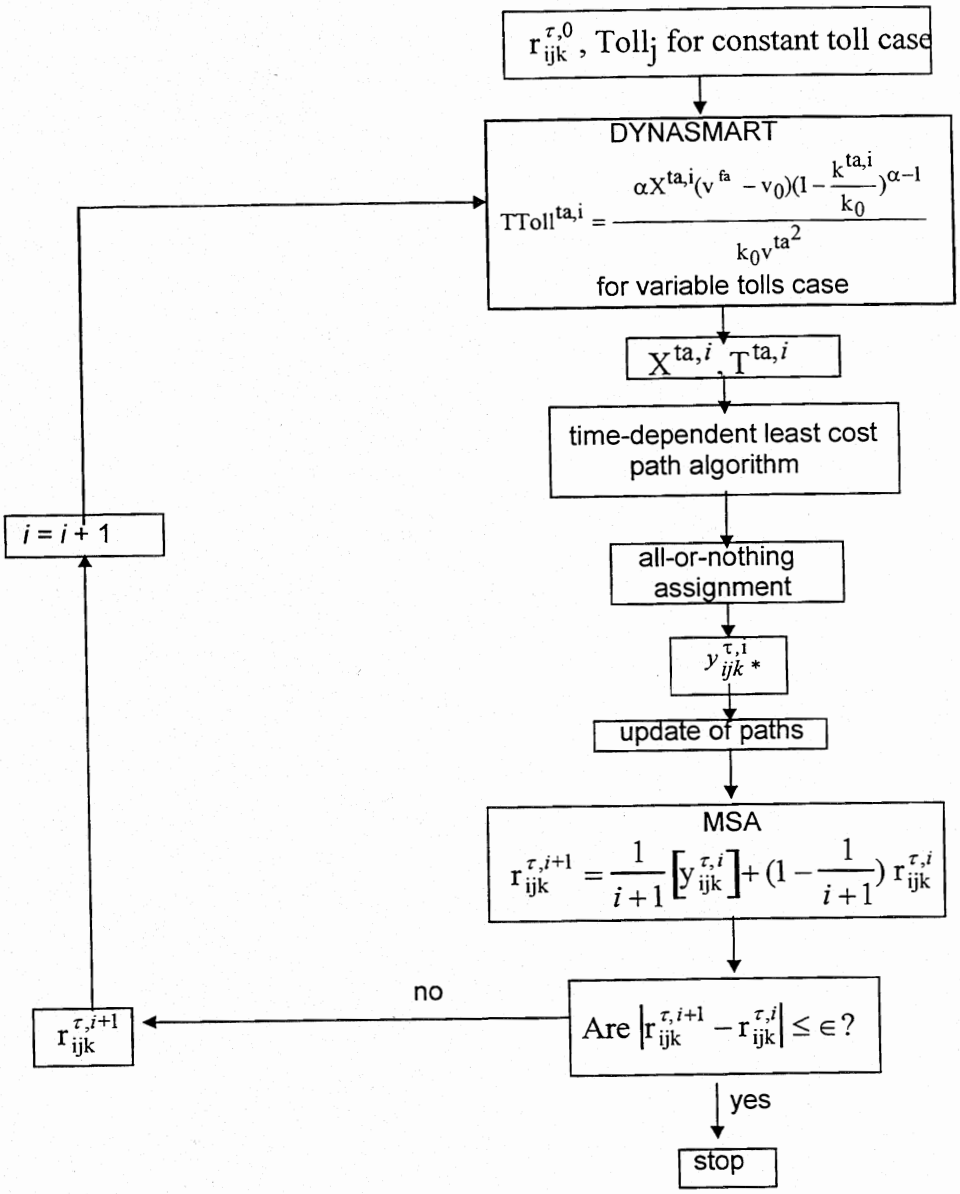

Figure 5.1 Algorithmic Framework for the UETDTA with Pricing Solution Algorithm.

(5) Update the set of paths by checking if $k^* \in K_{ij}^t$, and including it if it does not, $\forall$ i, j and τ. The path assignments for the next iteration $r_{ijk}^{\tau,i+1}$ are obtained through a convex combination of the current path assignments $r_{ijk}^{\tau,i}$ and the auxiliary path assignments $y_{ijk}^{\tau,i}$ using the Method of Successive Averages (MSA) $\forall$ i, j, k and τ:

$$r_{ijk}^{\tau,i+1} = \frac{1}{(i+1)}\left[y_{ijk}^{\tau,i}\right] + (1 - \frac{1}{(i+1)})\left[r_{ijk}^{\tau,i}\right] \qquad (5.16)$$

(6) Check the difference in the number of vehicles assigned to various paths over successive iterations. The path assignments for the next iteration $r_{ijk}^{\tau,i+1}$ are compared with the current path assignments $r_{ijk}^{\tau,i}$ $\forall$ i, j, k and τ.

$$\left| r_{ijk}^{\tau,i+1} - r_{ijk}^{\tau,i} \right| \leq \in \qquad (5.17)$$

The number of cases, N($\in$), in which their absolute difference is greater than a value $\in$ is recorded.

(7) (i) If N($\in$) $\leq \Omega$, where Ω is a pre-set upper bound on the number of violations of (5.22), convergence is assumed. Terminate the algorithm and output the path assignments $r_{ijk}^{\tau,i+1}$ as the solution to the UETDTA with pricing problem.

(ii) If N($\in$) > Ω, the convergence criterion is not satisfied. Update $i = i+1$. Go to Step 2 with the new current path assignments $r_{ijk}^{\tau,i+1}$.

Description of Experiment

As in chapter four, three pricing schemes are considered to study the effect of pricing under a UE assignment rule: (1) a fixed charge for using a link; (2) a fixed charge for crossing a boundary in a restricted zone of the network, and; (3) a variable price based on the level of congestion on the link.

The Test Network

This section describes the characteristics of the network used in the numerical experiments.

General Characteristics. The hypothetical test network (shown in Figure 5.2) is very similar to the one used for the experiments of chapter four, with the following characteristics: (1) all arcs in the network, other than the freeway links, are two-directional with two lanes in each direction; (2) the freeway links have three lanes; (3) the length of the entrance and exit ramp links is 1500 feet; the length for links 30-34, 31-25, 31-35, 34-30, 34-36, 35-31 and 36-34 is 3735 feet; all other links in the network are 2640 feet long; (4) free speeds are 35 mph for links 23-22, 23-24, 23-29, 24-18, 24-23, 24-30, 25-19, 25-26, 25-31, 26-20, 26-25, 26-27, 26-31, 27-21, 27-26, 27-28, 27-32, 27-39, 28-22, 28-27, 28-29 and 28-33, and 20 mph for all other links. Although the

free speeds may seem low and the network would represent more a local street configuration, they do not influence the effect of pricing in total link cost calculations.

As in chapter 4, the value of travel time is assumed constant and equal to six dollars per hour.

Demand Pattern. Figure 5.3 shows the loading profile used for the simulations of the effects of pricing under a UE assignment rule. Vehicles are generated over a 35-minute period. Statistics are collected for the vehicles generated after the first five minutes.

For the base demand level, the network is loaded with 421 vehicles during the first five-minute interval. In the second interval load is increased to 594 vehicles. In the third interval, demand is 719 vehicles. Demand peaks in the fourth with 826 vehicles, followed by a decrease to 756 vehicles in the fifth interval. In the sixth interval demand reduces to 590 vehicles, followed by 425 vehicles in the last loading interval.

Demand Levels. The loading factors (LF), defined as the ratio of the total number of vehicles, generated to a base value of 4331 vehicles, represent different levels of congestion for the network. The load factors used in the simulations for this research were low of 1.00, medium of 2.00 and high of 3.00. The corresponding number of generated vehicles is given in Table 5.1.

Constant Pricing Schemes Considered and Price Levels

The pricing schemes considered are the same as those of chapter four. The UE algorithm was run changing prices in ten-cent increments for the cases of (1) a pair of opposite direction links (9-15 and 15-9) with alternative routes and (2) a single link with no alternative routes for some of the vehicles. In the case of a restricted zone (zone 3 in Figure 5.2), the prices were changed in 20-cent increments. Toll increments were applied from zero up to the level at which vehicles using the priced arcs have no alternative but to pay the toll. In the case of arcs 9-15 and 15-9, the number of vehicles is zero. For the case of arc 38-37, the number depends on the loading factor used, which gives the number of vehicles that have node 37 as destination; similarly the case of zone 3.

Variable Pricing Schemes Considered

As in chapter four, the only variable pricing scheme consisted of applying pricing to all the links of the traffic network. The same test network for the constant toll experiments is used, with the same demand pattern and loading factors. Prices were updated every one minute interval. The congestion level thresholds in the variable pricing formula (Equation 4.11) were set at 0, 1,10, 25, and 50 percent.

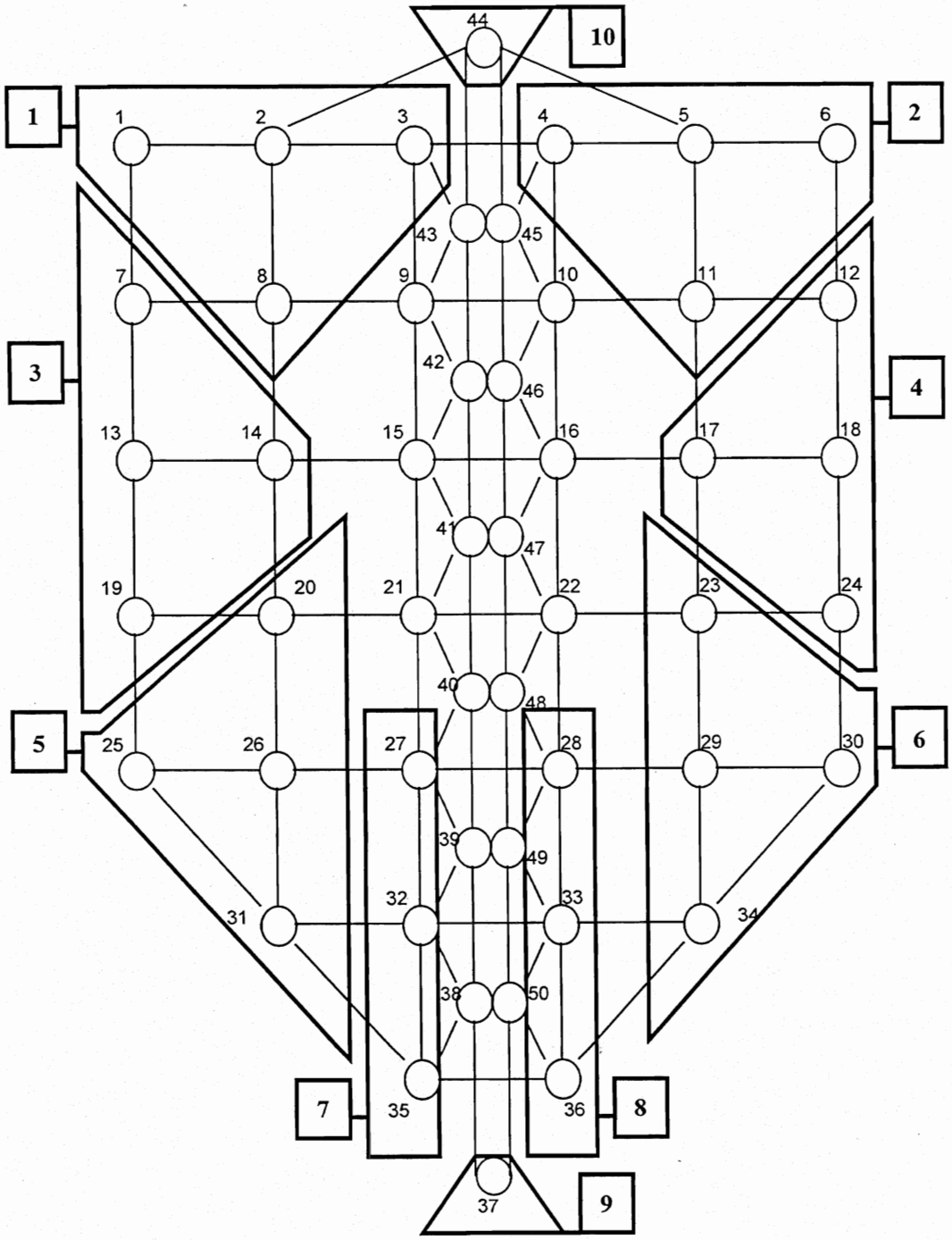

Figure 5.2 Test Network.

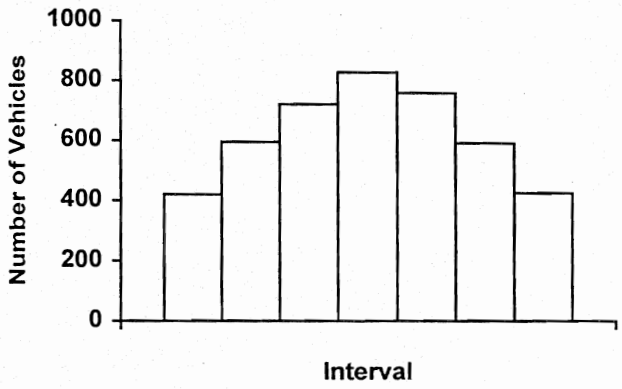

Figure 5.3 Loading Profile for Simulation Experiments of the Effects of Pricing under a UE Assignment Rule.

TABLE 5.1 NUMBER OF VEHICLES GENERATED FOR THE DIFFERENT LOAD FACTORS.

Load Factor (LF)	Number of Vehicles
1.00	4331
2.00	8711
3.00	13059

EXPERIMENTS RESULTS

This section discusses the effects of pricing under a UE assignment rule for all the tolling schemes considered. The presentation is divided in three parts. In the first part, the effect of pricing on travel times is presented. The second subsection discusses the effect of pricing on total trip distances, followed by the effect of pricing on revenues.

Effect of Pricing on Travel Times under a UE Assignment Rule.

The effect of pricing on travel times under a UE assignment rule for all the cases and load factors considered here is presented in tabular form in table 5.2. Table 5.2 presents the values of the total travel time along with the corresponding percentage difference relative to the base case of UE without tolling for each of the load factors. The percentage differences are also depicted graphically for each of the load factors in figures 5.4 to 5.6. The following convention is followed

to name each of the cases: CT denotes the case with constant tolls; the two-digit number following CT indicates the toll level in dollar cents (10,20,30, 40,60,80,100), with the last code indicating the link or zone that was tolled (i.e. 915 refers to links 9-15 and 15-9; 3837 refers to link 38-37, and Z3 refers to Zone 3 of the traffic network); VT denotes the variable toll case, followed by the percentage number that indicates the congestion level at which tolls are charged; DL stands for density level.

Figures 5.4 to 5.6 reveal the following trends for this particular network. For the low demand factor level, the use of constant tolls increases the travel times, but not significantly, when a single link is tolled. Only in one case is the travel time reduced, but this reduction is reversed when tolls are increased. When a zone in the traffic network is tolled, travel time increases much more than in the case of a single link and for all the cases. With a higher number of links tolled, the impact on the operation of the network is greater. For both cases, a single link and a tolled zone, the travel time increases with the toll level. Variable tolls do not affect significantly the operation of the network under the low demand level, as the higher density threshold values are not reached.

For the medium demand level, the use of constant tolls has a positive effect on the operation of the network when the toll levels are kept low. This is true for both cases of pricing a link or a zone of the network. However, as the tolls start to increase, the operational benefits for the network decrease, and at higher toll levels the operation of the network deteriorates with respect to the base case. For the case of variable tolls, the operation of the network improves through the use of pricing. In all cases, travel times are smaller than in the base case.

For the high demand level, the results are similar to the medium demand level. Small constant tolls benefit the operation of the network, but these benefits become smaller as the toll rises. However, the variable tolls improve the operation of the network in all cases.

Effect of Pricing on Trip Distances under a UE Assignment Rule

The effect of pricing on trip distances under a UE assignment rule for all the cases and load factors considered here is presented in tabular form in table 5.3. Table 5.3 presents the values of the total trip distances along with the corresponding percentage difference relative to the base case of UE without tolling for each of the load factors. The percentage differences are also depicted graphically for each of the load factors in figures 5.7 to 5.9.

The general trend observed in figures 5.7 to 5.9 is an increase in trip distances for all the load factors and tolling cases considered, both for constant and variable tolls. However, most of these increases are very small and in the range of less than half of one percent over the base distance for the case of UE with no tolling. The only cases in which the increases can be considered important are for the higher toll values for the restricted zone. The increase in travel

distance is explained by the assignment of vehicles to routes that are longer but that avoid the payment of tolls. The selection of the cheapest routes is not very much affected when a single link of the traffic network is tolled, as in the cases of links 9-15, 15-9 and 38-37, or when all the links in the network are tolled in a similar way, as in the case of variable tolls. When a number of links, but not all the links in the traffic network, are tolled, as in the case of zone three, cheapest routes will send vehicles to paths that avoid entering the restricted zone, resulting in longer routes, thereby increasing the total trip distance traveled.

TABLE 5.2 TOTAL TRAVEL TIME (HOURS) AND PERCENTAGE DIFFERENCES FOR DIFFERENT TOLLING CONDITIONS AND LOAD FACTORS UNDER A UE TRAFFIC ASSIGNMENT RULE.

Case	No. of Vehicles 4331	% Diff. in Travel Times	No. of Vehicles 8711	% Diff. in Travel Times	No. of Vehicles 13059	% Diff. in Travel Times
CT 10-915	460	0.27	973	-2.91	1564	-0.80
CT 20-915	461	0.57	982	-2.03	1537	-2.51
CT 30-915	459	-0.02	1010	0.76	1545	-2.02
CT 10-3837	458	-0.11	972	-2.97	1537	-2.53
CT 20-3837	461	0.43	979	-2.34	1541	-2.28
CT 30-3837	462	0.67	986	-1.57	1554	-1.42
CT 20-Z3	467	1.81	989	-1.27	1525	-3.26
CT 40-Z3	474	3.31	998	-0.40	1594	1.09
CT 60-Z3	474	3.31	1006	0.40	1592	0.99
CT 80-Z3			1015	1.29	1593	1.04
CT 100-Z3			1011	0.90	1600	1.47
VT 0% DL	459	0.12	983	-1.87	1515	-3.88
VT 1% DL	460	0.38	979	-2.34	1545	-1.99
VT 10% DL	460	0.30	978	-2.43	1555	-1.34
VT 25% DL	459	0.00	974	-2.79	1542	-2.20
VT 50% DL	459	0.00	977	-2.51	1522	-3.43
UE	459	0.00	1002	0.00	1577	0.00

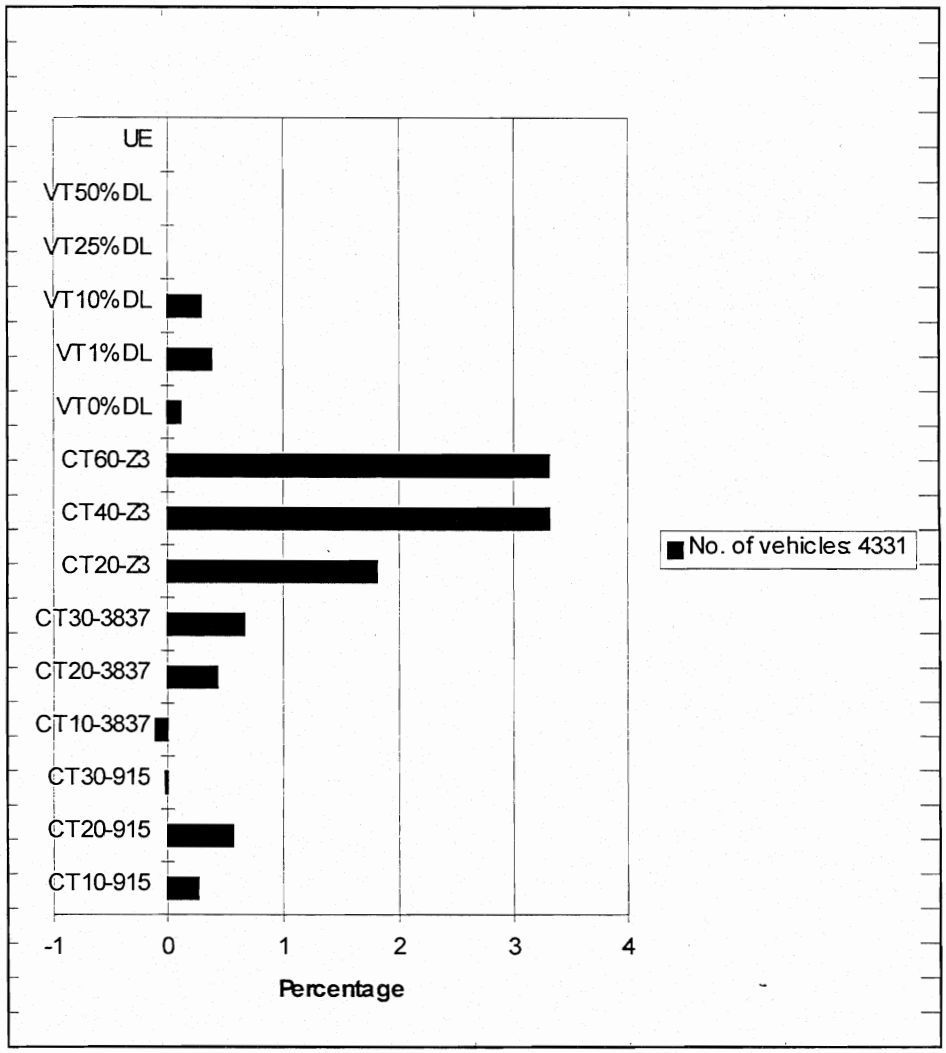

Figure 5.4 Percentage Difference in Total Travel Time for Different Tolling Conditions under a UE Traffic Assignment Rule for the Low Demand Scenario.

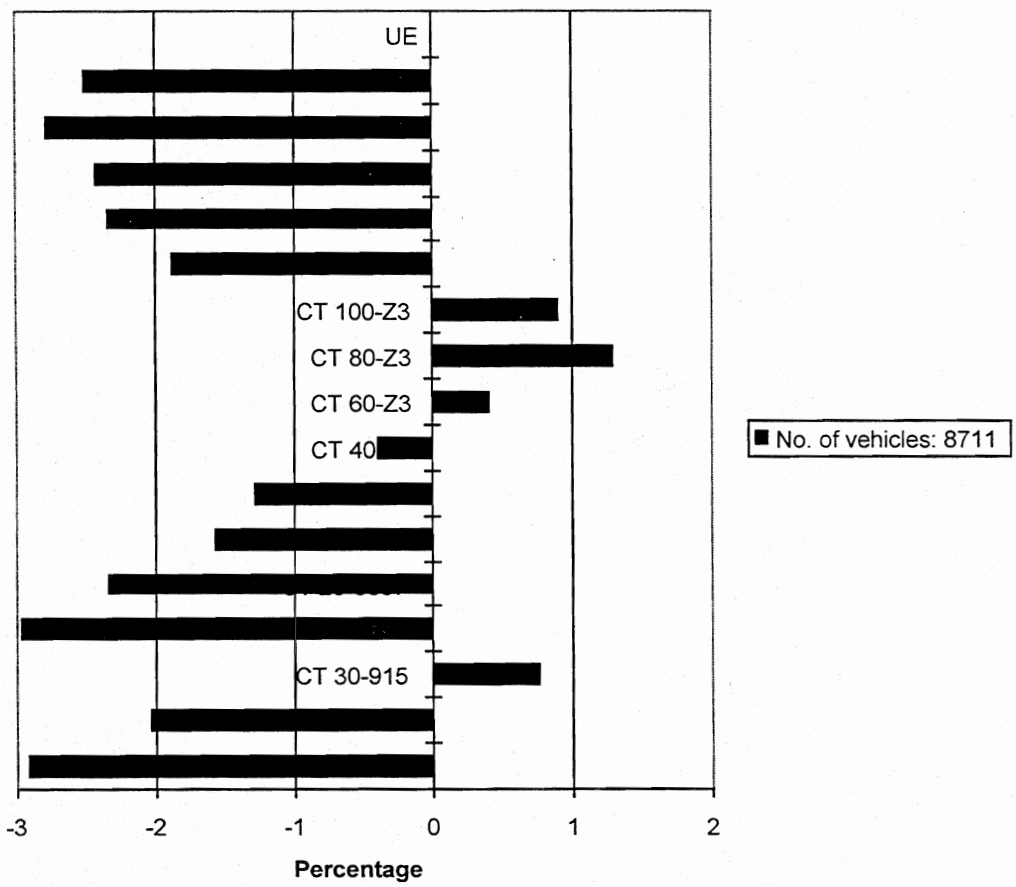

Figure 5.5 Percentage Difference in Total Travel Time for Different Tolling Conditions under a UE Traffic Assignment Rule for the Medium Demand Scenario.

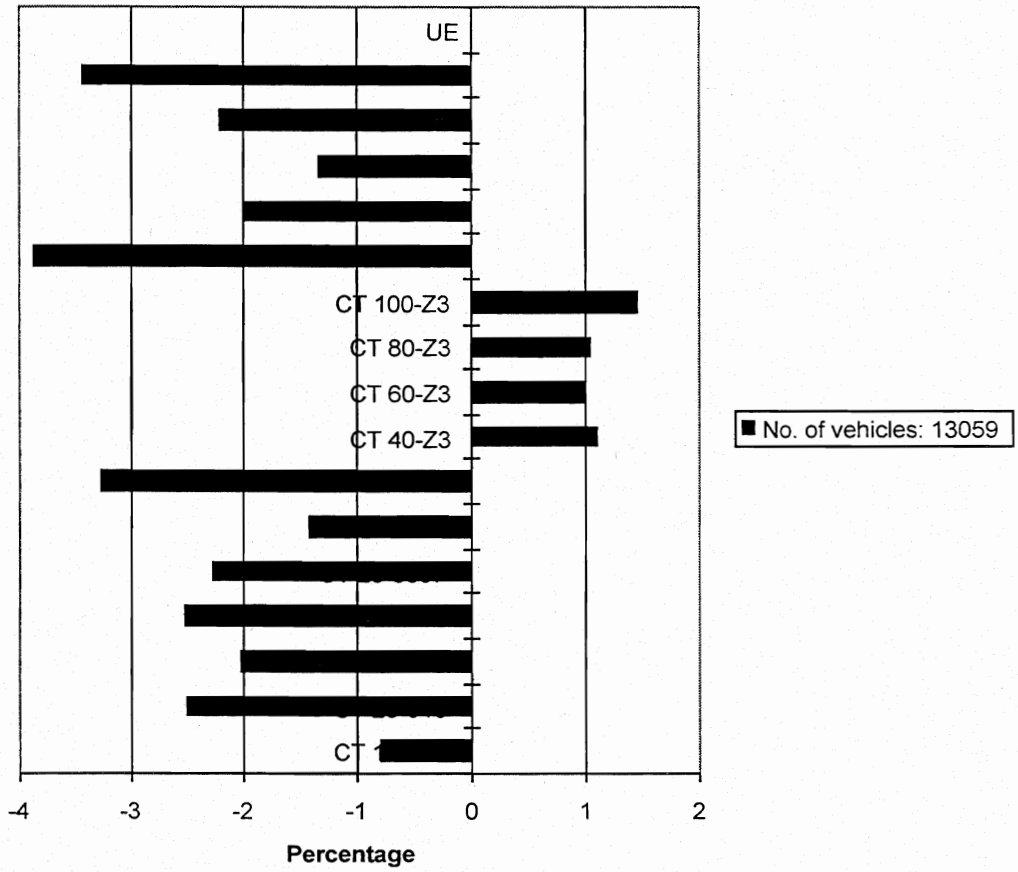

Figure 5.6 Percentage Difference in Total Travel Time for Different Tolling Conditions under a UE Traffic Assignment Rule for the High Demand Scenario.

TABLE 5.3 TOTAL TRIP DISTANCE (MILES) AND PERCENTAGE DIFFERENCE FOR DIFFERENT TOLLING CONDITIONS AND LOAD FACTORS UNDER A UE TRAFFIC ASSIGNMENT RULE.

Case	No. of Vehicles 4331	% Diff. in Trip Dist.	No. of Vehicles 8711	% Diff. in Trip Dist.	No. of Vehicles 13059	% Diff. in Trip Dist.
CT 10-915	8318	0.07	16629	0.17	24726	0.02
CT 20-915	8313	0.01	16609	0.05	24720	0.00
CT 30-915	8314	0.02	16621	0.13	24716	-0.02
CT 10-3837	8314	0.02	16620	0.12	24694	-0.11
CT 20-3837	8314	0.02	16625	0.15	24738	0.07
CT 30-3837	8317	0.06	16618	0.11	24709	-0.04
CT 20-Z3	8346	0.41	16688	0.53	24837	0.47
CT 40-Z3	8468	1.88	16925	1.96	25142	1.71
CT 60-Z3	8484	2.07	17027	2.57	25307	2.37
CT 80-Z3			17015	2.50	25354	2.56
CT 100-Z3			17038	2.64	25314	2.40
VT 0% DL	8315	0.04	16642	0.25	24770	0.20
VT 1% DL	8312	0.00	16642	0.25	24770	0.20
VT 10% DL	8315	0.04	16622	0.13	24784	0.26
VT 25% DL	8311	-0.01	16600	0.00	24721	0.00
VT 50% DL	8311	-0.01	16600	0.00	24697	-0.09
UE	8312	0.00	16600	0.00	24720	0.00

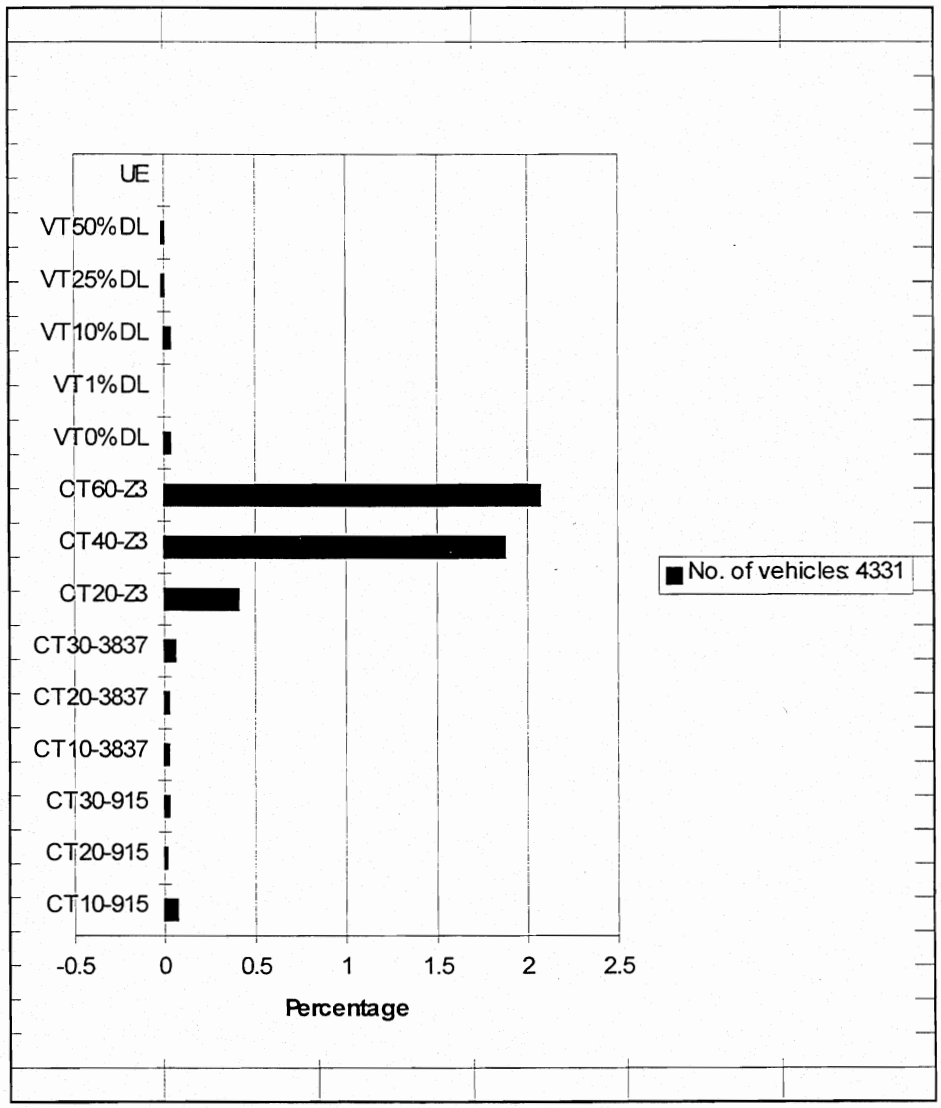

Figure 5.7 Percentage Difference in Total Trip Distance for Different Tolling Conditions under a UE Traffic Assignment Rule for the Low Demand Scenario.

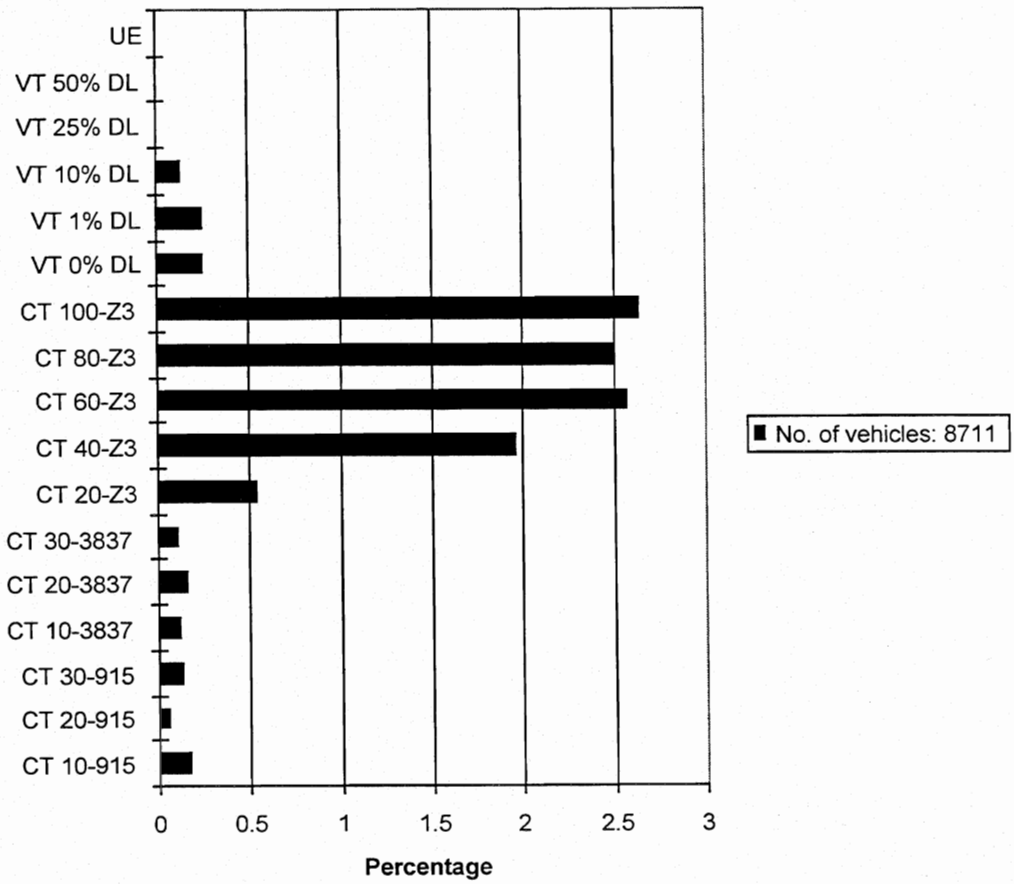

Figure 5.8 Percentage Difference in Total Trip Distance for Different Tolling Conditions under a UE Traffic Assignment Rule for the Medium Demand Scenario.

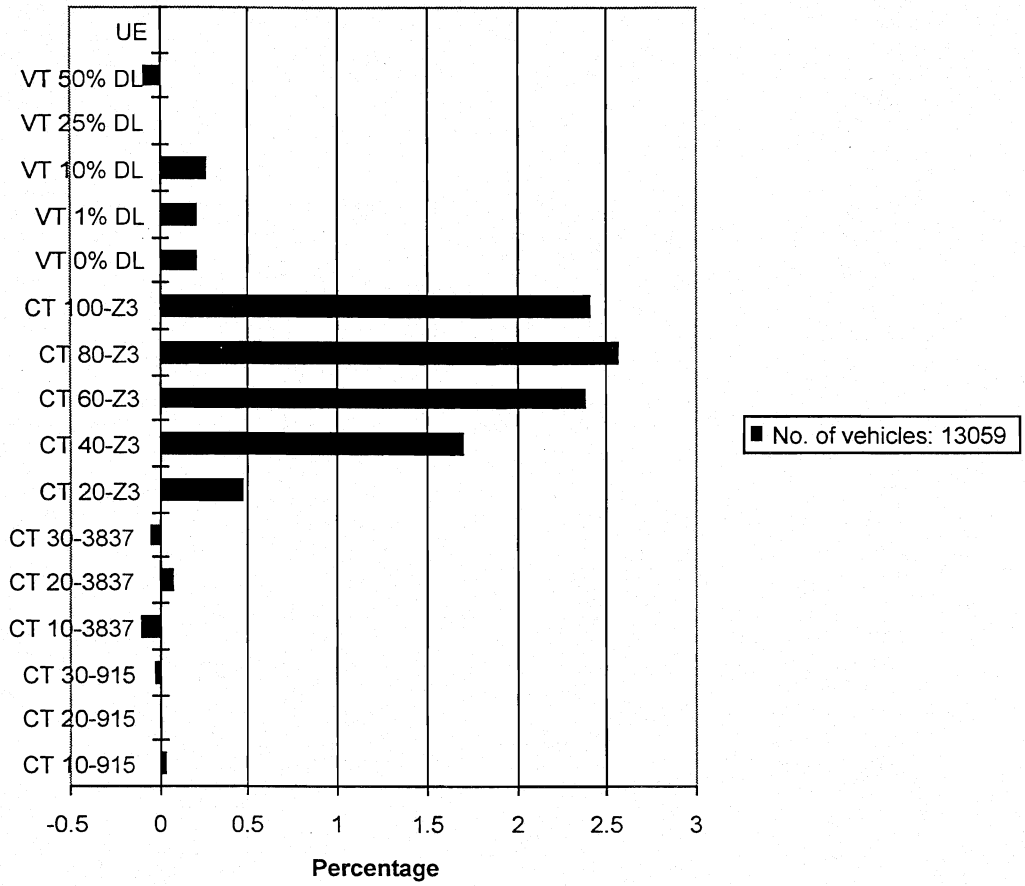

Figure 5.9 Percentage Difference in Total Trip Distance for Different Tolling Conditions under a UE Traffic Assignment Rule for the High Demand Scenario.

Effect of Pricing on Revenues under a UE Assignment Rule

The effect of pricing on revenues under a UE assignment rule for all the cases and load factors considered here is presented in a tabular form in tables 5.4, and graphically for each of the load factors in figures 5.10 to 5.12. The same convention as in section 5.4.1 is followed to name each of the cases.

Revenues for all the tolling cases considered in the experiments are rather small. This is a direct consequence of the levels of demand and of the geometric characteristics of the network used. The small number of vehicles using the tolled links in the case of constant tolls results in small revenues. Similarly, low densities in the case of variable tolls lead to low toll charges and small revenues.

However, the following trends can be observed in figures 5.10 to 5.12. In the case of constant tolls, as the toll and the load factor increase, so do the revenues collected, as expected, a captive population must pay the tolls regardless of their level. They cannot follow a different route to avoid tolled links. In the case of variable tolls, revenues are higher for low density thresholds but decrease sharply as the density level is increased. These higher density conditions occur only in a limited number of links.

TABLE 5.4 TOTAL REVENUE (DOLLARS) FOR DIFFERENT TOLLING CONDITIONS AND LOAD FACTORS UNDER A UE TRAFFIC ASSIGNMENT RULE.

Case	Number of Vehicles		
	4331	8711	13059
CT 10-915	0	0	2
CT 20-915	0	0	2
CT 30-915	0	0	0
CT 10-3837	44	91	137
CT 20-3837	87	182	273
CT 30-3837	131	274	410
CT 20-Z3	100	207	311
CT 40-Z3	153	317	508
CT 60-Z3	222	444	691
CT 80-Z3		592	895
CT 100-Z3		740	1115
VT 0% DL	65	270	616
VT 1% DL	64	266	628
VT 10% DL	1	74	350
VT 25% DL	0	0	18
VT 50% DL	0	0	0
UE	0	0	0

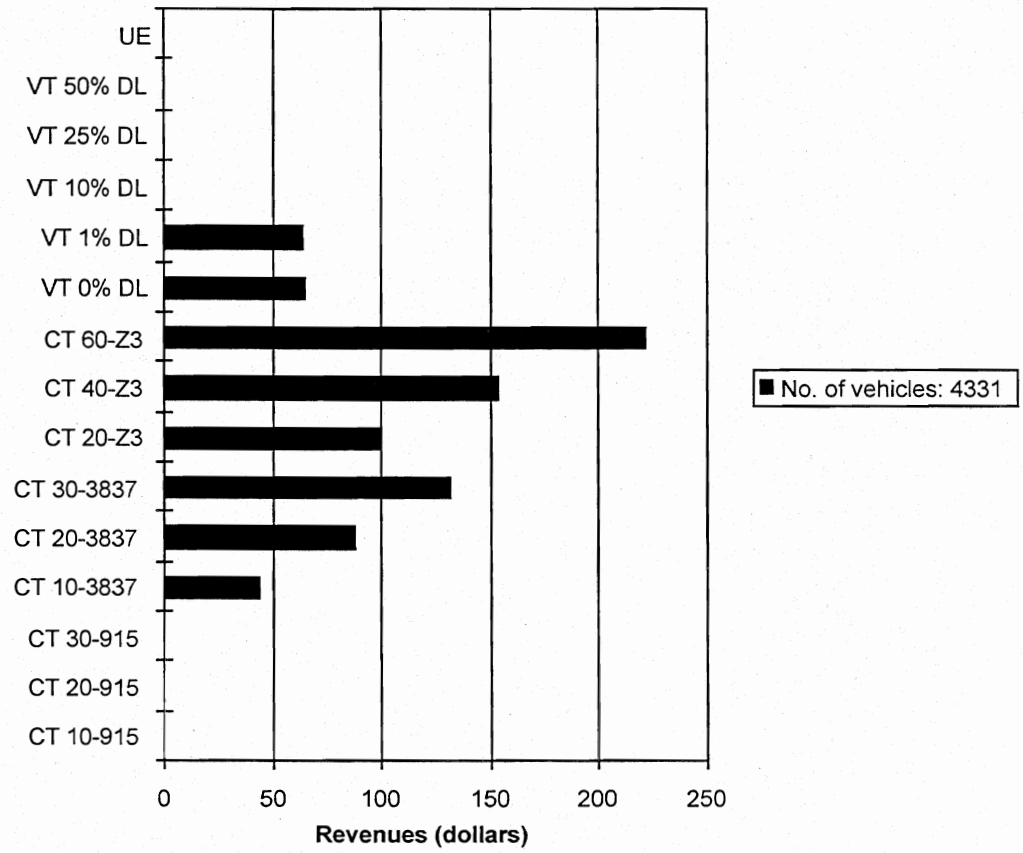

Figure 5.10 Revenues for Different Tolling Conditions under a UE Traffic Assignment Rule for the Low Demand Scenario.

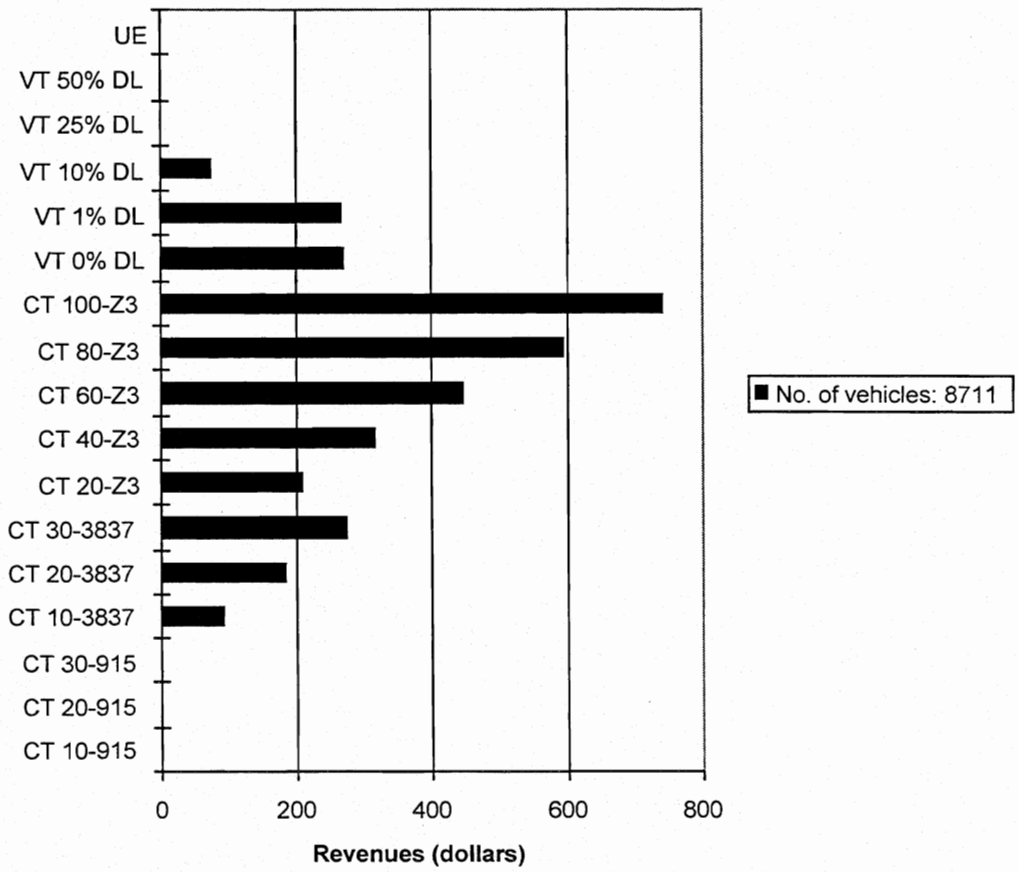

Figure 5.11 Revenues for Different Tolling Conditions under a UE Traffic Assignment Rule for the Medium Demand Scenario.

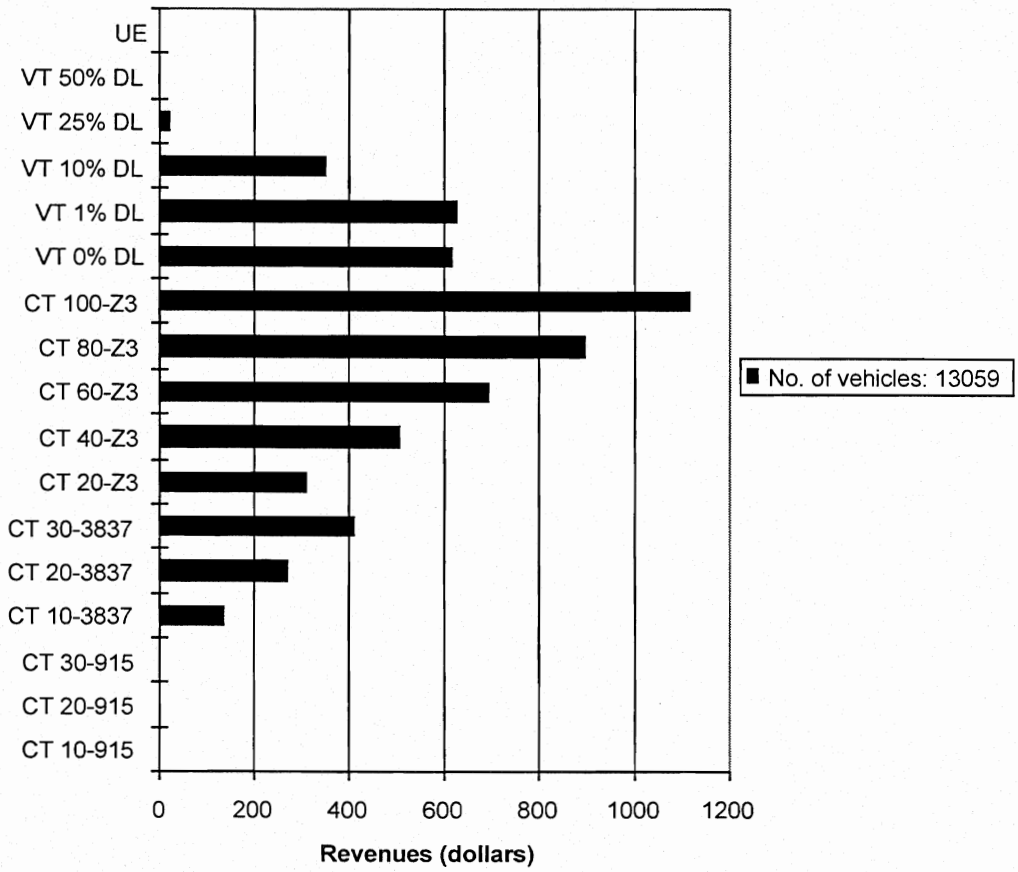

Figure 5.12 Revenues for Different Tolling Conditions under a UE Traffic Assignment Rule for the High Demand Scenario.

CONCLUSION

In this chapter a more realistic assignment rule than in previous chapter was investigated in the experiments. A UE assignment can be assumed to better represent the learning of drivers over the long run with respect to travel routes from their origin to their destination.

Constant tolls affect the operation of the network under the UE traffic assignment rule. Very small tolls have a positive impact in the operation of the network with respect to travel times. This positive impact decreases and is later reversed as the tolls increase. Contrary to the results of chapter four, the use of variable tolls was shown to benefit the operation of the network at higher demand levels.

Trip distances increase as a result of the use of tolls, but the increments are fairly small for the cases of pricing single links with constant tolls or all the links in the network with variable tolls. The trip distances are more significantly affected when access to a zone of the traffic network is tolled since vehicles with destinations away from the restricted zone will follow longer routes.

When a captive population exists, it is possible to generate revenue by means of tolling. In the case of constant tolls, any level will generate revenues. For the case of variable tolls, the revenue generation will depend on the congestion pricing scheme adopted. However, the incorporation of pricing needs to be further analyzed for the particular network in which is planned to be used. Furthermore, it is expected that prices perceived as unfair will encounter considerable public opposition, especially when they are not accompanied by measurable benefits for the paying users.

CHAPTER 6: ANALYSIS OF PRICING UNDER A DAY TO DAY DYNAMICS ASSIGNMENT RULE SET

Chapters four and five of this report presented the analysis of the effects of constant and variable pricing under two different assignment rules: one in which vehicles are assigned to the prevailing best route and follow a boundedly rational behavioral rule for path switching and the other a time-dependent user equilibrium rule. Both assignment rules consider only the routes followed from origin to destination but not changes in departure times. In this chapter a new assignment rule set that considers drivers' selection of both route and departure time is considered. This rule set seeks to capture the learning process of drivers in their daily search for routes and departure times that lead them to their destinations at their desired arrival times.

In the analysis, pricing is incorporated into a simplified version of the day to day dynamics algorithm developed by Hu and Mahmassani (1995) in which only changes in route and departure times are considered. The chapter is structured as follows. The first section presents the concepts of day to day dynamics, and the analysis framework with the incorporation of pricing under this assignment rule set. The second section describes the experimental design to illustrate the effects of pricing in a day to day analysis framework. The third section presents the results of the experiments with constant prices, followed by the experiments with variable prices in the fourth section. The final section presents the conclusions of this chapter.

DAY TO DAY DYNAMICS

The problem is formulated as the assignment of traffic flows to a network represented by a directed graph G={N, A}, where N is the set of nodes N={1,2,...,q,...} and, A represents the set of directed arcs joining the nodes. A node can be a trip origin and/or a destination and/or a junction of physical links. A network with multiple origins $q \in Q$ and destinations $j \in J$ is considered for generality. Each driver i departs from origin q to destination j and wishes to arrive at his/her preferred arrival time PAT_i, $\forall i \in D$, the set of all users. The number of users is assumed to be fixed. The preferred arrival time is assumed to be fixed for any particular user. Driver i chooses his/her departure time $DT_{i,t}$ and route $RC_{i,t}$ for day t as a function of a process described by the equations:

$$RC_{i,t} = f_r(X_i, Z_{i,t}, Y_{i,t}|\theta_r, TOLLS) \quad (6.1)$$

$$DT_{i,t} = f_{dt}(X_i, Z_{i,t}, Y_{i,t}|\theta_{dt}, TOLLS) \quad (6.2)$$

where,

$RC_{i,t}$: route for user i on day t,

$DT_{i,t}$: departure time for user i on day t,

$f_r(.)$: function that represents the route-choice process,

$f_{dt}(.)$: function that represents the departure time process

X_i: vector of driver characteristics,

$Z_{i,t}$: vector of endogenous information characteristics for driver i up to day t,

$Y_{i,t}$: vector of exogenous information characteristics for driver i up to day t,

θ_r, θ_{dt}: parameter vectors to be calibrated.

TOLLS: Constant or variable out of pocket costs.

The aggregated number of vehicles, departing from each origin to each destination in any time interval, corresponding to each departure time selection, form a three dimensional time-dependent origin-destination (OD) matrix. The selection of routes defines the distribution of flows over the network. Let $r_{q,j}^{\tau,k}$ be the number of vehicles that depart from origin q with destination j at time τ and following route k and $r_{q,j}$ the number of vehicles that go from origin q to destination j. For the case of constant total demand, denoted by D, the following equations need to be satisfied:

$$\sum_{\tau}\sum_{k} r_{q,j}^{\tau,k} = r_{q,j} \quad \forall q,j \tag{6.3}$$

$$\sum_{q}\sum_{j}\sum_{\tau}\sum_{k} r_{q,j}^{\tau,k} = |D| \tag{6.4}$$

The day-to-day dynamics problem consists in finding the distribution of network flow patterns associated with the $r_{q,j}^{\tau,k}$ on day t in such a way to reproduce the day to day evolution of the users' selection of departure time and route choice. The problem is a recursive search where drivers, based on their personal experience and outside information sources, select each day new routes and departure times as described by equations (6.1) and (6.2).

Figure 6.1, in which the terms departure time and route can be interchanged, is based on previous work by Mahmassani and his colleagues (Chang and Mahmassani, 1988; Mahmassani, 1990; Mahmassani and Chang, 1985, 1986a, 1986b; Tong et al., 1987; Tong, 1990). It provides the framework for the analysis of the day-to-day dynamics under pricing problem. According to the framework, drivers evaluate the utility they had with their previous route and departure time choice and then decide to search, within a boundedly-rational limit, for a new route or departure time if they are not satisfied. The search continues until no change in the route or departure time is sought.

Mahmassani and co-workers have shown that commuters consider travel time and "schedule delay" as the two main factors in the departure time decisions for their home to work trip. If pricing is incorporated, the total cost of travel will now be considered instead of travel time only. Travel cost is defined as travel time plus any out of pocket payment, converted to the same units, preferably time units. Schedule delay (SD_{it}) for a particular individual i at day t is defined as the difference between the preferred arrival time (PAT_i) and the actual arrival time (AT_{it}) or $SD_{it} = PAT_i - AT_{it}$. If a tripmaker is not satisfied with a specific choice of route or departure time, i.e., it is not within his/her indifference band of tolerable delay, he/she will look for a new route and/or change his/her departure time. This indifference band has been viewed as the main behavioral factor in the day to day responses of drivers to congestion.

The following rules are used to express the boundedly rational process of route and/or departure time switching decisions. According to the rules, the user would not look for new routes or departure times as long as the currently followed routes and departure time remain within the driver's indifference bands, as:

$$\gamma_{i,t} = 0, \text{ if } 0 \le ESD_{it} \le EBD_{it} \text{ or } -LBD_{it} \le LSD_{it} \le 0 \qquad (6.5)$$

$$\gamma_{i,t} = 1, \text{ otherwise}$$

$$\psi_{i,t} = 0, \text{ if } 0 \le ESD_{it} \le EBR_{it} \text{ or } -LBR_{it} \le LSD_{it} \le 0 \qquad (6.6)$$

$$\psi_{i,t} = 1, \text{ otherwise}$$

where $\gamma_{i,t}$ is an indicator variable equal to 1 if user i decides to switch departure time for the following day after the trip on day t (i.e. for the commute on day t+1); $\psi_{i,t}$ is defined in the same form for route switching. ESD_{it} is earlier schedule delay, equal to $MAX(PAT_i - AT_{i,t-1}, 0)$ and LSD_{it} is late schedule delay, equal to $MAX(AT_{i,t-l} - PAT_{i,t-l}, 0)$. EBD_{it} and LBD_{it} are the respective departure time indifference bands of tolerable schedule delay corresponding to early and late arrivals for day t and EBR_{it} and LBR_{it} denote the early and late indifference bands that define route switching. The early and late components were defined by the studies of Mahmassani and Chang (Mahmassani and Chang, 1985 and 1986a).

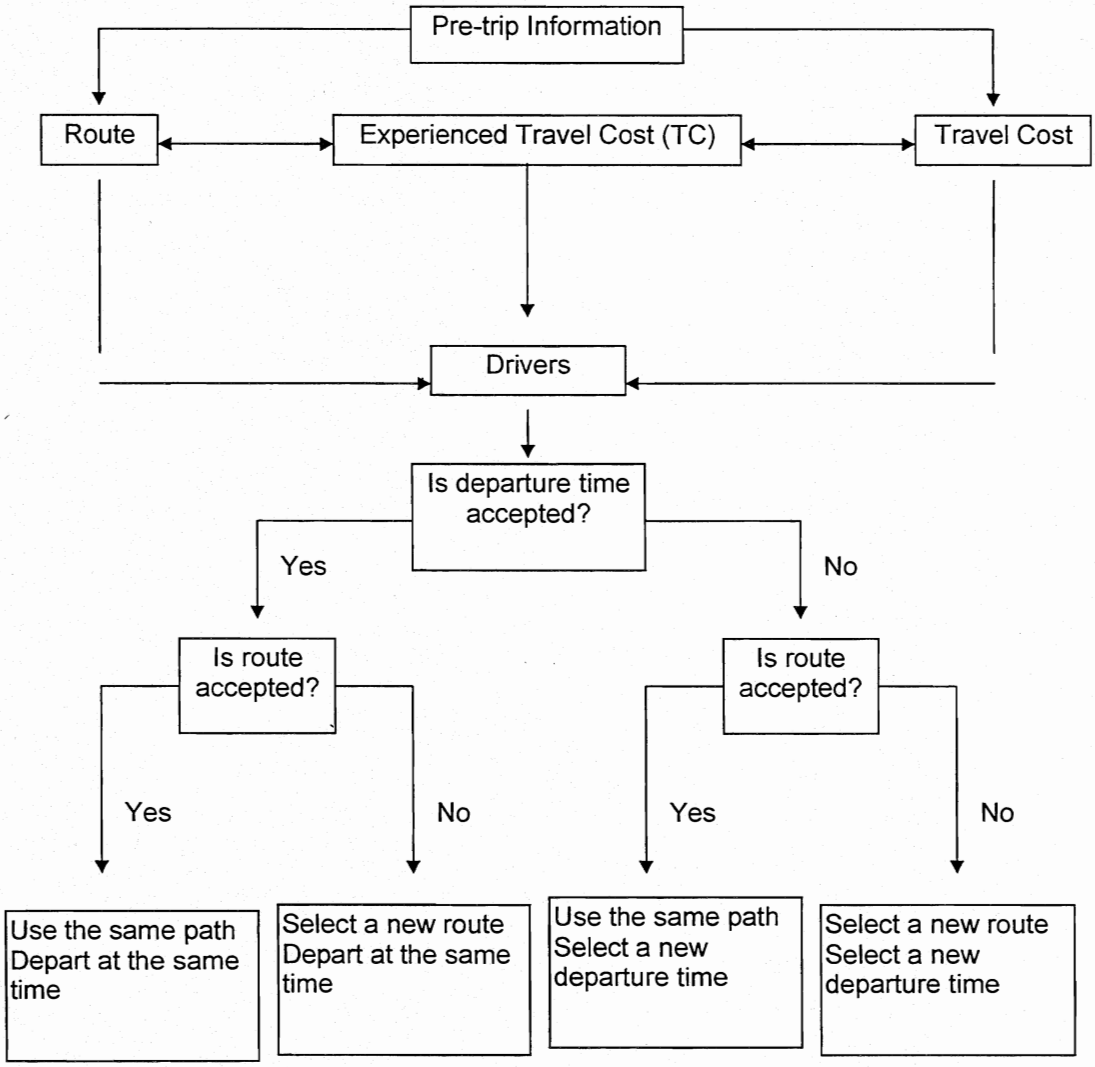

Figure 6.1. Day-to-Day Pre-trip Decision Making Process (Hu, 1995).

The indifference bands are random variables corresponding to each individual and cannot be observed nor measured directly. They take different values according to the day or the user (Tong, 1990) and will be inferred from actual observations of commuters' decisions. IBDT and IBRC are the indifference bands for departure time and route choice respectively. A model for the indifference band of the departure time has the following functional form (Jou et al., 1992):

$$IBDT_{it} = W_{it}\,\beta_1 + (1-W_{it})\,\beta_2 \qquad \text{Initial Bands} \qquad (6.7)$$

$$+ W_{it}\,\beta_3\,AGE_i + (1-W_{it})\,\beta_4\,AGE_i \qquad \text{Socio-economic Component}$$

$$+ W_{it}\,\beta_5\,GENDER_i + (1-W_{it})\,\beta_6\,GENDER_i$$

$$+ W_{it}\,\beta_7\,NFAIL_{i,t}^{\beta_8} + (1-W_{it})\,\beta_9\,NFAIL_{i,t}^{\beta_{10}} \qquad \text{Dynamic Component}$$

$$+ W_{it}\,\beta_{11}\,\delta_{i,t}\left(\frac{\Delta TR_{it}}{\Delta DT_{it}}\right) \qquad \text{Myopic Component}$$

$$+ \varepsilon_{it} \qquad \text{Unobserved Component}$$

where,

$\beta_1 ... \beta_{11}$: parameters to be estimated,

AGE and GENDER: individual's characteristics,

$NFAIL_{it}$: the number of unacceptable early and late arrivals until day t,

ΔTR_{it}: the difference between travel costs (in time units) of commuter i on day t and t-1,

ΔDT_{it}: the departure time that commuter i has adjusted between day t and t- 1,

W_{it}: a binary indicator variable, equal to 1 if $Sd_{it} > 0$ (early-side) and equal to 0 if $Sd_{it} < 0$ (late side)

δ_{it}: a binary indicator variable equals to 0 if $DT_{i,t} = DT_{i,t-1}$; otherwise 1, and

ε_{it}: error term for commuter i on day t.

A model for the indifference band of the route choice has the following functional form (Jou, 1992):

$$IBRC_{it} = W_{it}\,\beta_{12} + (1- W_{it})\,\beta_{13} \qquad \text{Initial Bands} \qquad (6.8)$$

$$+ W_{it}\,\beta_{14}\,STDTR_{it} + (1- W_{it})\,\beta_{15}\,STDTR_{it} \qquad \text{Dynamic Component}$$

$$+ W_{it}\,\beta_{16}\,NFAIL_{it} + (1- W_{it})\,\beta_{17}\,NFAIL_{it} \qquad \text{Dynamic Component}$$

$$+ \upsilon_{it}$$

where

$\beta_{12}, ..., \beta_{17}$: parameters to be estimated,

$STDTR_{it}$: the standard deviation of travel time up to day t,

$NFAIL_{it}$: the number of unacceptable early and late arrivals until day t,

W_{it}: a binary indicator variable, equal to 1 if $SD_{it} > 0$ (early-side) and equal to 0 if $SD_{it} < 0$ (late side), and

υ_{it}: error term for commuter i on day t.

For the incorporation of pricing, travel time is replaced, as in previous chapters, by a generalized travel time function (Equation 4.10), that considers travel time plus out of pocket payments, converted to time units by using the VOT in the case of constant tolls. For variable tolls, the time equivalent of the toll is directly added to the travel time. Terms corresponding to β_{14} and β_{15} parameters of Equation (6.8) will now consider the standard deviation of travel cost (in time units) up to day t.

Tong (1990) and Mahmassani (1990) describe in detail the estimation procedure for Equations (6.7) and (6.8). ε_{it} and υ_{it} are random terms assumed to be jointly and normally distributed over days and across commuters, with zero means and general covariance matrix Σ, or MVN $(0,\Sigma)$). Σ is expressed as:

$$\begin{bmatrix} \Sigma_{\varepsilon} & \text{cov} \\ \text{cov} & \Sigma\nu \end{bmatrix}$$

Entries Σ_{ε} and Σ_{υ} are TxT matrices that capture the serial correlation across the elements of the sequence $(\varepsilon_{it}, t=1,...,T)$ and $(\upsilon_{it}, t = 1,...,T)$. The cov entries reflect the correlation that might exist between the route and departure time bands for a given user.

Using data from a survey in the Dallas, Texas area, Jou et al (1992) calibrated the models presented in Equations (6.7) and (6.8). The values they found for the parameters are presented in Table 6.1.

TABLE 6.1 VALUES FOR PARAMETERS OF EQUATIONS OF IBDT AND IBRC (JOU ET AL., 1992).

	IBDT		IBRC
β_1	23.26	β_{12}	27.22
β_2	17.82	β_{13}	18.76
β_3	7.61	β_{14}	8.87
β_4	4.51	β_{15}	4.37
β_5	-5.59	β_{16}	8.95
β_6	-6.57	β_{17}	9.13
β_7	5.49		
β_8	1.16		
β_9	4.36		
β_{10}	0.78		
β_{11}	4.17		

The selection of a route by an individual user is a discrete choice process due to the limited number of possible choices. On the other hand, the user can select from a theoretically infinite number along a continuum of departure time alternatives. However, for practical purposes, the number of alternatives can be discretized. Random utility models are used to describe the selection of new alternatives, conditional upon the decision to change from the current route and/or departure time. Users are assumed to evaluate all the available alternatives and select the one perceived to be the best. Among the models that have been calibrated using this framework are those by Abkowitz (1981), Hendrickson and Plank (1984), Small (1982) and Tong (1990). Abkowitz's model is for the choice of departure time and mode; Hendrickson and Plank's is for the joint choice of departure time and mode; Small's is for the choice of departure time. A model for the joint choice of departure time and route is expressed in a functional form as:

$$U_{i\tau k} = \alpha_1 + \alpha_2 \, PERSON_{i,\,\tau,k,t} + \alpha_3 \, ET_{i,\,\tau,k,t} + \alpha_4 \, ESD_{i,\,\tau,k,t} + \alpha_5 \, LSD_{i,\,\tau,k,t} + \pi_{i\tau k} \qquad (6.9)$$

where $\alpha_1,\ldots,\alpha_5$ are parameters to be estimated, $U_{i\tau k}$ is the utility of departure time τ and route k, denoted as (τ,k), for individual i, $PERSON_{i,\tau,k,t}$ are the tripmaker characteristics, $ET_{i,\tau,k,t}$ is the

anticipated travel cost (in time units) of alternative (τ,k), and ESD and LSD are early and late schedule delay respectively.

Small's (1982) model of departure time is based on a sample of 527 auto commuters from the San Francisco Bay area. A simplified version of Small's model adapted to purposes of this research is:

$$U_{i\tau} = - 0.106\ TR_{i\tau} - 0.065SDE_{i\tau} - 0.254SDL_{i\tau} - 0.58\ DlL_{i\tau} \qquad (6.10)$$

where

$U_{i\tau}$: utility of departure time alternative τ to individual i

TR: travel time in minutes

SDE=Max{-SD,0}: the early schedule delay for individual i under alternative τ

SDL_Max{SD,0}: the late schedule delay

D1L: a late dummy variable, equals 1, if SD$\geq$ 0; 0, otherwise, and

SD: schedule delay, arrival time minus official work start time in minutes.

Negative values of the coefficients indicate that the tripmaker utility is negatively affected by long travel time, early arrival and late arrival. Schedule delay has the strongest effect on commuters' decisions.

Hendrickson and Plank's (1984) model of departure time adapted to this research has the following expression:

$$U_{i\tau} = - 0.021TR_{i\tau} - 0.00042SDE_{i\tau} - 0.148SDL_{i\tau} + 0.0014\ SDL_{i\tau}^2 \qquad (6.11)$$

where the variables are as defined above. For Hendrickson and Plank's model the late arrival has a very high penalty. This model is used for the numerical experiments of this chapter, replacing travel time by travel cost.

A simulation approach is followed to evaluate the evolution of the network under the day to day dynamics framework described here. The simulation approach is presented in the next section.

Day to Day Dynamics Algorithm

The algorithm for the day-to-day dynamics when pricing is incorporated is presented graphically in Figure 6.2. A network user i, with associated behavioral attributes and making his/her selection based on to his/her own recent and accumulated experience and/or perceived information, departs from origin q to destination j and wishes to arrival his/her preferred arrival time (PAT_j), $\forall i \in D$. The procedure simulates the dynamic behavior of the user i from day to day and obtains the daily system performance from individual trip information statistics. The procedure is as follows (Hu, 1995):

Step 0: *Initialization.* Generate vehicles' attributes and historical paths. Obtain a set of paths from origin q to destination j for each discrete departure time interval τ on day t, denoted as $K_{q,j,\tau,t}$. Assign to each tripmaker i a set of static and run-time attributes, and a set of behavior attributes, B_i. Set iteration counter I=1.

Step 1: *Network Loading.* For each tripmaker i, assign a path k_i; from q to j, $k_i \in K_{q,j,\tau,t}$ an initial departure time, and a loading location, i.e. a generation link. For each day, the number of vehicles for each time interval τ and for each path k, denoted as $r_{q,j}^{\tau,k}$, is generated to form a three-dimensional matrix over both space and time.

Step 2: *Traffic Simulation.* Simulate network performance using DYNASMART. From the simulation, obtain an updated vehicle file with time-dependent travel cost (in time units) information for links and movements.

Step 3: *Information' Update.* Update the historical path information in terms of travel cost (in time units), add new paths or delete obsolete paths from the historical path file.

Step 4: *Day to Day Behavior: Indifference Bands* Calculate the departure time and route choice indifference bands for the tripmaker i according to his/her set of behavior attributes, B_i. Determine values of the switching indices $\gamma_{i,t}$ and $\psi_{i,t}$ to find out changes in departure time and route switching.

Step *5:* *Convergence test.* If convergence criterion is satisfied (at least 90% of tripmakers are satisfied with their current selections), stop. Otherwise, continue.

Step *6:* *Selection of Departure Time and Route.* If changes in departure time, route or both occur, update departure time and/or route choice

Step *7.* *Resequence and Feedback.* Resequence vehicles according to their respective departure times and generation links. Obtain a time-dependent origin destination matrix.

Set I=I+1 and go to step 1.

Convergence Criteria

Followings Wardrop's first principle (1952), and extending the concept to incorporate the boundedly-rational behavior exhibited by drivers (Mahmassani and Chang, 1987), the convergence criteria for the day to day dynamics with pricing is set as follows:

(1) The travel costs on all the used routes fall within the boundedly-rational indifference bands for all the drivers, and are less than the cost on all the routes that carry no vehicles.

(2) The average travel cost is a minimum. Drivers will recursively search for the route and departure time selections that satisfy their requirements. To achieve a hundred percent satisfaction a great number of iterations may be needed. As such, an operational limit is set in the numerical experiments. It requires that a fraction of drivers be satisfied with their current choices. For the experiments of this chapter this fraction is set to 90%.

EXPERIMENTS DESCRIPTION

Experiments were conducted to evaluate the effect of constant and variable prices under the day to day dynamics assignment rule set. Two different pricing schemes were considered: (1) a fixed charge for crossing a boundary in a restricted zone of the network, and; (2) a variable price for all the links of the network based on the prevailing level of congestion on the link.

The effect on the flow patterns, total travel times, total trip distances and revenues is addressed. This section describes the assumptions and characteristics of the network used in the simulations, pricing schemes considered, demand patterns, demand levels and experimental factors.

The Test Network

The same network used in experiments of chapter four (Section 4.4.1.2) is also used to conduct the experiments with pricing under the day to day dynamics assignment rule set. The network is depicted again in Figure 6.3 for convenience.

Experimental Factors

For these experiments, the preferred arrival time (PAT_i) is assumed fixed for all the drivers and equal to the work schedule time, 8:30 AM. The loading profile used for the simulations is assumed to be evenly distributed over a period of 25 minutes (from 8:05 AM to 8:30 AM) on the first day. From the second day, drivers can select a departure time from 8:00 AM to 8:40 AM, discretized in 40 one-minute intervals. Historical paths are those given by the time-dependent system optimal procedure of Mahmassani and Peeta (1992). The maximum number of paths for each origin-destination pair and for each departure time is 5.

Anticipated travel cost for each driver is updated by the combination of recent and historical travel time information as follows:

$$ETC_{i,\tau,k,T} = \sum_{t=1}^{T-1} w_{i,t} PTC_{i,\tau,k,t} \qquad (6.12)$$

where $ETC_{i,\tau\ k,T}$ is the anticipated travel cost for tripmaker i on route k at time τ on day T, $PTC_{i,\tau,k,t}$ is the travel cost on any day t. $\sum w_{i,t}$ equals 1, and is used to express the

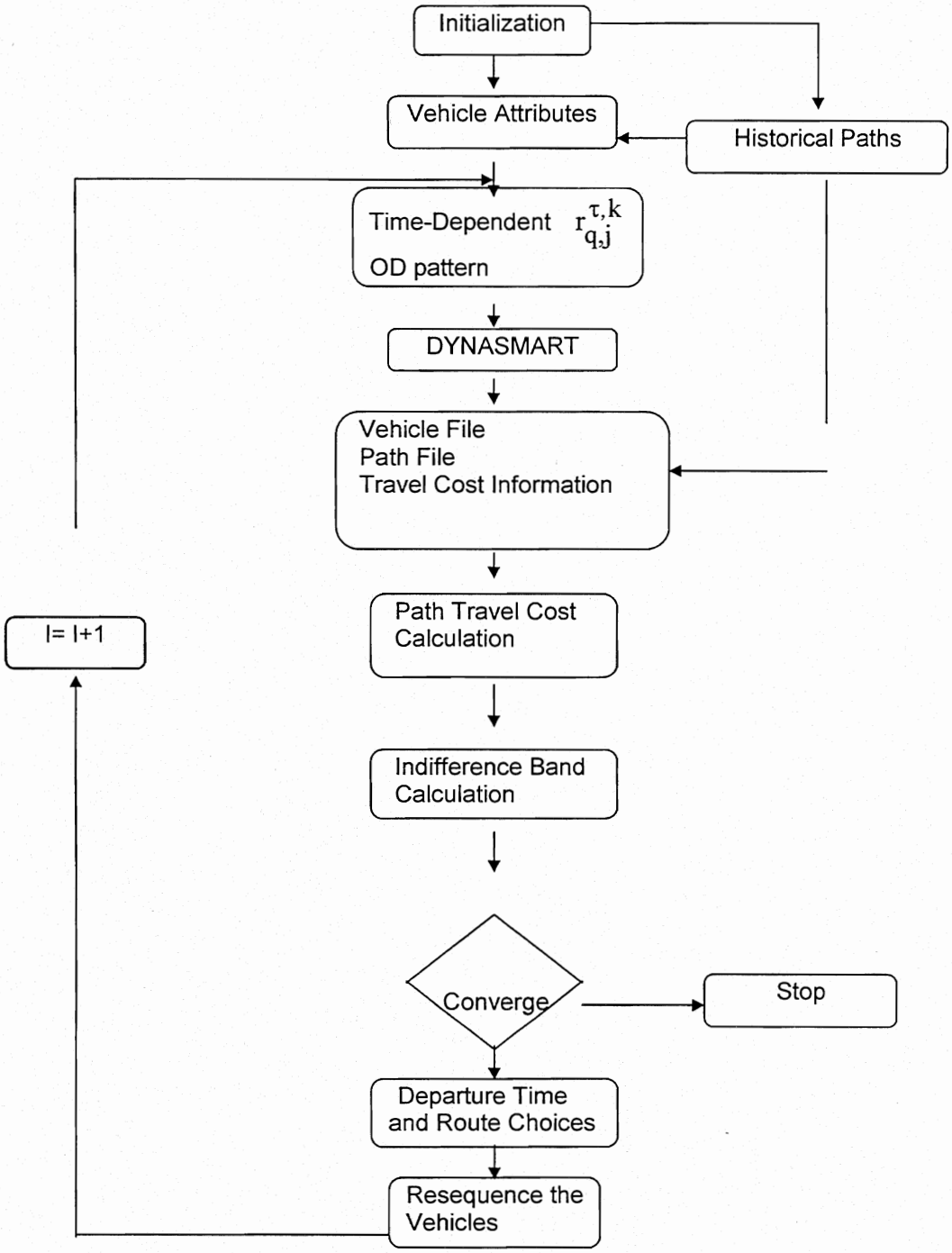

Figure 6.2 Flowchart of the Day to Day Dynamics Algorithm (Hu, 1995).

importance of historical travel cost information. The particular values used are $w_{i,T-1}=1$, and $w_{i,1}=0,....,.w_{i,T-2}=0$. In this way only the information of the previous day is considered.

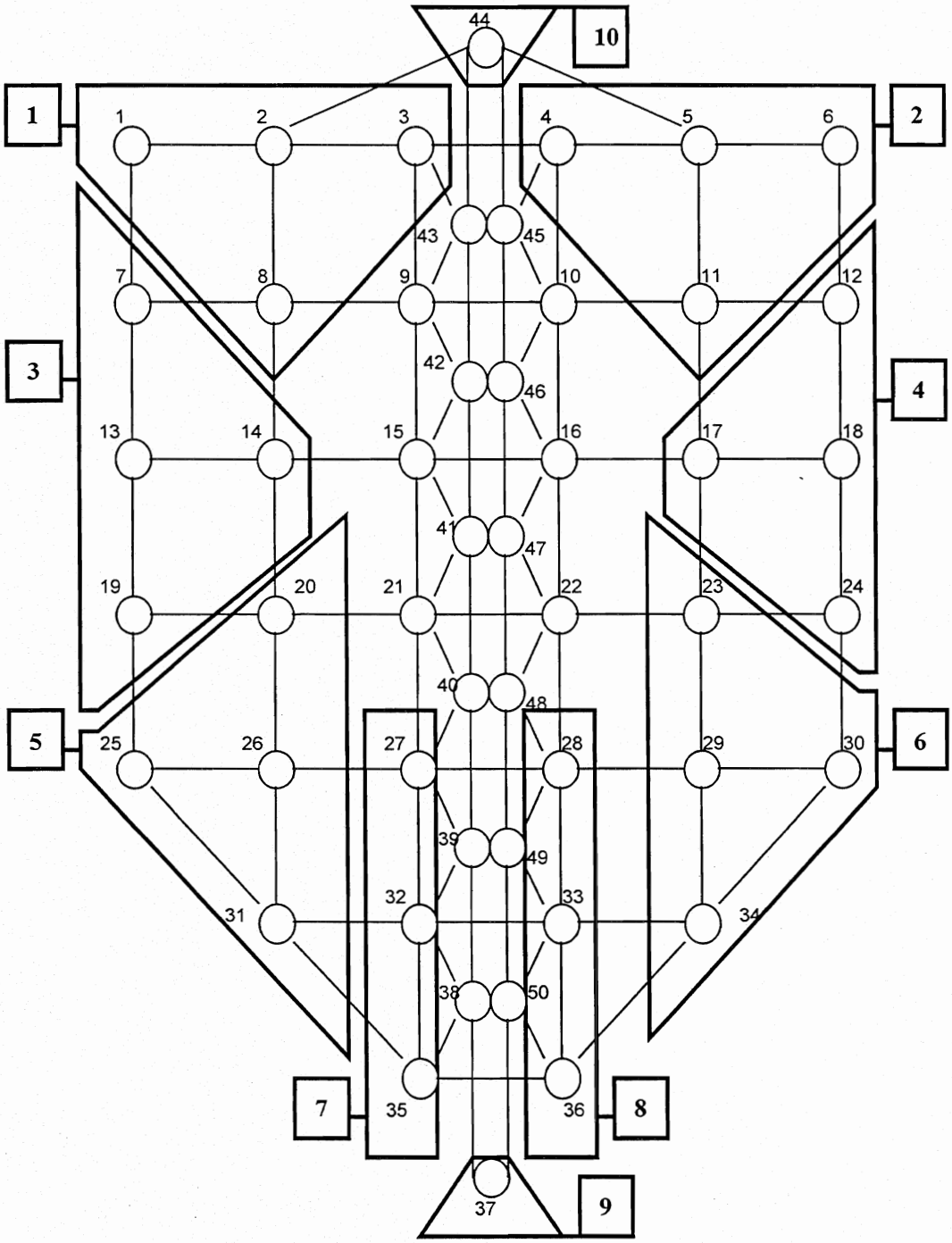

Figure 6.3 Test Network.

The departure time model proposed by Hendrickson and Planck (1984) (equation 6.11) is used in the simulation experiments. All vehicles in the simulation are assumed to be non-equipped. Drivers select routes from the historical paths given by the time-dependent system optimal algorithm of Mahamassani and Peeta (1992).

CONSTANT PRICING SCHEME AND PRICE LEVELS

Taking into account the results of chapter five and the minimal effect that the pricing of a single link had in the overall operation of the test network, only one pricing scheme was considered for the application of constant pricing. The only case considered was that of a fixed charge for entering a restricted zone of a traffic network. As in the previous assignment rules, the restricted area of the traffic network is zone 3 in Figure 6.3. Vehicles entering the zone will be forced to pay the corresponding toll.

The value of travel time, as in chapter 4, is assumed to be constant and equal to six dollars per hour.

Price Levels

Prices were increased from 25 cents to one dollar. In that way four different prices were considered 25, 50, 75 cents and one dollar.

Experiment Results

Figures 6.4, 6.5 and 6.6 show the temporal flow patterns for days 2, 5 and 7 for the base case (no toll) and the different constant toll levels. As in Chapter five CT means constant toll and the number that follows indicate the amount in cents for the toll.

It can be observed in the Figures that the constant tolls applied do not greatly affect the distribution of temporal flow patterns. The drivers' departure time choice behavior observed in the base case does not change significantly when tolls are applied. Drivers leave their origin with a very similar pattern without being affected too much by the toll levels.

The total travel time, shown in Figure 6.7, increases (relative to the no toll base case) for all the toll levels, and increases as the toll level increases. Vehicles follow routes that go around the tolled zone, thereby increasing their travel time.

The use of longer routes is further illustrated in Figure 6.8, which depicts the total trip distance. The latter increases as the toll increases. Longer routes to avoid the toll payment are used. More and more vehicles avoid the tolled zone day after day of the simulation.

Revenues, shown in Figure 6.9, reach the highest values in the first days of the simulation. Then, they start to fall as the simulation continues. Since the number of vehicles entering the restricted zone is decreasing, revenues also decrease. At the end of the simulation only vehicles that have no alternative but to pay the toll are the ones entering the restricted zone.

Convergence for the conducted experiments is achieved relatively fast. For the fraction of satisfied drivers used (90%), the simulation stops after seven iterations for the base case and also for the cases with constant tolls. No significant difference is observed in the convergence patterns for the base case and the cases with constant tolls. The relative low value of the parameter corresponding to travel cost used with respect to that of late scheduled delay (equation 6.11) makes the effect of tolls almost non significant. If higher values of travel time were used, the relative weight of travel cost would be more significant and the effects of pricing would be more noticeable.

VARIABLE PRICING SCHEMES CONSIDERED

As in chapters four and five, the only variable pricing scheme considered was the case of pricing all links of the traffic network. The general characteristics of the test network are the same of the experiments with constant tolls. The same demand pattern was considered. Prices were updated every one minute interval. The congestion level thresholds in the variable pricing formula (Equation 4.11) were set at 0, 1,10, 25, and 50 percent.

Experiment Results

With the variable tolls, changes in the temporal flow pattern are more evident when a density level of 10% is used. For other density levels considered, no significant difference in the temporal flow pattern is observed (Figures 6.10, 6.11 and 6.12). In this example, drivers do not appear to be very much affected by the application of variable tolls. The main reason being the low levels for the tolls. Since a small number of vehicles is loaded into the network, densities stay at low levels and tolls behave accordingly.

Figure 6.13 shows that the travel time is about the same with no discernable effect of the density level used to find the tolls. No significant difference in travel times can be observed when variable tolls are used for the number of vehicles loaded.

Travel distances, shown in Figure 6.14, are almost identical for all the toll levels considered. Differences of less than two per cent in travel distances are observed. Again the low number of vehicles loaded into the network does not affect too much the routes followed by the vehicles.

Toll revenues, shown in Figure 6.15, increase as the drivers search for routes and departure times that lead them to their destination at their preferred arrival times. They are not too concerned about the toll levels since due to the low densities in the links, the average tolls are only of about 25 cents per vehicle.

Results for variable tolls reflect similar characteristics to those of the constant tolls, the relative low importance of travel cost in departure time and route selections compared with schedule delay is reflected in the solutions.

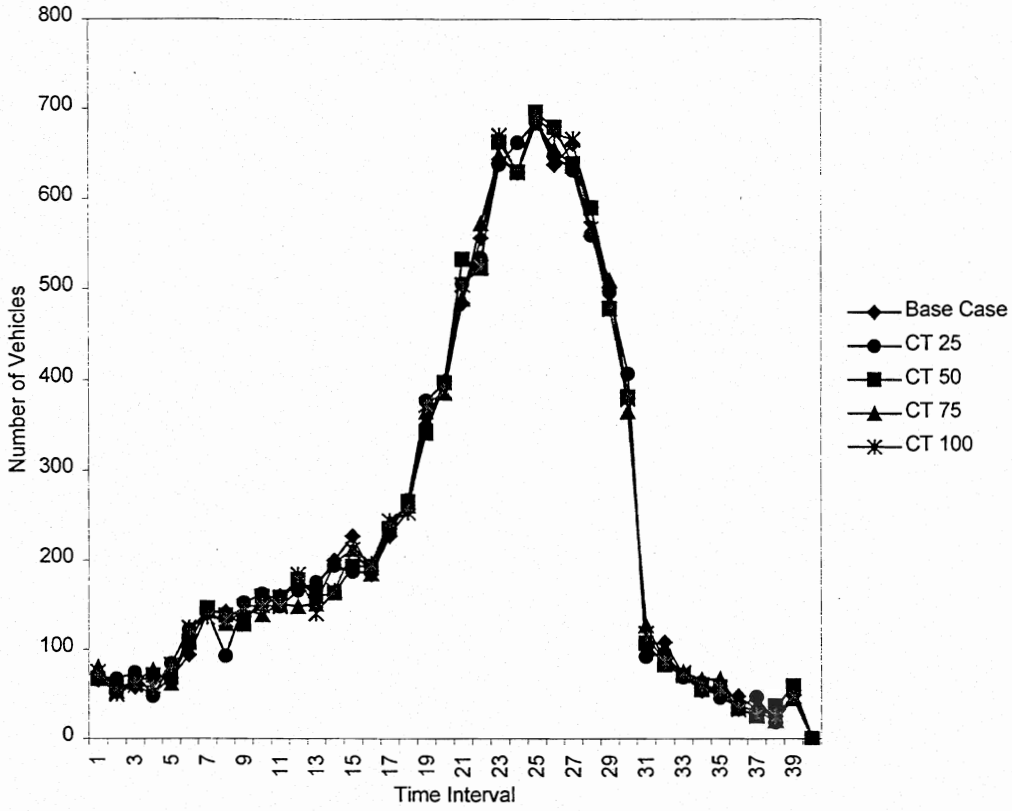

Figure 6.4 Temporal Flow Pattern for Day 2. Constant Tolls.

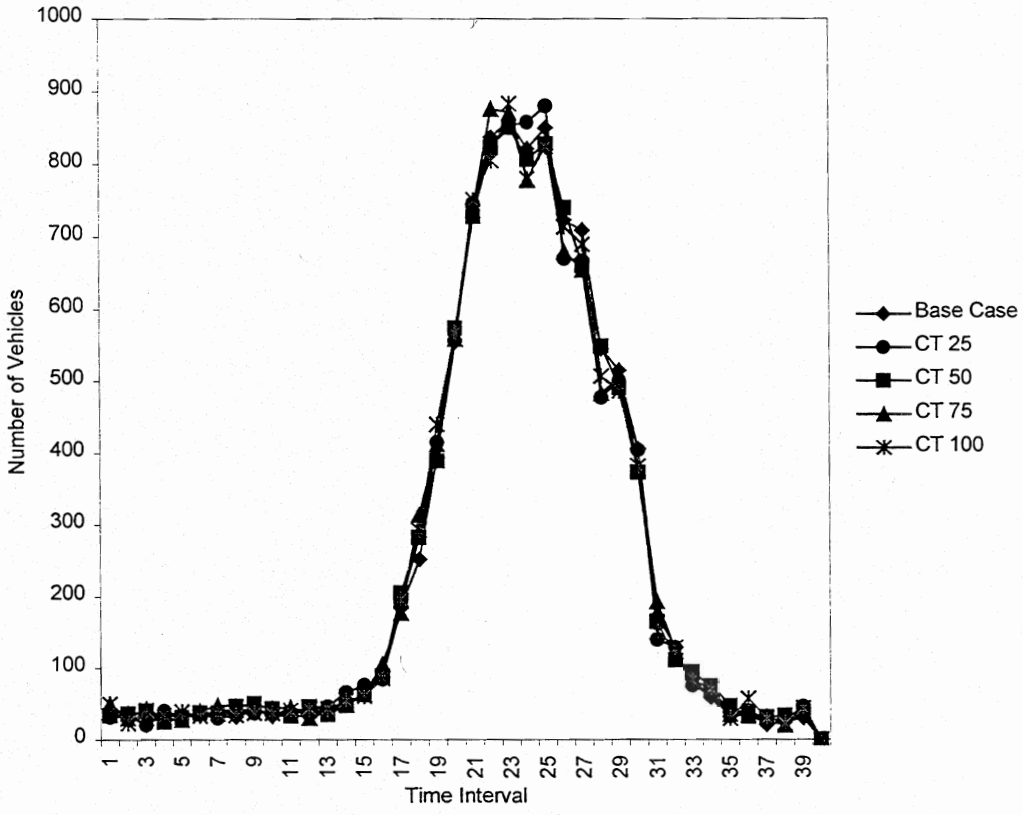

Figure 6.5 Temporal Flow Pattern for Day 5. Constant Tolls.

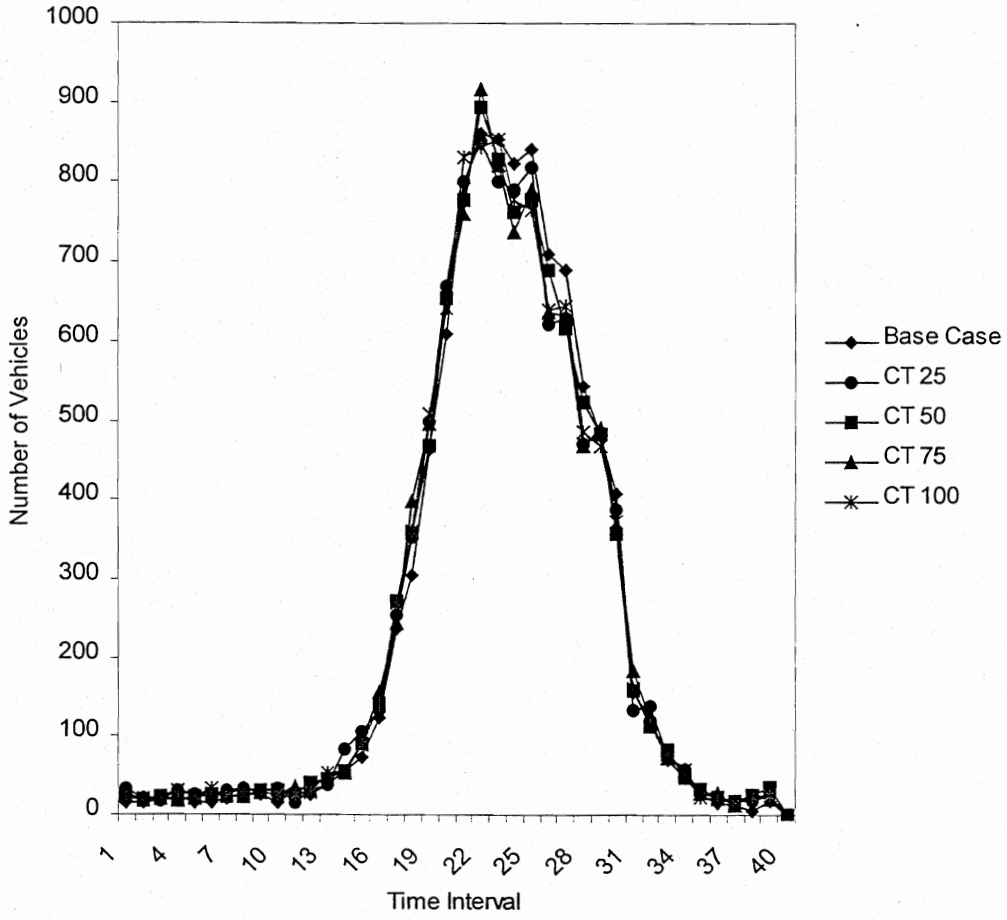

Figure 6.6 Temporal Flow Pattern for Day 7. Constant Tolls.

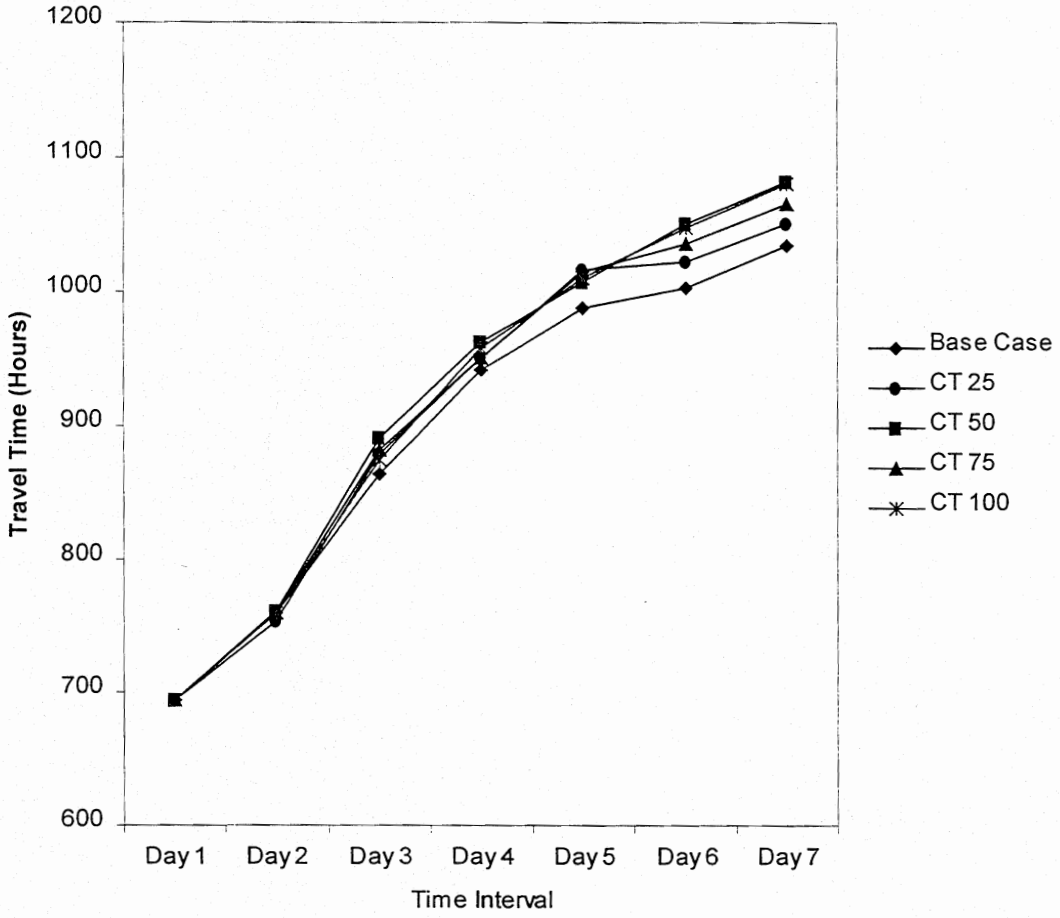

Figure 6.7 Daily Evolution of Total Travel Times for Constant Tolls.

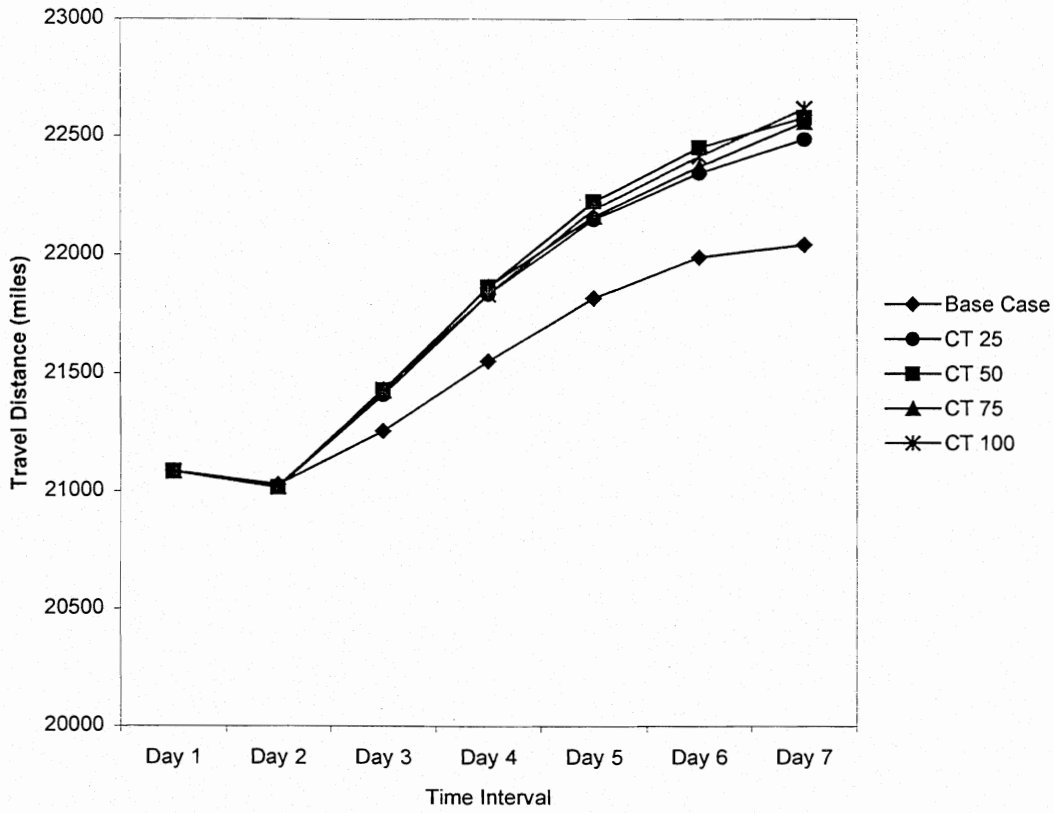

Figure 6.8 Daily Evolution of Total Travel Distances for Constant Tolls.

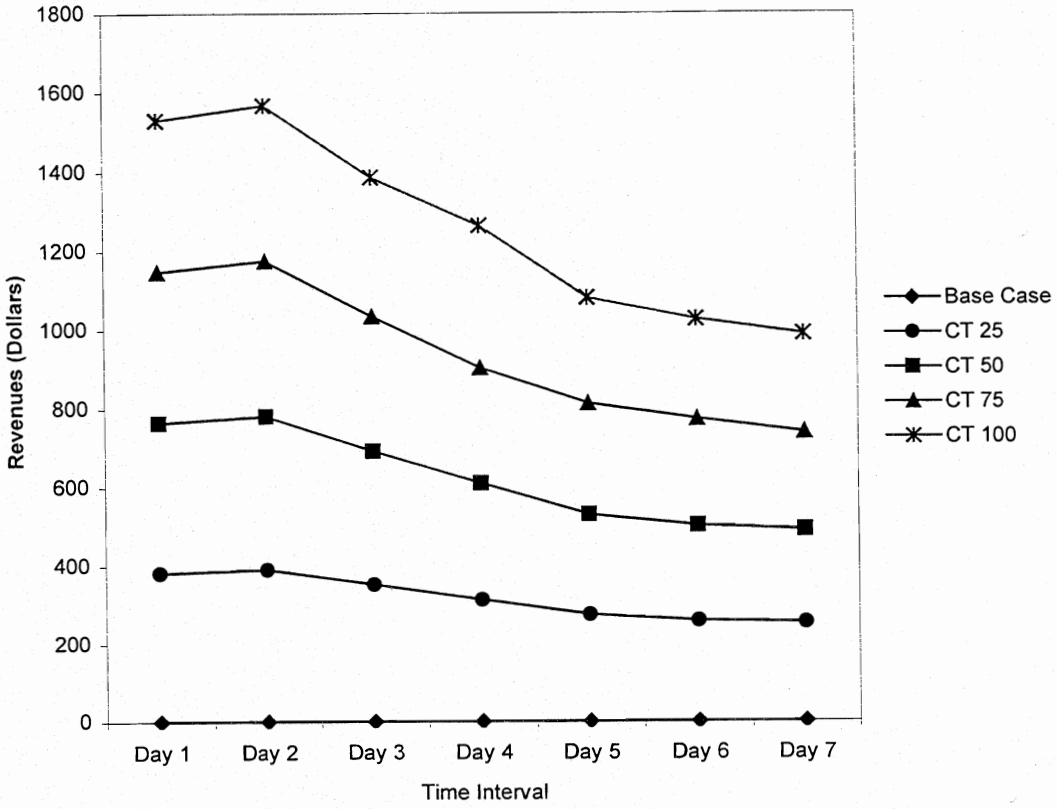

Figure 6.9. Daily Evolution of Revenues for Constant Tolls.

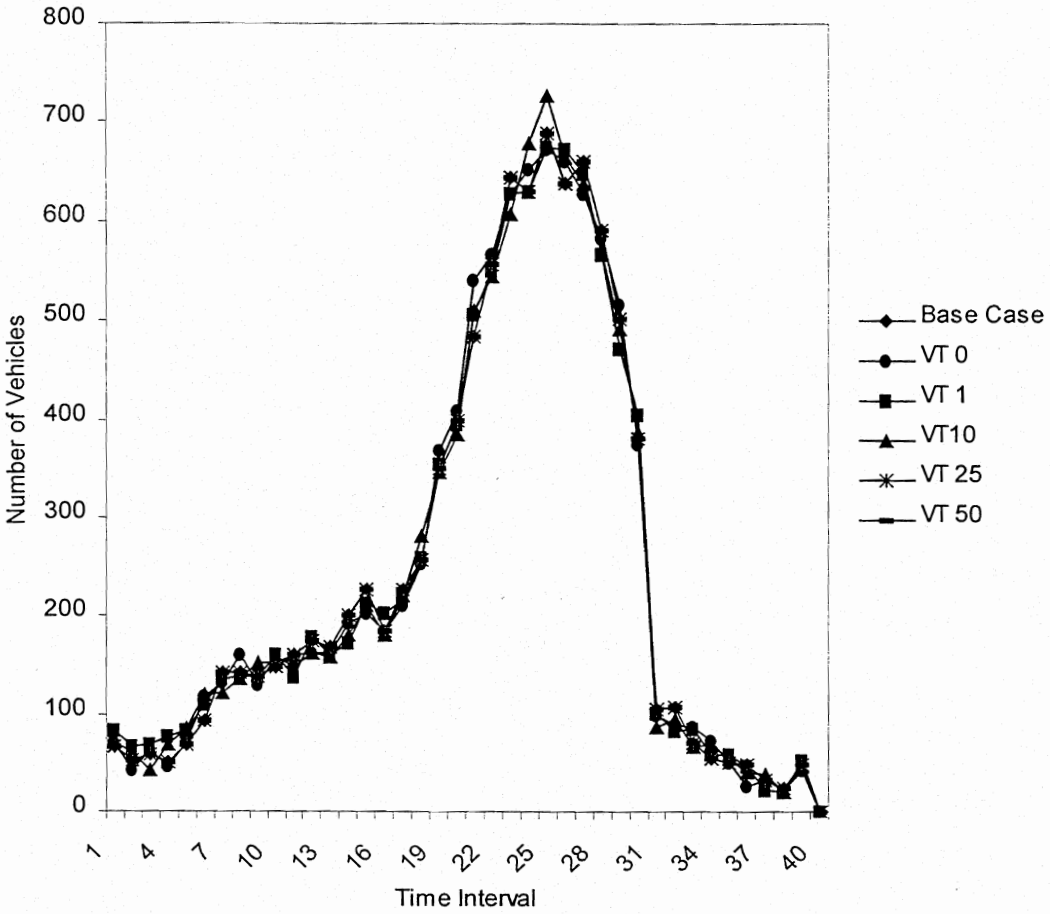

Figure 6.10 Temporal Flow Pattern for Day 2. Variable Tolls.

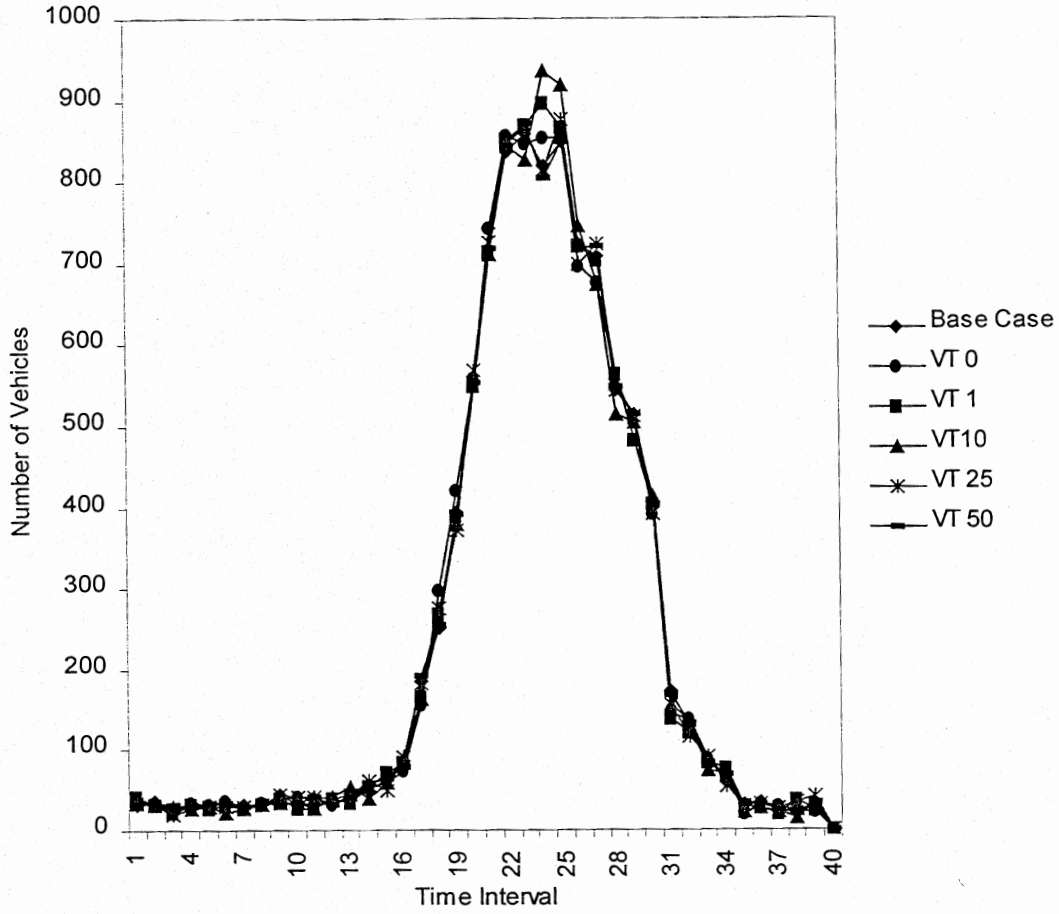

Figure 6.11 Temporal Flow Pattern for Day 5. Variable Tolls.

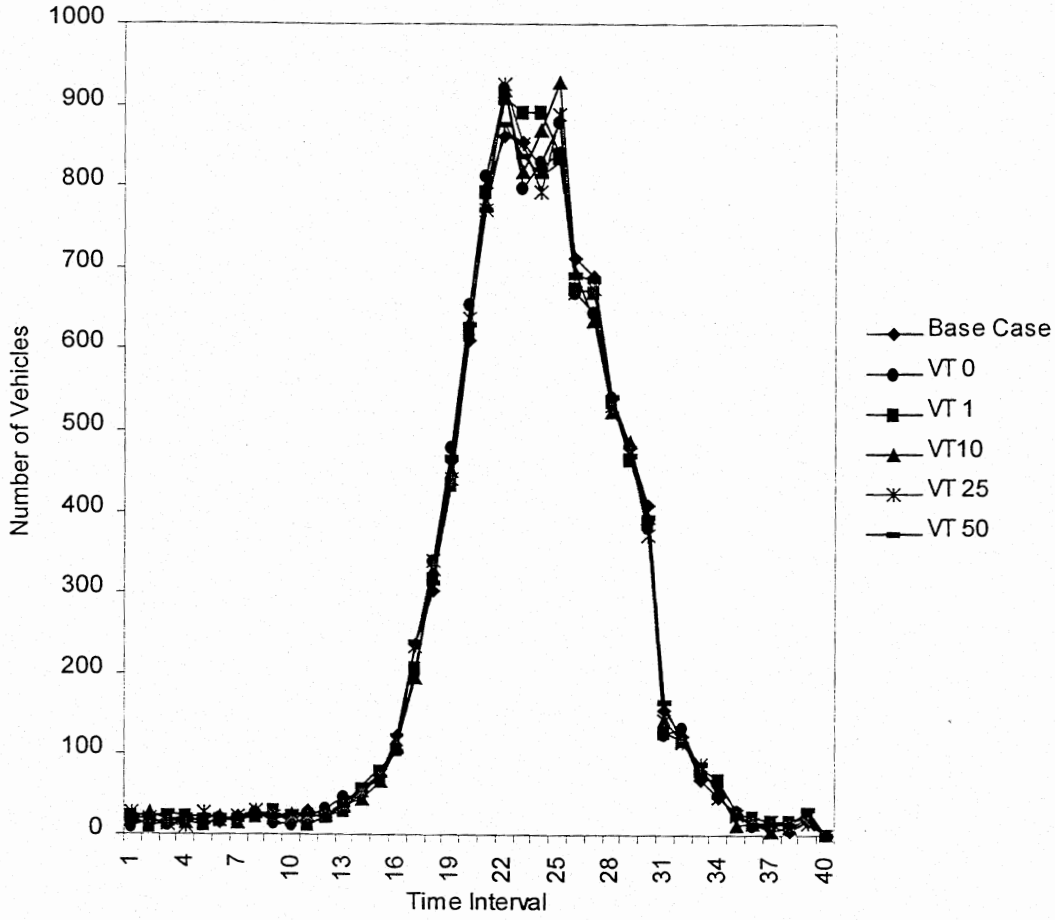

Figure 6.12 Temporal Flow Pattern for Day 7. Variable Tolls.

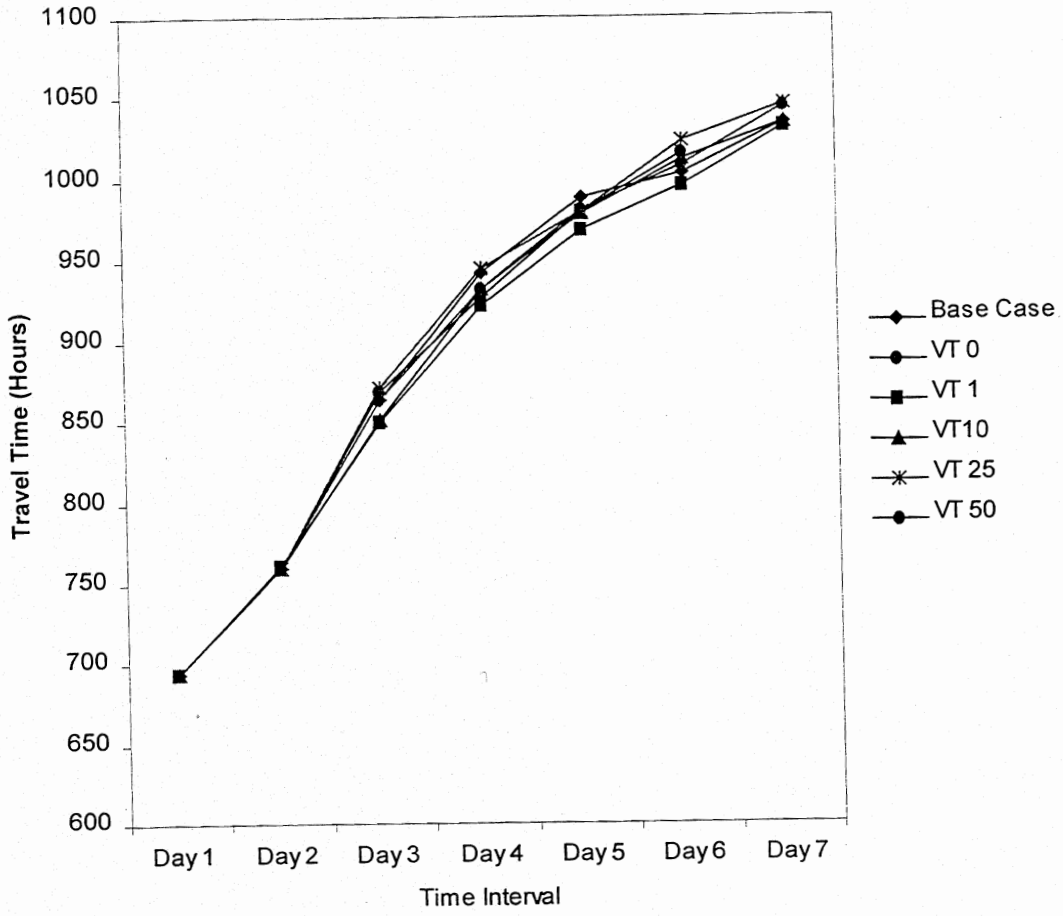

Figure 6.13 Daily Evolution of Total Travel Times for Variable Tolls.

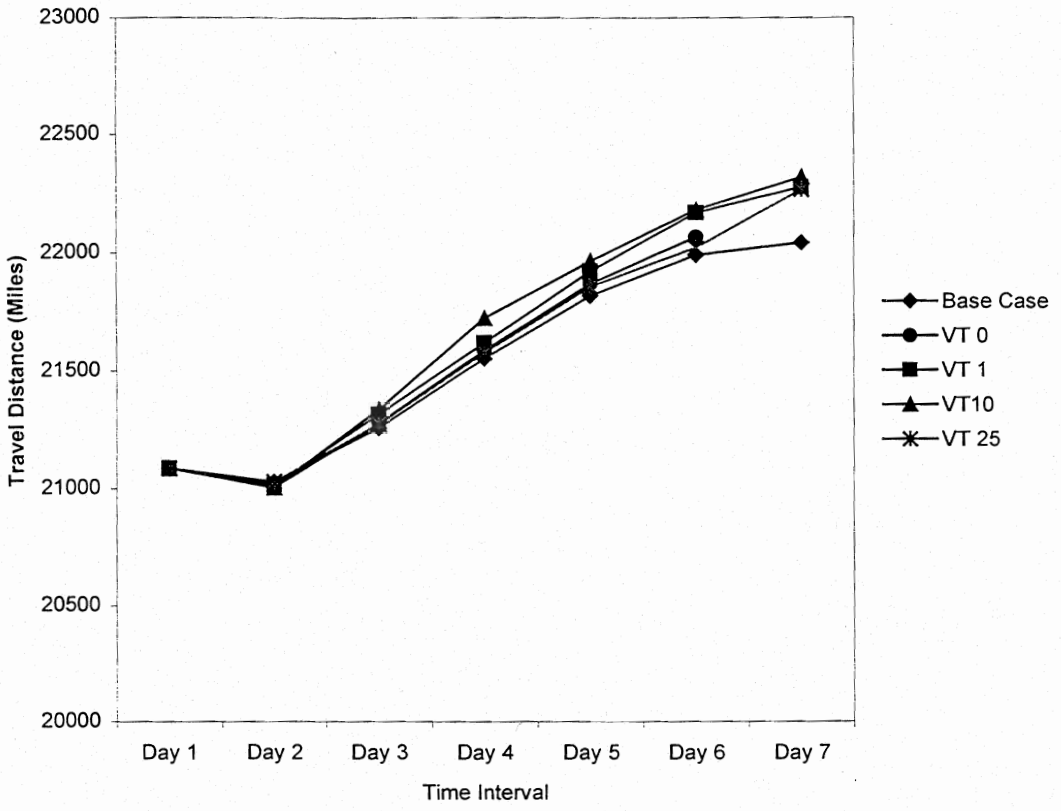

Figure 6.14 Daily Evolution of Total Travel Distances for Variable Tolls.

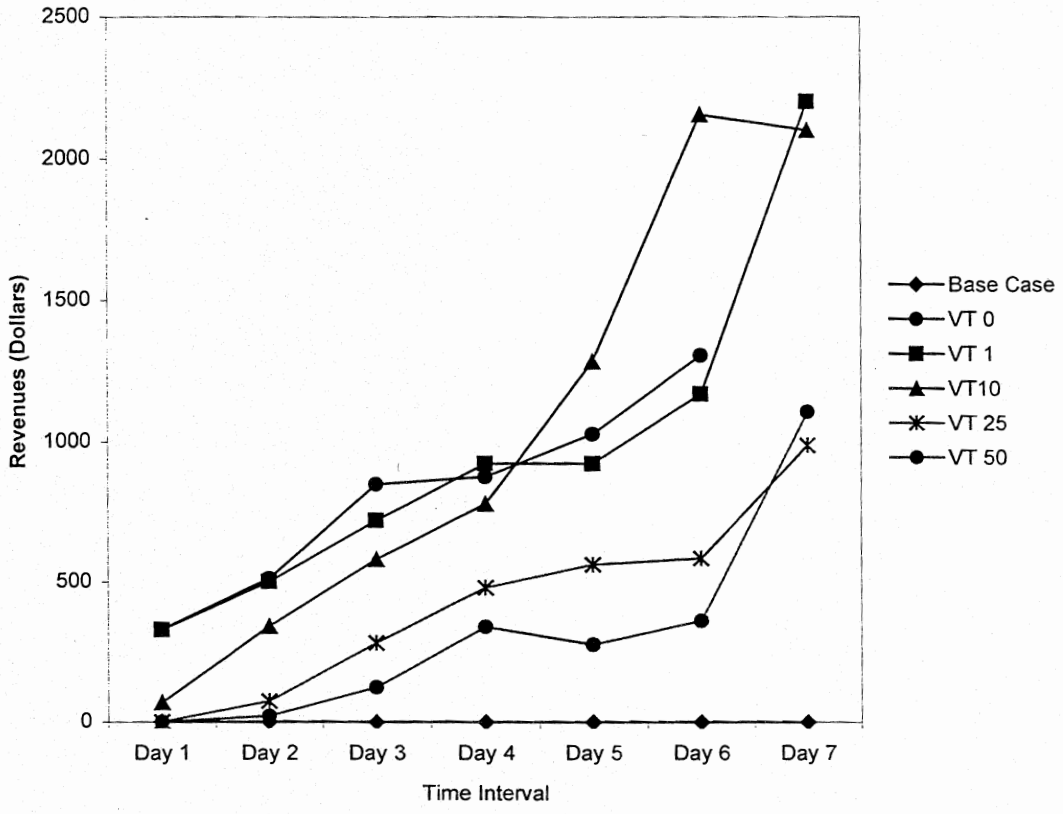

Figure 6.15 Daily Evolution of Revenues for Variable Tolls.

CONCLUSION

The application of tolls under a day-to-day dynamics assignment framework has confirmed results from previous chapters. Constant tolls affect negatively the operation of the network since drivers in their search for routes without out of pocket expenses follow longer routes which lead them to higher travel times but lower payments.

Regarding variable tolls, results are still uncertain. With the low number of vehicles loaded into the network, no clear effect on the variables considered was observed.

CHAPTER 7: CONCLUSIONS

This chapter presents final concluding comments on this report and suggests directions for future research in the area. General conclusions are presented first, contributions of this work are described second, and. last are possible directions for future research.

GENERAL CONCLUSIONS

Developments in electronic vehicle identification and fee payment technologies have changed the role of congestion pricing in transportation from a theoretical tool to control congestion to a feasible element of a transportation plan. Previously confronted technical difficulties for its implementation have been overcome by the development of new electronic devices such as smart cards.

Congestion pricing, first implemented in Singapore, is now under consideration by a growing number of communities both in the US and abroad. Congestion pricing is being used in both urban networks and rural roads. Its adoption is likely to grow over the next few years.

Congestion pricing may have a role to play in the continuing deployment of intelligent transportation systems. It will help to achieve the benefits of these technologies by managing the spatial and temporal distribution of vehicles admitted into the network in conjunction with the implementation of the ITS technologies. Drivers that might be priced off the improved roads could also benefit via improved transit services or direct subsidies.

At a time when it is difficult to secure alternative sources for the financing of new roads and traffic improvements, including ITS, and new taxes are systematically opposed, congestion pricing appears to provide an attractive financial alternative in addition to its potential operational uses. Support for congestion pricing increases when users of the road network realize the benefits they could achieve with an improved transportation system. This is particularly true when congestion pricing is used in conjunction with the opening of new facilities, on which drivers are not used to travel for free, and are hence more willing to pay for such use. Users that cannot afford to pay for the use of a new facility can continue using the old roads. They would benefit through demand redistribution between the old and new roads.

However, it is still uncertain if the benefits of congestion pricing are achievable to their theoretical limit. Although it is technically possible to charge users differentiated fees according to the marginal costs that they impose on the system, and privacy issues are now essentially resolved through new technologies, the correct determination of the optimal prices in a time-

dependent context can only be achieved in an approximate manner. Current mathematical tools are limited in this regard, though they could be improved with directed research.

This research has shown that, under the assumption of inelastic demand, and for the limited number of pricing schemes considered, constant prices applied in a limited number of links of a network or in a restricted zone, could affect negatively the operation of the entire network, regardless of the assignment criteria considered. Although such schemes could improve traffic conditions in the restricted zone or in the affected links by reducing the number of vehicles using the tolled arcs, vehicles that are priced off the tolled links may have to follow longer routes with higher travel times. Higher tolls increase revenues only due to the number of vehicles that belong to the captive population. However, there may be cases where the drivers that remain using a given road greatly benefit from the reduced number of users, and the operation of the whole network also improves.

Variable prices, unlike constant prices, show different effects for the schemes considered here, depending on the assignment criteria followed. Under very congested conditions, when vehicles are assigned to the prevailing shortest route, total travel times are smaller for tolls updated at short intervals and low densities. Higher density levels or larger update times increase the total travel time. Total trip distances are for all the cases higher than for the no toll condition. However, this increase in trip distances is not particularly large. Revenues behave in a more predictable way, decreasing as the toll update intervals are lengthened or density levels for tolling are increased.

When a user equilibrium assignment criterion is followed, the use of variable tolls reduces the total travel times for congested conditions. In the case of limited congested conditions, the total travel times are very similar to the user equilibrium with no toll operation. Trip distances are not very much affected by the use of variable tolls at any of the load factors considered. They only increase marginally at medium and congested conditions. Revenues decrease, as in the case of the assignment to the prevailing shortest route, as the density threshold level is increased.

For the day to day assignment rule set, variable tolls, for the loading factor and the value of the parameters considered, do not have a significant effect on the travel times or traveled distances. These increase, but only marginally. Revenues also increase as the iterations progress but remain generally at a very low average level for the cases considered in the experiments conducted here. Essentially, the tolls did not influence drivers decisions

significantly, as they were kept fairly low and the effect of travel times are less important than the schedule delay.

RESEARCH CONTRIBUTIONS

The most important contribution of this research is the development of a methodology for the study of the effects of constant and variable pricing in a road network under different traffic assignment criteria including current best path, user equilibrium and day to day dynamics. More generally, the methodology can evaluate various pricing schemes under different user behavior rules governing the response to pricing, information and pricing control.

This study has also developed an improved simulation-based numerical approximation to the global marginals; this approach incorporates intertemporal terms in previously estimated local marginals. Nonetheless, obtaining an exact expression for the global marginals and consequently for the optimal prices, remains a difficult and challenging task.

Within the methodology developed, the research has also shown how prices can be incorporated into a dynamic traffic simulator to reproduce a congestion pricing scheme, and how different assignment criteria can be followed to analyze the effect of pricing. This can be particularly helpful to evaluate proposed schemes before they are actually implemented.

FUTURE RESEARCH

The application of congestion pricing and the estimation of its effects are still in need of further research. Research is needed about ways to make congestion pricing more palatable to the general public. One major concern is to find an attractive redistribution of the revenues from congestion pricing. Perceived equity concerns continue to drive the public against expanded application of congestion pricing.

An approach to estimate the marginal travel times and externalities has been developed in Chapter 3. However, this estimation continues to be local in nature and as such it is still incomplete. The computation of additional terms for the global marginals can lead to better approximations and a more accurate estimation of the true costs of additional vehicles onto a network. Evaluation of pricing schemes that consider the local externalities found in this form can be an interesting avenue of further research.

Results for this study are from a computer simulation model. This can limit the credibility of the research. However, the practical implementation of a congestion pricing scheme and its evaluation is not an easy task due to its cost. It is important to take advantage of places where

congestion pricing is being implemented for the collection of information needed to evaluate future schemes.

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