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16. Abstract

Since transportation infrastructure projects have a lifetime of many decades, project developers must consider not only the current demand for the project but also the future demand. Future demand is of course uncertain and should be treated as such during project design. Previous research for Southwest Region University Transportation Center (Report 167556) explored the impact of uncertainty on roadway improvements and found neglecting uncertainty to lead to suboptimal network design decisions. This research is extended in the current work by considering not only motor vehicle traffic, but other modes as well.

The first half of this report examines the problem of network flexibility in the face of uncertainty when constructing a potentially revenue-generating toll road project. Demand uncertainty and network design are considered by way of a bilevel stochastic recourse model. The results from a test network, for which a closed form solution is possible, indicate that the value of network flexibility directly depends on initial network conditions, variance in future travel demand, and toll pricing decisions.

The second half of this report integrates Environmental Justice into the transit frequency-setting problem while considering uncertainty in travel demand from protected populations. The overarching purpose is to improve access via transit to basic amenities to: 1) reduce the disproportionate burden transit dependent populations' experience; and 2) increase the financial security of low-income households by giving them a feasible option to reduce their dependence on autos. The example application illustrates the formulation successfully increases access to employment opportunities for residents in areas with the high percentages of low-income persons, as well as demonstrates the importance of considering uncertainty in the locations of low-income populations and employment.

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Multimodal Network Models for Robust Transportation Systems

by

Jennifer Duthie

Erin Ferguson

Avinash Unnikrishnan

S. Travis Waller

Research Report SWUTC/09/167867-1

Southwest Region University Transportation Center

Center for Transportation Research

University of Texas at Austin

Austin, Texas 78712

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ABSTRACT

Since transportation infrastructure projects have a lifetime of many decades, project developers must consider not only the current demand for the project but also the future demand. Future demand is of course uncertain and should be treated as such during project design. Previous research for Southwest Region University Transportation Center (Report 167556) explored the impact of uncertainty on roadway improvements and found neglecting uncertainty to lead to suboptimal network design decisions. This research is extended in the current work by considering not only motor vehicle traffic, but other modes as well.

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The second half of this report integrates Environmental Justice into the transit frequency-setting problem while considering uncertainty in travel demand from protected populations. The overarching purpose is to improve access via transit to basic amenities to: 1) reduce the disproportionate burden transit dependent populations' experience; and 2) increase the financial security of low-income households by giving them a feasible option to reduce their dependence on autos. The example application illustrates the formulation successfully increases access to employment opportunities for residents in areas with the high percentages of low-income persons, as well as demonstrates the importance of considering uncertainty in the locations of low-income populations and employment.

EXECUTIVE SUMMARY

Since transportation infrastructure projects have a lifetime of many decades, project developers must consider not only the current demand for the project but also the future demand. Future demand is of course uncertain and should be treated as such during project design. Previous research for Southwest Region University Transportation Center (Report 167556) explored the impact of uncertainty on roadway improvements and found neglecting uncertainty to lead to suboptimal network design decisions. This research is extended in the current work by considering not only motor vehicle traffic, but other modes as well.

The first half of this report examines the problem of network flexibility in the face of uncertainty when constructing a potentially revenue-generating toll road project. A multi-stage stochastic recourse model based on network user equilibrium travel behavior was formulated in this research to answer an important question in toll road development: How does network flexibility affects project decisions? A key component of the model is the consideration of network-based managerial flexibility under travel demand uncertainty to maximize the value of toll road projects. The formulated model is a complex bilevel optimization model with an upper level (the leader) value maximization problem and a lower level (the follower) user equilibrium problem. The results from an experimental analysis on a single-origin and single-destination network, for which closed form solution was possible, show that optimal network improvements include not only capacity increases on the toll road, but also on feeder routes. Results show that as the variance of demand increases, it is better to limit the initial capacity and add capacity if it is warranted. Various toll rates were considered and the initial capacity assigned to the toll road was found to be inversely proportional to the toll rate. A zero-recourse model was also developed for comparison, and it was found that the value of recourse (or improvement of the optimal solution found using the recourse model over the zero-recourse model) is directly proportional to variance of travel demand.

The second half of this report integrates Environmental Justice into the transit frequency-setting problem while considering uncertainty in travel demand from protected populations. The need to incorporate equitable access in transit service design is demonstrated by documenting the current and anticipated continued rise in transportation and energy expenditures, growing awareness of transportation as a social equity issue, and a review of transit network design

problems. The purposes of improving equity in transit access to basic amenities are to: 1) reduce the disproportionate burden transit dependent populations' experience; and 2) increase the financial security of low-income households by giving them the option to reduce their dependence on autos. This research contributes to the transit design literature by integrating equitable access into the transit frequency-setting problem. The formulation was shown to be successful at increasing access to employment opportunities for residents in areas with the high percentages of protected persons by applying it to a mid-sized metropolitan area. The example application also reflects the financial constraints and inherent tradeoffs necessary in reaching equitable access across multiple origin-destination pairs. Finally, results indicate the higher the level of uncertainty, the more important its consideration is in order to achieve robust bus route frequencies.

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Chapter 1: Introduction

It has long been recognized among transportation researchers that future travel demand cannot be known with certainty. Several highly variable factors are central to such predictions including future political and economic conditions, spatial demographics, and development patterns. Despite this, future demand most commonly is treated as deterministic, yielding a single portrayal of future congestion upon which decisions involving project funding are made. Several reasons are given for neglecting uncertainty in future demand, including the lack of data to quantify the uncertainties present, a lack of political will to complicate the decision making process, the computational intractability of network analysis techniques applied to realistic sized networks for multiple scenarios, and a lack of knowledge as to how incorporating uncertainty can improve predictions. A previous SWUTC report focused on the last issue by developing methodologies for treating demands between each pair of origin-destination (O-D) pairs as uncertain. The current report extends this work to the case of multiple modes: toll road users versus non-toll road users, and transit.

Considering multiple modes gives us more flexibility to improve the transportation system under uncertainties. This research was originally motivated by the question: "Which mode is the most 'robust' given the uncertainties planners face?" The current work takes a step towards answering this question by developing methods to plan for robust transportation systems that include modes other than only the automobile. Chapter 2 focuses on the automobile, allowing for methodologies that are similar to the ones developed in the previous SWUTC report, but consider toll road users separately from non-toll road users and working towards the objective of maximizing profit in a public private partnership toll road. The goal is to find the optimal capacity for initial construction of the toll road as well as the amount of capacity to add once the true demand level has been realized. Chapter 3 focuses on the transit mode, specifically buses. The goal of this problem is to set transit route frequencies such that equity across various population groups (e.g., grouped by income level) is maximized. While the research in this report does not answer the initial question about the most robust mode of transportation, the methods developed herein for considering robustness within models of various different modes is a necessary step towards answering this question.

Chapter 2: Network Flexibility in Toll Road Projects

2.1 Introduction

Networks are fundamental to the analysis of many engineering systems. Typically, infrastructure systems constitute networks that are distributed over wide geographic areas and composed of links with different characteristics and utilization patterns. Network representation is particularly apparent in transportation and energy systems where individual links (e.g., highways or high-voltage transmission lines) interconnect to create a complex network topology that delivers services to the public. In such settings, where each link interacts with the other elements of the system, project valuation is inherently tied to network conditions.

Revenue generating highway projects (i.e., toll roads) are increasingly being used to lessen the pressure on strained public finances while meeting the growing demand for transportation system capacity. The concept of toll roads (either privately or publicly-owned) is not new. While many similarities can be drawn from the past experiences, there is at least one distinct difference between modern toll roads and those of the 1800s: the complexity of the existing roadway network. Early toll roads were the very beginnings of an emerging network and their revenue was affected by little else than the point-to-point demand (and of course the toll evasion rates). Toll roads today in any metropolitan area are very much affected by the extensive network topology and modal interconnectivity.

The strategic position of a toll road in the surrounding transportation network is an important factor in assessing long-term project revenue risks (Schaufelberger and Wipadapisut, 2003; Tiong, 1990). While economic development and population growth – both uncertain in the long-term – are primary determinants of the future origin-destination (O-D) demand, the actual traffic distribution (or assignment) is determined by the network topology and link conditions changing over time. Such conditions include increased congestion, deterioration of roadway condition, changes in toll pricing strategies, as well as construction of new network links that lead to changes in network topology.

Hence, recognizing the time-dependent nature of the problem, it is important to consider multiple stages of decision making – the initial decision to build the toll road, as well as the opportunities to influence network parameters in each time period following the initial decision. In reality this phenomenon has already been observed as some project developers have proposed to finance improvements of publicly-owned and operated roadways to stimulate traffic growth on their toll roads. By exploring the opportunity to alter network topology to re-assign the traffic and increase the toll road demand, as well as by specifying contractual limitations on what the public sector can do to their part of the network (e.g., non-compete clauses), long-term project revenue risks can be reduced and managed. In such settings, it is clear that managerial flexibility goes beyond project-specific actions (such as option to defer or abandon project), and includes network flexibility in both temporal and spatial domains (e.g., which network link to improve and at what time). While the term 'flexibility' can be used to describe the management's ability to enact a variety of options, here we will focus only on actions to the network (e.g., adding a lane on a feeder link) that occur after the uncertainty associated with future conditions is realized. Such actions are also known as "network recourse" actions.

Even though the implications of transportation network flexibility are important and far-reaching, to the best of the authors' knowledge, the early investigative efforts to value toll road projects have been limited to project-specific representations of demand and flexibility. The objective of this research is to develop a model capable of evaluating the effects of uncertainty in future demand and network flexibility on project development decisions. The model is capable of capturing temporal and spatial interactions between project decisions, debt obligations, and network structure, answering a fundamental question in toll road development: *How does network flexibility affect project decisions*? In other words, will projects be developed differently if management is given the ability to alter initial project decisions (e.g. toll link capacity) in response to the realizations of uncertainty in future travel demand and opportunity to change the capacity on the surrounding network links following the initial decision?

The remainder of this chapter is organized as follows. The next section summarizes the state-of-the-art in modeling uncertainty in toll road demand. Background literature on project valuation and traffic demand modeling is reviewed in context of the objective of this research. Then, two new stochastic programming models are presented. The first allows for initial decisions on the toll road capacity as well as recourse actions on the network after uncertain parameters are realized. The second model - formulated for comparison and quantifying the value of recourse -allows only for initial toll road capacity decisions. Following the model formulations, numerical analysis is conducted on a test network for which a closed-form solution is available. Finally, the chapter concludes with a discussion of managerial implications and limitations of this study.

2.2 Modeling Toll Road Demand

Representation of uncertainty in future demand is an integral component of project valuation. Typically, uncertainty in project revenue is assessed on the project (link) level using a mathematical representation of discrete- or continuous-time stochastic processes (Garvin and Cheah, 2004; Zhao and Fu, 2006; Ford, 2002). Huang and Chou (2006) used a generalized Wiener process to model toll road operating revenues, while Zhao and Fu (2006) developed a state-dependent revenue function within the framework of a stochastic dynamic programming model. Ho and Liu (2002) implemented a lognormal process to solve for equity value using a binomial lattice approach, and Chiara *et al*. (2007) investigated multiple-exercise real options in infrastructure projects. While these models allow for representation of revenue uncertainty for valuing project-specific managerial flexibility including option to defer (Luehrman, 1998), abandon (Dixit, 1989), and alter the operating scale (Tregeorgis, 1993; Dixit, 1989), they fall short of accounting for complex network interactions and the effects of *network-based real options* on managerial decisions. The research presented here fills this gap by treating travel demand uncertainty directly within the framework of network and routing under equilibrium conditions.

In practice, there is no standard method for predicting toll road demand. Kriger *et al*. (2006) describe the three most common methods used to estimate toll road demand: 1) assignment of users to all roads – tolled and "free" – in a route choice model, 2) treatment of the choice to use toll roads explicitly within a modal split model, and 3) application of diversion curves to calculate the new facility's share of the existing traffic. The model presented here is based on the first option, converting monetary units to time *via* a "value of time" parameter. Unlike diversion curve methods, which do not capture traveller behavior, and treating toll roads as a separate mode, which requires iteration between the modal choice and route choice models, the method used in this research requires the use of only one model and captures traveler behavior through a generalized cost approach to Wardrop's first principle (1952). Since deterministic analyses are not able to measure risk, the network model developed herein is stochastic.

Uncertainty in future demand was considered by a number of researchers. Lam and Tam (1998) used Monte Carlo simulation methods to study the impact of uncertainty in traffic and revenue forecasts for road investment projects. The authors assumed normal distributions for each of several uncertain parameters including population and demand elasticity. Chen *et al*. (2001) implemented a network model to evaluate measures of profit and risk for a new investment, and Chen *et al*. (2003) solved for the optimal toll rate and capacity to maximize expected profit. While these efforts considered network structure and uncertainty in demand, they were limited to static project development conditions, neglecting the impact of temporal and spatial managerial flexibility.

Yang *et al*. (2004) defined a model to optimize toll rates between each pair of entry and exit points along a roadway such that maximizes total revenue and social welfare, while Subprasom *et al*. (2003) solved a similar problem using a genetic algorithm and expanded the uncertainty set to include not only demand but also cost estimates and value of time. From the perspective of network design, previous research has focused on toll rate setting (Ho *et al*., 2005; Mun *et al*., 2005; Joksimovic *et al*., 2005), competition among private toll road operators (Calcott and Yao, 2005; Xiao *et al*., 2007), and the interactions between pricing, capacity improvements and financing in a single time period problem (Verhoef and Rouwendal, 2004). Unlike previous literature, the model presented here is a multi-stage model.

2.3 Model Formulations

Two stochastic programming models are presented in this section. TRND-R (Toll Road Network Design with Recourse) is a stochastic recourse model designed to evaluate the effects of network flexibility on toll road development decisions. TRND-NR (Toll Road Network Design with No Recourse) is presented for comparison, to allow a value to be placed on the ability to take network recourse actions as future uncertainties are realized. Stochastic programming models are used to determine optimal decisions under risk and uncertainty. A thorough treatment of stochastic programming can be found in Birge and Louveaux (1997).

Stochastic programs are anticipative since a decision (e.g., initial toll road capacity) must be chosen by using the *a priori* probability measures of the uncertain parameters, $\xi(\omega)\ \forall\ \omega \in \Omega$ where Ω is the set of all realizations, without the opportunity of making additional observations. Consider now the situation when corrective – second stage actions, known as *recourse actions*, can be taken after uncertain parameters are realized. A formulation that incorporates both *a priori* actions as well as corrective adjustment actions in a single mathematical model is known as a *stochastic recourse model*.

2.3.1 TRND-R model

A stochastic recourse model, TRND-R, is introduced to evaluate the effects of using managerial flexibility to change a transportation network's link conditions on toll road development decisions. To simplify the analysis, but also to gain insights into model behavior, the developed model is limited to two stages. Stage I is a speculative stage in which initial investment decisions such as toll road capacity are made. Hence, actions taken in the first stage (building the toll road with initial capacity x), denoted by x, are anticipative since they are taken before the realization of the uncertain parameters (future demand) $\xi(\omega)$. On the other hand, Stage II is a corrective stage in which the environment ω is observed and recourse actions (adding capacity to the network links, such as feeder links), denoted by $y(\omega)$, are selected. In this formulation, action x precedes the recourse actions $y(\omega)$ given uncertainty in demand between pairs of origins and destinations, d ($\xi(\omega)$=d(ω)).

Project debt servicing obligations are assumed to be proportional to the Stage I investment, x. To account for credit risk, we consider a Stage II penalty. If the project fails to meet a predefined revenue threshold (assumed proportional to the initial investment by a factor Ψ) during any time period, a penalty term is applied. Although the model presented here does not account for recovery given default, it can be easily extended to take into account stochastic recovery rate and more complex credit risk assessment methods.

Let z_R defines the Stage I objective function value for TRND-R as follows:

$$z_R = \max_{x \geq 0} E_\xi[Q_R(x, \xi(\omega))] - \gamma x \tag{1}$$

where γ represents the unit cost of capacity and $E_{\xi}[Q_R(x,\xi(\omega))]$ represents the expected recourse function. The Stage II objective function value for a given realization of demand $\xi(\omega)$ is maximization over the set of recourse actions $y(\omega)$ and can be defined as:

$$Q_R\left(x,\xi(\omega)\right)=\max\left\{q_R\left(x,y(\omega),\xi(\omega)\right)\middle|\,y(\omega)\geq 0\right\} \tag{2}$$

where the second-stage objective function $q_R(\cdot)$ is defined as:

$$q_R\left(x,y(\omega),\xi(\omega)\right)=\sum_{t=1}^{T}\frac{\phi_t(x,y(\omega))}{(1+i)^t}-\rho\max\left\{0,L_t(x,y(\omega))\right\}-\gamma y(\omega) \tag{3}$$

It is important to note that Stage II is a bilevel problem. To obtain the flow on the toll road, $v^*_{l=tollroad}$, the lower level user equilibrium problem (UE) must be solved. Solving this problem uniquely determines the flows for every link l given the initial capacity on the toll road, x, and network capacity additions $y(\omega)$. Finally, given a toll rate r, toll road revenue $\phi_t(\cdot)$ and losses $L_t(\cdot)$ in time period t for $\forall\,\omega\in\Omega$ are defined as:

$$\begin{aligned}\phi_t\left(x,y(\omega)\right)&=r\times v^*_{l=tollroad,t}\left(x,y(\omega)\right)\\ L_t\left(x,y(\omega)\right)&=\Psi\gamma x-\phi_t\left(x,y(\omega)\right)\end{aligned} \tag{4}$$

The UE conditions are defined in equations 5-7, similarly to Beckmann *et al*. (1956). UE was first stated by Wardrop in what is now commonly referred to as Wardrop's first principle: *the journey times in all routes actually used are equal and less than those which would be experienced by a single vehicle on any unused route* (Wardrop, 1952). The time index is ignored since the problem is solved in the same way for each year.

$$v^*_{l=tollroad}(x,y(\omega))=\arg\min\left\{f(v)\middle|\,v\in V\right\} \tag{5}$$

where the constraint set V is defined in equation (6) with parameters defined as follows: **A** is the link-path incidence matrix, **B** is the origin-destination pair/path incidence matrix, h is the path flow vector, and $d(\omega)$ is realization ω of the O-D pair travel demand matrix;

$$V=\left\{v\middle|\,v=\mathbf{A}h,d(\omega)=\mathbf{B}h,h\geq 0\right\} \tag{6}$$

and $f(v)$ is the UE objective function defined in equation (7):

$$f(v) = \sum_{l \in \mathrm{L}} \int_{\kappa=0}^{v_l} c_l\left(\kappa, x, y(\omega)\right) d\kappa \tag{7}$$

where $l \in \mathrm{L}$ represents a set of links, and $c_l(\cdot)$ is the vector of generalized link costs. The solution varies by time period due to changes in the O-D travel demand, which can be represented by an autoregressive function:

$$d_t(\omega) = \alpha + \beta d_{t-1}(\omega) \tag{8}$$

For the purposes of this research, only the initial travel demand realization is considered uncertain. Demand in all succeeding years follows the autoregressive function. Since we assume recourse occurs only in Stage II, the following equality holds for the vector of capacities in each time period: $\mu^{t=1}(\omega) = \mu^{t=2}(\omega) = \ldots = \mu^{t=T}(\omega),$ such that $\mu^{t=1}(\omega) = \mu^{t=0} + x + y(\omega)$, where $\mu^{t=0}$, is the vector of initial link capacities (equals zero for toll road).

2.3.2 TRND-NR model

Consider now a situation in which a project developer has no network recourse and must make all capacity decisions before the realization of uncertainty. Then, the problem becomes a bilevel stochastic program with only one capacity improvement vector, x. The value (recourse) function $Q_R(x)$ reduces to $Q_{NR}(x)$ as follows:

$$E_{\xi}[Q_{NR}(x, \xi(\omega))] = \sum_{\omega=1}^{\Omega} \sum_{t=1}^{T} \frac{r \times v^{*}_{l=tollroad,t}(x, \omega)}{(1+i)^t} - \rho \max\left\{0, L_t(\omega)\right\} \tag{9}$$

This simple recourse function $Q_{NR}(x, \xi(\omega))$ represents an expectation over the set of possible realizations $\omega \in \Omega$ without considering optimal corrective recourse action $y(\omega)$ in the next stage; hence, the objective function of the no-recourse problem can be expressed as:

$$z_{NR} = \max_{x \geq 0} E_{\xi}[Q_{NR}(x, \xi(\omega))] - \gamma x \tag{10}$$

2.3.3 Summary of models and value of network recourse

Table 2.1 summarizes and contrasts differences between TRND-R and TRND-NR model. As it can be observed from Table 2.1, the hierarchical structure of TRND-R and TRND-NR is apparent. Both models have an upper-level or "leader" (maximization) problem, and a constraint defined by a lower-level, or "follower" equilibrium (minimization) problem. This type of problem, equivalent to *Stackelberg game* problem in mathematical game theory, is referred to as *bilevel optimization problem* (Bard, 1998).

Table 2.1. TRND-R and TRND-NR Model Summary

	Stage I	Stage II
TRND-R	$z_R = \max E_\xi[Q_R(x,\xi(\omega))] - \gamma x$ $s.t.$ $x \ge 0$	$E_\xi[Q_R(x,\xi(\omega))] = \max \sum_{\omega=1}^{\Omega}\sum_{t=1}^{T} \frac{r \times v^*_{l=tollroad,t}(x, y(\omega))}{(1+i)^t} - $ $- \rho \max\{0, L_t(x, y(\omega))\} - \gamma y(\omega)$ $s.t.$ $v^*_{l=tollroad,t}(x, y(\omega)) = \arg\min\{f(v) \mid v \in V\}$ $V = \{v \mid v = \mathbf{A}h, d(\omega) = \mathbf{B}h, h \ge 0\}$ $y(\omega) \ge 0$
TRND-NR	$z_{NR} = \max E_\xi[Q_{NR}(x,\xi(\omega))] - \gamma x$ $s.t.$ $x \ge 0$	$E_\xi[Q_{NR}(x,\xi(\omega))] = \sum_{\omega=1}^{\Omega}\sum_{t=1}^{T} \frac{r \times v^*_{l=tollroad,t}(x,\omega)}{(1+i)^t} - $ $- \rho \max\{0, L_t(\omega)\}$ $s.t.$ $v^*_{l=tollroad,t}(x,\omega) = \arg\min\{f(v) \mid v \in V\}$ $V = \{v \mid v = \mathbf{A}h, d(\omega) = \mathbf{B}h, h \ge 0\}$

In general, bilevel problems are non-convex, non-differentiable, and **NP** -hard problems (Bard, 1991; Deng, 1998). A special case of bilevel problems in which lower-level control action set is limited to a fixed number of variables has been solved using polynomial approximation (Liu and Spencer, 1995); however, to the best of authors' knowledge, a general polynomial approximation scheme has not been reported in literature. In his seminal book about bilevel optimization, Bard (1998) provides insights into the behavior of this class of models and summarizes the relevant solution approaches.

While finding a solution to general bilevel problems is quite complex, TRND-R and TRND-NR reduce to smooth nonlinear optimization problems if the lower-level equilibrium problem is convex and the network path-flows are uniquely determined. To gain insights into the model behavior and its managerial implications, in the next section we consider a sample network with unique flows in which identification of feeder and competing routes is trivial.

To assess the value network flexibility, the value of network recourse (*VNR*) is introduced. This index represents a percentage of increase in project value if network recourse is allowed, and is defined as follows:

$$VNR = \frac{z_R}{z_{NR}} - 1 \tag{11}$$

2.4 Analysis of Network with Unique Path Flows

To test the effects of network recourse on project value, the formulations presented in the previous section are implemented on the four-link test network shown in Figure 2.1. Each link in the network is labeled with the corresponding link flow (v) and path flow (h) variables. Link 3 is tolled and demand for travel, d, exists between nodes A and C. The transportation network topology is such that links 1 and 4 compete with the tolled link, and link 2 acts as a feeder. In general, this formulation can be easily extended to consider other O-D demand pairs. However, in such cases a clear connection between the feeder link 2 and the toll road link 3 would not be apparent.

The path flows are uniquely determined by the link flows and the generalized cost function is linear, so a closed-form solution to the UE problem can be derived. While these assumptions do not hold for general transportation networks, especially for larger networks, making them allows insights to be gained into the problem.

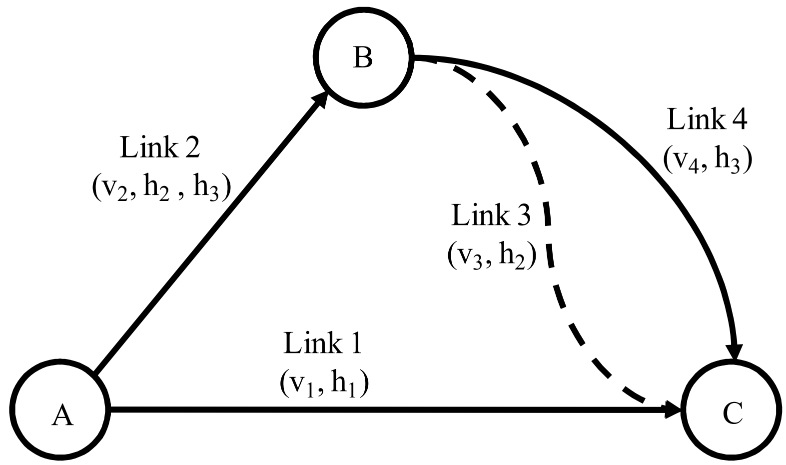

Figure 2.1. Test network

Let $c = \theta_1 + \theta_2 (x/\mu) + r$ represent the generalized link cost function where *r* is a vector of link tolls (equals zero for all non-toll roads), and θ_1 and θ_2 are parameters. The value of time is implicitly assigned to unity, but other values could be easily implemented. Integrating the cost function over link flows can be expressed as:

$$\int_{x=0}^{v_l} c_l(x)dx = \theta_1 v_l + \frac{\theta_2}{2\mu_l} v_l^2 + r v_l \quad \forall l \in \mathrm{L} \tag{12}$$

Since the resulting cost-flow functions are quadratic, the objective function of the lower-level UE problem is also quadratic. Hence, the problem becomes convex with linear constraints. The unique flow solution of this problem can be found using a Lagrangian formulation and the first order necessary and sufficient conditions.

The closed form path flow solutions are as follows. Flow on the toll road, path 2 is:

$$h_2 = \left(\frac{\mu_3}{\mu_4} h_3 - \frac{\mu_3 r_3}{\theta_2} \right) \tag{13}$$

where flow on the complementing (quasi-competing) path (path 3) flows is defined as:

$$h_3 = \frac{\frac{\theta_2 d}{\mu_1} - \theta_1 + \frac{\mu_3 r_3}{\mu_2} + \frac{\mu_3 \mu_4}{\mu_1}}{\theta_2 \left(\frac{\mu_3}{\mu_4 \mu_2} + \frac{\mu_3}{\mu_4 \mu_1} + \frac{1}{\mu_2} + \frac{1}{\mu_4} + \frac{1}{\mu_1} \right)} \tag{14}$$

and where the flow on the toll road's competing path (path 1) is defined as:

$$h_1 = d - h_2 - h_3 \tag{15}$$

The above closed form solution of path flows confirms expected user behavior. The higher the toll rate, the lower the flow on path 2, which consists of the tolled road link 3 and a feeder link 2. While link 1 clearly defines a competing route, link 4 can be considered a quasi-competing, or complementing link. This can be observed from equations 13 and 14. The flow on path 2 depends not only on $[\mu_3 / \mu_4]$, but also indirectly on $[(\mu_3 \mu_4)/\mu_1]$. This indicates that even for simple networks with unique path flows, interconnectivity effects can be significant and sometimes counterintuitive.

With a closed form solution of the lower level - network user equilibrium problem, the TRND-R model presented in the previous section reduces to a nonlinear optimization problem. While the Stage I formulation does not change, Stage II for the considered network with unique path flows and linear cost functions can be defined as follows:

$$\begin{aligned}
& E_\xi[Q_R(x,\xi(\omega))] = \max \sum_{\omega=1}^{\Omega} \sum_{t=1}^{T} \frac{r_3 \times h_2^t(x, y(\omega))}{(1+i)^t} - \\
& \qquad\qquad - \rho \max\{0, L_t(x, y(\omega))\} - \gamma y(\omega) \\
& s.t. \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (16) \\
& h_2^t(x, y(\omega)) = \left(\frac{\mu_3^t(x, y(\omega))}{\mu_4^t(x, y(\omega))} h_3 - \frac{\mu_3^t(x, y(\omega)) r_3}{\theta_2} \right) \qquad \forall \omega \\
& y(\omega) \geq 0 \qquad \forall \omega
\end{aligned}$$

In contrast to Stage II of the TRND-R model presented in equations 2-8, toll road demand in the Stage II formulation in equation 16 is explicitly defined. Hence, the earlier bilevel problem is reduced to nonlinear stochastic optimization model, which can be represented as a nonlinear optimization model for a finite number of demand realizations. Since the TRND-NR model for the unique path network is a straightforward extension of the TRND-R model, its formulation is omitted in this report.

Sensitivity of toll road demand with the respect to lower-level user equilibrium solution is tested first. The parameters used for analysis are as shown in Table 2.2, unless otherwise noted. While units are omitted for purpose of generalizing results, local conditions (such as unit cost of capacity, risk-free discount rate and value of time parameters) can be easily incorporated.

Table 2.2. Model Data

θ_1	θ_2	r_2	Ψ	ρ	γ	i
1	5	3	0.5	3	0.8	0

Figure 2.2 presents the sensitivity of toll road link flow with respect to marginal increases in capacity of either the feeder link or the toll link, given fixed demand, *d=1000*. The *y*-axis indicates a difference in toll road link flow (h_2) given an equal increase in capacity of either the toll road or the feeder road. The legend gives the initial capacities on each of these links before any additional capacity increases are made. For simplicity, the initial capacity on link 2 $\left(\mu_{l=2}^{t=0}\right)$ is shown as μ(2,0), while the initial capacity on the toll road, link 3 $\left(\mu_{l=3}^{t=0}\right)$ is shown as μ(3,0).

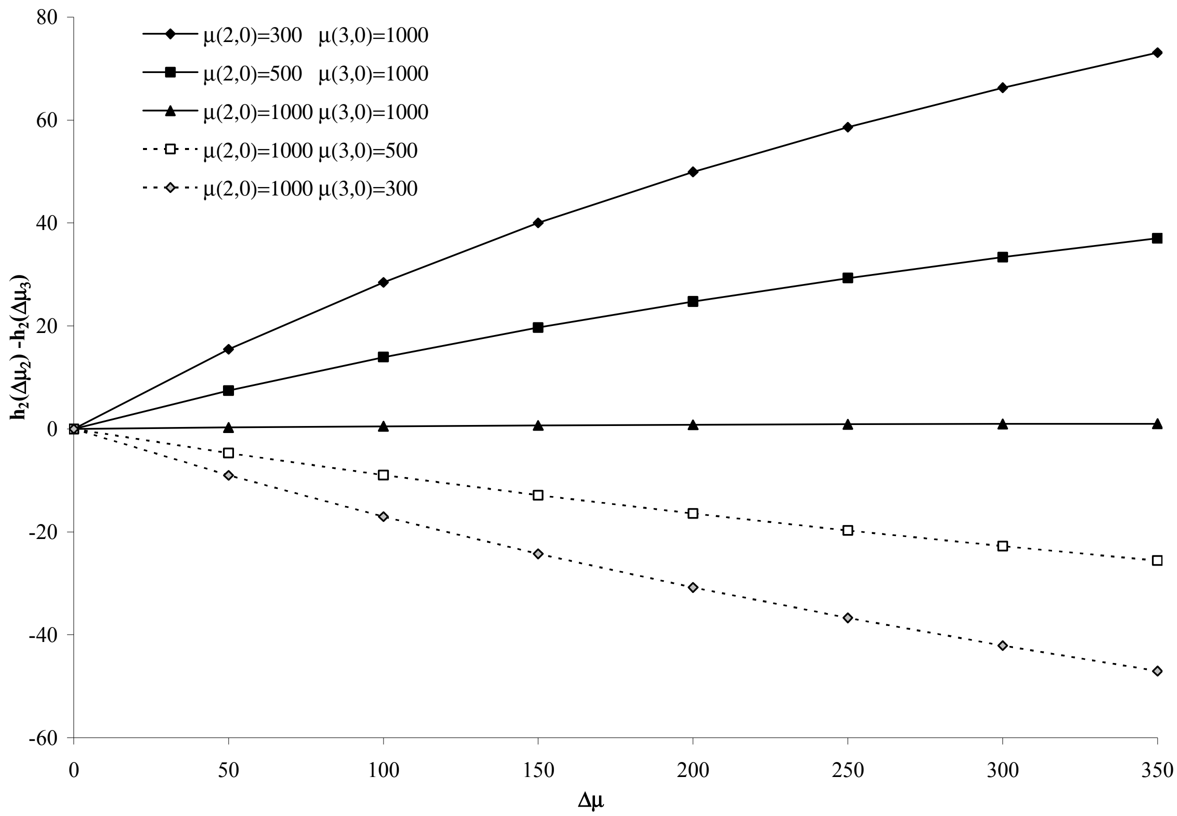

Figure 2.2. Toll road flow vs. change in initial capacity

When the toll road capacity is initially high, capacity improvements to the feeder link, link 2, have a greater impact than capacity increases to the toll road. When the feeder link capacity is initially high, the reverse is true. When both links have high initial capacities, the impacts of increasing their capacities are approximately the same. Hence, it is important to consider the capacity on feeder links in toll road project development. If the feeder route represents a bottleneck, it is more cost-effective to improve feeder links on the public part of the network, than to add new capacity on the toll road.

Uncertainty in demand is modeled using realizations of three potential scenarios: $d(\omega 1)=1000$, $d(\omega 2)=1500$, and $d(\omega 3)=2000$. The probability of realizing each demand value, $\Pr(d(\omega))$, is varied to test the sensitivity of the solution with the respect to changes in variance of demand, calculated as:

$$V_{\xi}(d) = \sum_{\omega \in \Omega} \Pr(d(\omega))\left[d(\omega) - E_{\xi}(d(\omega))\right]^2 \tag{17}$$

Figure 2.3 depicts the optimal first stage decision (initial capacity installed on the toll road) X^*, and the optimal second stage decision Y^* (expansion of the capacity on the network links after realization of demand uncertainty $d(\omega)$). The values of the optimal decision variables X^* and Y^* are determined for a range of toll rates *r*. First, it can be observed from the figure that non-zero recourse was observed only for higher demand scenarios $\omega 2$ and $\omega 3$. This indicates that for low realizations of traffic demand between origin-destination pairs A and C, network flexibility has no value. This is also intuitive and expected. If there is a low demand between the origin and destination, feeder links will not represent a bottleneck; hence, network recourse will not be warranted.

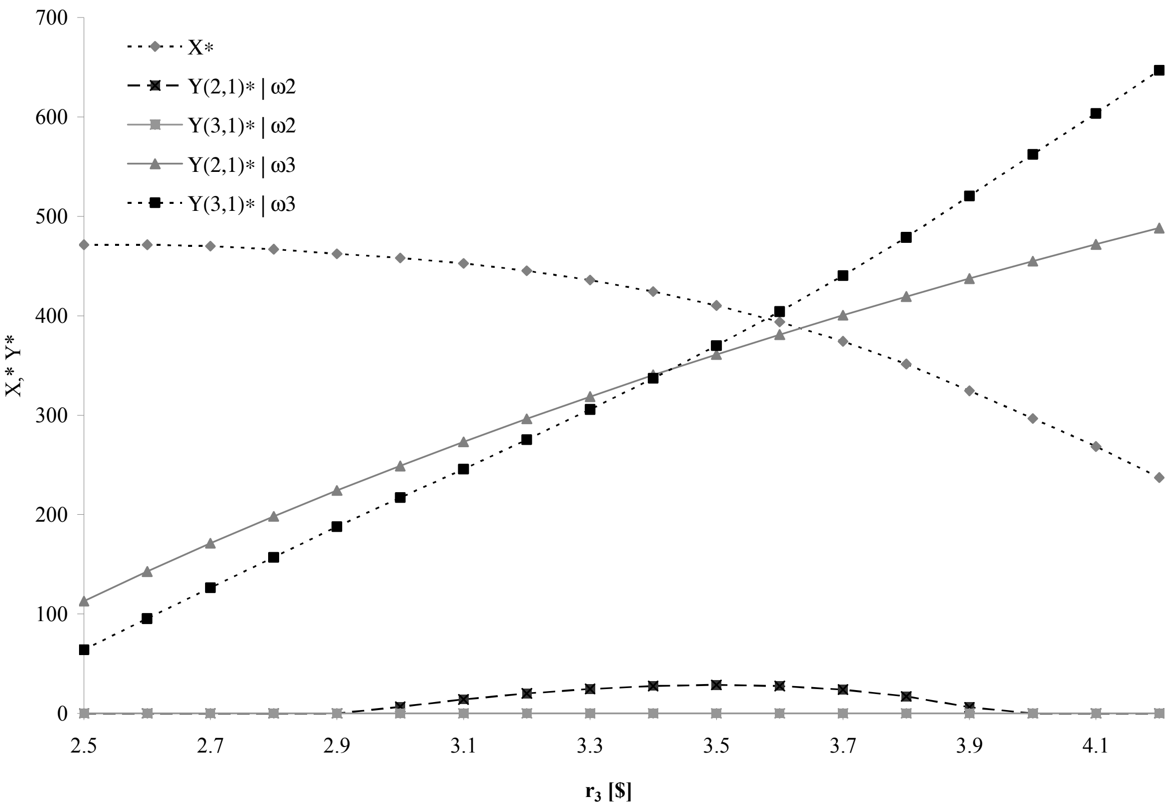

Figure 2.3. First and second stage capacity vs. toll rate

Second, it can be observed from Figure 2.3 that the optimal value of first stage decision X^* (optimal initial capacity on the toll road) decreases with an increase in toll rate *r*. Hence, with consideration of higher toll rates, capacity decisions should be postponed to the next stage when the uncertainty is resolved. In other words, if a developer considers high toll rates, the optimal development strategy is "wait and see". In the first stage (initial development phase) avoid overinvestment in toll road capacity; then, in the second stage, if realization of demand warrants it, install additional capacity on the feeder and tolled link. In the network illustrated in Figure 2.1, with initial link capacities of competing and quasi-competing links 1 and 4 given respectively as $\mu(1,0)$=500 and $\mu(4,0)$=300, the impact of second stage improvements in capacity on a feeder link is greater than the impact of improvements in capacity on the tolled link. The values of the capacity improvements can be also interpreted as shadow prices. In other words, following a realization of demand, a shadow price for the feeder link is higher than a shadow price for the tolled link. For demand realization $d(\omega 3)$, this holds only when the toll rate is less than \$3.40. This indicates that for higher toll rates, improvement in capacity of the tolled link becomes more cost-effective. Therefore, the optimal second stage decision is to reduce path cost by adding additional capacity on the tolled link rather than on the feeder link.

Figure 2.4 shows the sensitivity of the optimal first and second stage decision variables, X^* and Y^*, with respect to the increase in uncertainty measures - $V_\xi(d)$. Keeping the rest of the model's parameters constant, the results indicate an anticipated trend: with an increase in demand uncertainty, $V_\xi(d)$, and risk penalty, ρ, the optimal level of initial installed capacity on the tolled link, X^*, decreases.

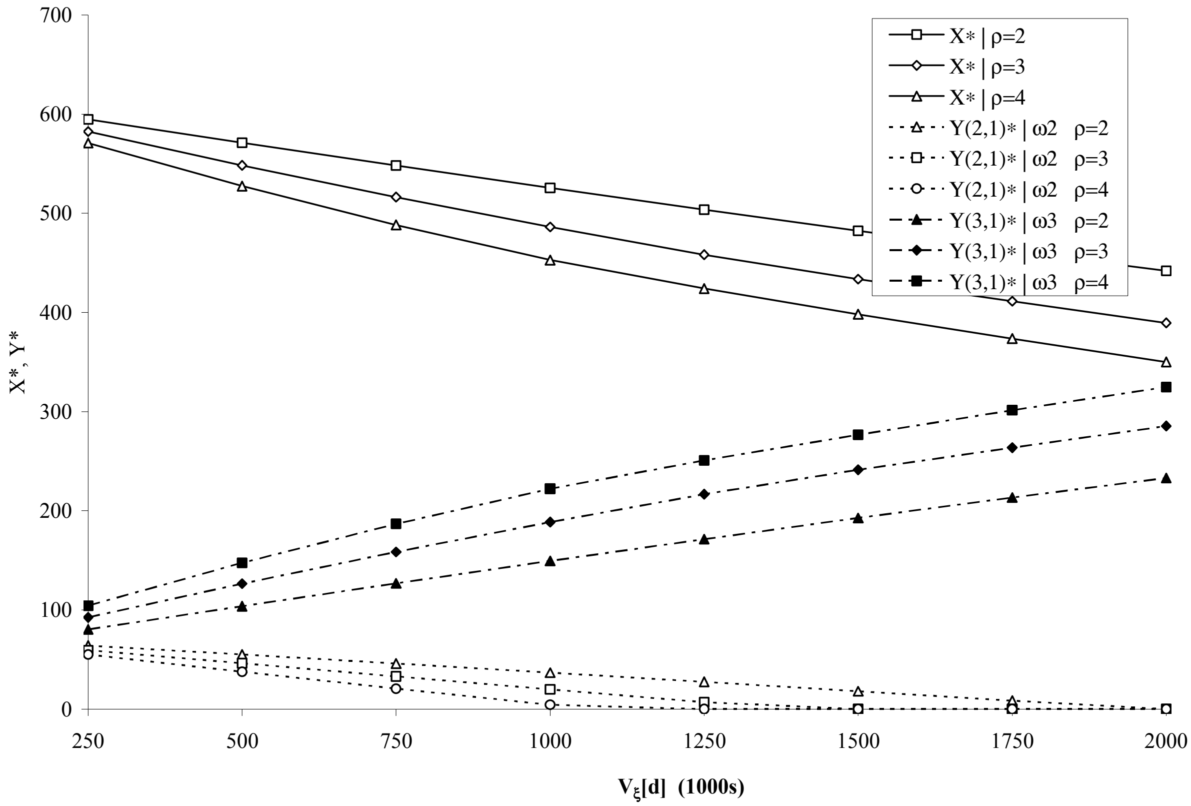

Figure 2.4. First and second stage capacity vs. variance in demand and risk penalty

Figure 2.4 further shows the optimal level of recourse variables Y^* given three different scenarios ω, a penalty term ρ, and a variance $V_\xi(d)$. As previously discussed, for scenario $d(\omega 1)$, where demand realization is low, recourse actions have no value. For demand scenario $d(\omega 2)$, network recourse has a value only for capacity improvements on the feeder link 2. This is mainly to initial specification of the network in which the feeder link 2 represents a bottleneck for traffic to be routed on the tolled link 3. For demand realization $d(\omega 3)$, the model indicates the following: a constant capacity improvement on the feeder link independent of changes in variance $\left(\mathrm{Y}(2,1)^*\middle|\omega 3 = 250\right)$, and an increase in capacity added to the tolled link 3 in the second stage $\mathrm{Y}^*(3,1)$ with an increase in variance. Note that for clarity of presentation of the results, the line indicating the constant value of recourse of feeder link 2 under scenario 3 is omitted.

The value of recourse actions must be observed in relation to the first stage decisions. With increasing variance and risk penalty, it becomes optimal to limit initial investment and act opportunistically as uncertainty is resolved: the greater the uncertainty – the greater the value of next stage network recourse. This is clearly illustrated for demand scenario $d(\omega 3)$ when there is high demand realization. Here, the model shows that the bottleneck in capacity on the feeder link 2 is compensated regardless of the $V_{\xi}(d)$, while additional value of network flexibility is gained by increasing the capacity on the tolled link.

Figure 2.5 illustrates VNR with respect to changes in $V_{\xi}(d)$ and shows that recourse is more valuable for projects associated with larger uncertainty, an expected result. VNR is initially higher for the lower values of initial capacity on competing link 1. As $V_{\xi}(d)$ increases, however, the trend reverses. This indicates that the impact of network flexibility and the VNR are not only conditional on $V_{\xi}(d)$, but they are also affected by the initial specification of the network and link capacities.

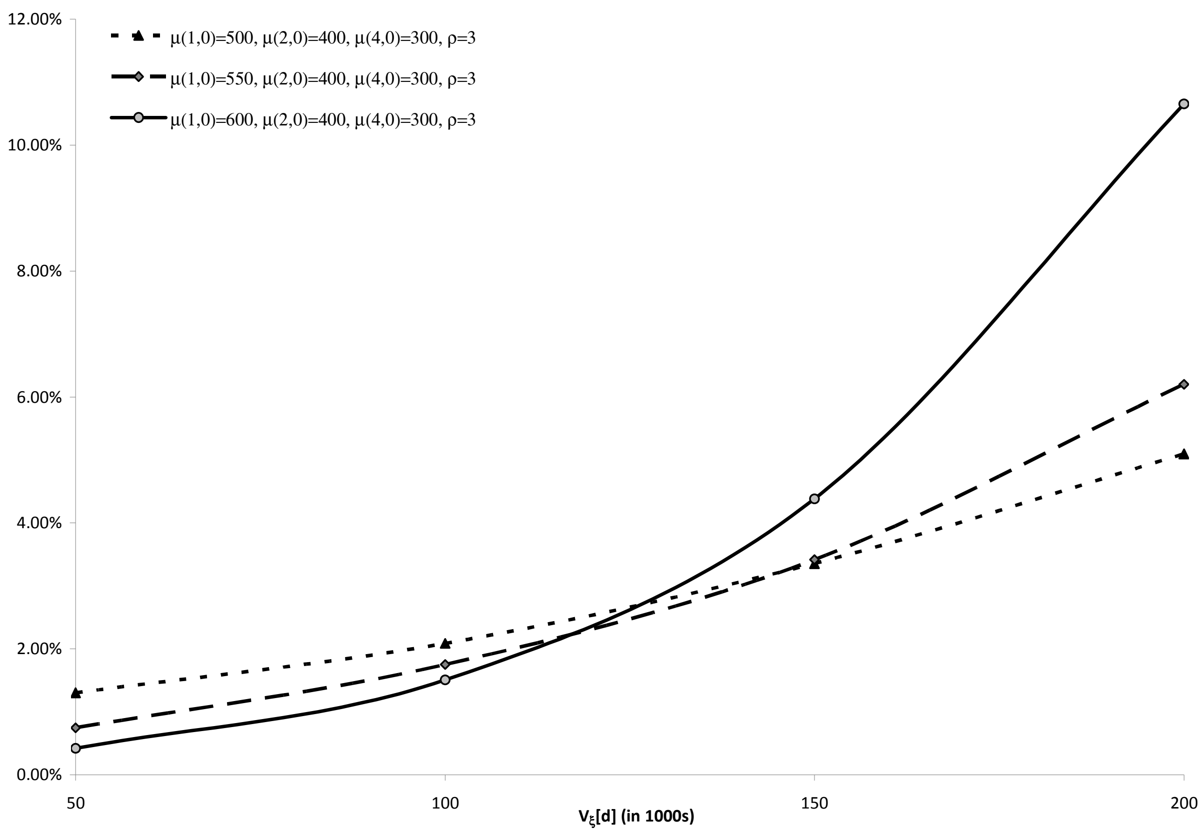

Figure 2.5. Value of network recourse vs. variance of demand with increase in competing link

Similar trends can be observed in Figure 2.6. The results show that VNR decreases as the initial capacity on feeder link 2, $\mu(2,0)$, is increased. In contrast to the observed effects on VNR of increasing $V_{\xi}\left(d\right)$ and the capacity on competing link 1, shown in Figure 5, VNR increases proportionally with a decrease in feeder link capacity. The value of network recourse for transportation networks with heavily constrained feeder links grows exponentially with increase in variance. This is an important conclusion as provision of network flexibility could result in consideration of toll road projects that are initially not feasible.

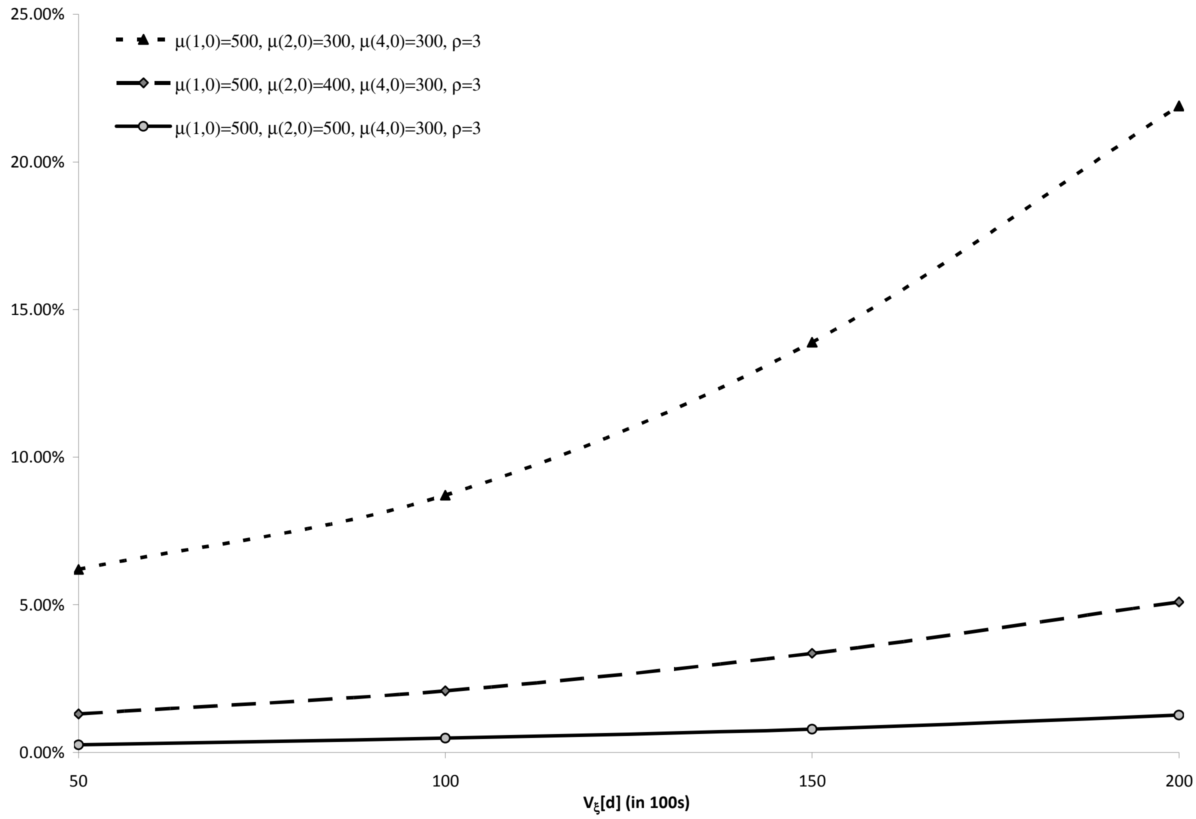

Figure 2.6. Value of network recourse vs. variance of demand with increase in feeder link capacity

2.5 Managerial Implications, Limitations and Model Extensions

The results presented in this research highlight several managerial implications. First, failure to account for network-based managerial flexibility in interconnected projects such as toll roads can result in significant asset undervaluation. The value of this recourse comes from two sources: stage-based installment of new capacity, and capacity improvements on the links operated by the public-sector. Even though this research did not explicitly consider the effects of public sector lack of flexibility to improve conditions on its part of the network (e.g., "non-compete" clauses, budget constraints), the analysis techniques and results can be extended to consider such situations. Here, it is lack of public sector flexibility (to expand the link capacity and minimize total system travel time) that can potentially increase project value for the private sector. Further, network recourse is of particular importance for long-term contracts where the network structure is bound to be changed affecting the flows and the project's competitive position in the network. In such settings, both preventive "non-compete" clauses on competing links and contractual options to improve feeder links in the surrounding network assure that the project's strategic position is preserved.

The second important managerial implication resulting from this research is that network flexibility affects initial design. This implication is equivalent to stage-based design, where design strategy expands to include part of the network links affecting the flow on a toll road project. The values of network flexibility and recourse increase with an increase in uncertainty of model parameters, complexity of network structure, and the consequence of failure to meet debt obligations.

Third, network flexibility implicitly affects credit risk. Rating agencies examine the effects of dynamic changes in network conditions based on the ability of the appropriate agency to repay project debt. In general, this assessment is based on predicting broad effects of the state and federal capital improvement programs as well as the plans of regional and local transportation commissions. For example, the likelihood that competing links will be built or improved is based on an assessment of the degree of cooperation between various levels of government agencies and toll operators and authorities. If such cooperation exists, the likelihood that competing links will be developed is lessened. The TRND-R model formalizes this process to account for marginal impacts of specific network design options to manage project risks.

Fourth, toll pricing decisions affect optimal design strategy. Project developers would benefit from realizing that pricing decisions can limit their ability to explore network-based options as risk management tool.

There are, of course, limitations to the models presented in this chapter. While TRND-R represents an effective framework that can account for recourse, its computational tractability is constrained by nonlinearity and non-convexity. The lower level problem of TRND-R, UE with static link flows) is inherently non-linear, thus requiring iterative solution methods. Other methods for assigning are available and could also be explored, specifically the stochastic user equilibrium model where user's perception of travel time is assumed imperfect. The Method of Successive Averages (MSA) and the Frank-Wolfe (FW) algorithm (Frank and Wolfe, 1956) are two common approaches.

TRND-R can be easily extended to multi-stage situations in which the next stage recourse would be nested in the previous stage structure, increasing the complexity of the problem. To address this issue, different bounding, approximation and simulation methods can be considered including Sequential Approximation Method (SAM), Monte Carlo simulation, and augmented Lagrangian algorithm. Further, recent advances in the application of metaheuristics to non-convex non-smooth problems promises new solution approaches for this class of bilevel problems.

Another way to extend TRND-R is to change the lower level problem from static UE to a dynamic traffic assignment (DTA) model. Using DTA would allow for analysis of time-dependent tolling, provision of real-time information, and evacuation strategies. Several DTA formulations are available in the literature (see Peeta and Ziliaskopous (2001) for an overview), and simulation-based models are the most popular due to their applicability to large-scale networks. However, altering TRND-R in this way would further increase its complexity.

It is also important to note that TRND-R considers network flexibility isolated from other managerial options. In reality, a number of real options are available to project developers. Hence, interactions among different real options with network-based options can exist both in spatial and temporal domain.

TRND-R can be easily extended to quantify the value of network-based real options in other inherently interconnected projects, such as transmission lines and pipelines. The bilevel structure of the model for other interconnected projects is unlikely to change. The lower level network behavior model typically involves optimization or solution to systems of nonlinear equations. Finally, the concept of considering network interconnectivity in project development can be even further expanded to account for multi-layered networks and modal transfer points.

2.6 Conclusions

A multi-stage stochastic recourse model based on network user equilibrium travel behavior was formulated in this research to answer an important question in toll road development: *How does network flexibility affect~~s~~ project decisions*? A key component of the model is the consideration of network-based managerial flexibility under travel demand uncertainty to maximize the value of toll road projects. The formulated model is a complex bilevel optimization model with an upper level (the leader) value maximization problem and a lower level (the follower) user equilibrium problem. The results from an experimental analysis on a single-origin and single-destination network, for which closed form solution was possible, show that optimal network improvements include not only capacity increases on the toll road, but also on feeder routes. Results show that as the variance of demand increases, it is better to limit the initial capacity and add capacity if it is warranted. Various toll rates were considered and the initial capacity assigned to the toll road was found to be inversely proportional to the toll rate. A zero-recourse model was also developed for comparison, and it was found that the value of recourse (or improvement of the optimal solution found using the recourse model over the zero-recourse model) is directly proportional to variance of travel demand.

Chapter 3: Incorporating Environmental Justice into the Transit Frequency-Setting Problem

3.1 Introduction

The objectives of this chapter are to: 1) illustrate the need for a new methodology to consider issues of equitable access and mobility in transit route and service design; 2) present a new methodology for addressing equitable access from a transit route and service design perspective; and 3) demonstrate the proposed approach by applying the methodology to transit service in a mid-sized metropolitan area.

Public transportation is called upon to fulfill many roles from decreasing pollution levels and reducing United States' dependence on foreign oil to sparking economic development and redevelopment in urban areas (Bailey, 2007; Center for Clean Air Policy, 2009; Center for Transit Oriented Development, 2009). One of transit's original roles and a motivation for transit subsidies has been to serve individuals who are unable to travel by private auto. This role is increasing in importance in light of recent volatility of gasoline prices and a rise in household expenditures on transportation. Transportation expenditures recently reached the second highest share in American household budgets (Haas and Makarewicz, 2008). Coupled with the increase in transportation costs is an increase in housing prices, which is compounded by inadequate growth of the average American household income. From 2000 to 2005, average household income rose 10.3% with transportation costs increasing by 13.4% and housing costs increasing 15.4% (Lipman, 2006). As a result, the percent of the population facing the real possibility of no longer being able to afford travel by personal auto is on the rise, particularly with the national unemployment rate reaching 9.5% in June 2009 (Bureau of Labor Statistics, 2009).

One opportunity for households to improve their financial security is by relying more on transit for access to basic needs (e.g., employment, supermarkets, medical service). Bailey (2007) found households in proximity to transit service that choose to get rid of a personal auto saved on average $6,251 annually. However, this elevates an existing issue facing many U.S. metropolitan areas and transit agencies: regular transit users, partially or fully dependent on transit, experience the lowest levels of mobility among all U.S. population segments (Giuliano, 2005). Therefore, to make transit service an attractive and feasible mode to reach basic amenities, and in-turn a reasonable option for low-income households to become more financially secure, the levels of accessibility and mobility experienced by regular transit users needs to be improved.

Current development in the U.S. is biased towards the personal auto so much so those unable to travel by auto, or have limited access to an auto, experience undue burdens and restrictions on their ability to meet basic needs. Travel by transit in many U.S. metropolitan areas tends to be restrictive in when and where one can travel and time consuming. This combination results in reduced access mobility for individuals partially or completely dependent on transit. These individuals carry an inequitable burden in satisfying basic needs and in some instances forgo needs, such as medical services, due to the absence of feasible transportation options, or cut expenditures on other necessities, such as food, to make travel by auto financially possible (Giuliano, 2005; Wallace et al., 2005; Gicheva et al., 2007). Previous research has illustrated workers from low-income households dependent on transit are also disproportionately limited in employment opportunities with reduced access to their regional economy (Center for Clean Air Policy, 2009). Land use patterns, segregated by type and distributed inequitably across neighborhoods, contribute to the burden of reduced access experienced by transit dependent populations. For example, the distribution of supermarkets has been found to vary based on socio-demographics with low-income and minority neighborhoods home to 25% to 70% fewer supermarkets than middle income and predominantly white neighborhoods (Powella et al., 2007), requiring some residents to travel on average an additional 1.1 miles to reach a grocery store (Zenk et al., 2005).

Collectively, these studies and recent events point to the need for improved transit and land use planning considering equitable levels of access to basic amenities via different transportation modes. This chapter focuses on equitable levels of access from a transit service design perspective; it is applicable under current and evolving land use patterns (by treating the size of the protected population and the number of destinations as uncertain). Addressing equity and access issues in a transit service design problem contributes to a gap in current related research. Transit network design problems predominantly focus on minimizing user and operator cost without considering equity or access for disadvantaged populations (Kepaptsoglou and Karlaftis, 2009). The proposed approach aims to produce transit service resulting in equitable opportunities for at-risk or protected populations (e.g., low-income) to access basic needs. In the context of this research, accessibility is defined in terms of transit service frequency, number of potential bus or rail lines available for a trip, and the attractiveness of the destination based on travel time and concentration of amenities (e.g., number of supermarkets). The methodology compares access via transit to access via personal auto for selected origins and destinations. The objective function in the problem formulation minimizes the difference between access via transit and access via personal auto across the selected origins and destinations. During this process, access by transit is improved using access by personal auto as a yardstick for a base level of travel convenience and flexibility necessary to meet basic needs in an auto-oriented society. The result is transit service designed to provide an equitable difference between access via transit and access via personal auto across the selected origins and destinations within a given set of operating constraints.

The following section discusses traditional transit route and service design methodologies and the degree to which equity and accessibility have been considered. Subsequent sections present the formulation and solution method for the equitable access transit service design problem, numerical analysis with the formulation applied to data from a mid-sized U.S. metropolitan area and a summary noting potential future research and applications to current planning practices.

3.2 Background

Equity entered the public transport dialogue as early as the 1970's focusing on fiscal equity in funding public and mass transit systems through government subsidies (Kain and Meyer, 1970; Wachs, 1989; Hodge, 1988). Executive Order 12898 entitled Federal Actions to Address Environmental Justice (EJ) in Minority Populations and Low-Income Populations was a definitive catalyst in igniting awareness regarding the potential disproportionate impacts transportation projects could have on disadvantaged populations. Executive Order 12898 brought about the need for EJ analysis methodologies capable of determining whether an action or project would cause disproportionately high and adverse human health or environmental impacts on minority or low-income populations. Research studies and papers aimed at formulating methods to identify at-risk or disadvantaged populations, define disproportional impacts, quantify impacts, and explore potential appropriate mitigations followed (see Sanchez, 1998; Lane et al., 1998; Dixon et al., 2001; Purvis, 2001; Forkenbrock and Sheeley, 2004; Duthie et al., 2007). These EJ and equity considerations helped raise awareness and foster a climate for discussing and exploring transport as a social issue with respect to public transportation provision and land use patterns.

The attention of the urban planning and transportation profession has recently focused on the social implications of transportation, specifically on levels of access and mobility experienced by low-income, minority, and transportation disadvantaged population groups. Researchers are exploring the possible existence of and finding relationships between individuals' levels of mobility and accessibility and their employment rates and personal health (Sanchez, 1999; Sanchez 1998; Cervero et al., 2002; Wallace et al., 2005). The prominence of case studies considering low-income and/or minority communities' levels of access and mobility is also increasing. For example, Lucas et al. (2002) evaluate the United Kingdom's Urban Bus Challenge Fund's effectiveness at improving economic opportunities and quality of life in four low-income neighborhoods. Curriea et al. (In press) and Lucas and Fuller (2005) also use case studies to understand and investigate impacts varying levels of accessibility and mobility have on disadvantaged populations. Finally, researchers and practitioners are reviewing policy and planning processes from a social equity perspective looking for effective existing policies and opportunities to develop new strategies through continued applied research (see Sanchez and Wolf, 2005; Lucas, 2006; Lucas et al., 2007; Lucas and Stanley, 2008; Sanchez, 2008; Ward, 2009).

The above research has set the stage for explicitly incorporating equity into transit network design problems. Equity objectives and constraints are already being integrated into road network design problems with applications to spatial allocation of road improvements, highway investment, intergenerational equity, toll prices, congestion pricing, and cordon pricing (see Yang and Zhang, 2002; Meng and Yang, 2002; Antunes et al., 2003; Chen and Yang, 2004; Connors et al., 2005; Szeto and Lo, 2006; Maruyama and Sumalee, 2007; Santos et al., 2008; Duthie and Waller, 2008). In contrast, researchers focused in the area of transit network design do not appear to have explicitly incorporated issues of equity and access.

As noted above, transit route and service design problems have traditionally focused on minimizing user and operator costs (Kepaptsoglou and Karlaftis, 2009). For example, Pattnik et al. (1998) select routes for a transit network with the objective of minimizing cost to the operator and total travel time experienced by the user with constraints on total operating costs and vehicle fleet size. Zhao et al. (2005) focus on minimizing transfers and maximizing service coverage to develop routes for a public transportation network. Borndorfer et al. (2007) consider line planning in public transportation focusing on finding lines and frequencies such that a given travel demand is satisfied while minimizing operating costs for agencies and minimizing travel times for passengers. Schobel et al. (2009) address public transit stop locations, formulating an objective to cover all given demand points with minimal additional travel time. These problem definitions and solution approaches reflect the pervasive attention allocated to maximizing operating efficiencies to benefit the user and transit agency. While operating efficiencies are beneficial and critical to consider, equity issues are also pertinent to address in transit network design. This is particularly valid given the reduced mobility and accessibility experienced by the transportation disadvantaged and the resulting burden they experience meeting their basic needs.

Historically, some transit route and service network design problem formulations and solution approaches have included access to the transit system, but not the level of access the system provides to users. For example, Wu and Murray (2005) consider the quality of service provided based on transit travel time, frequency of stops, and access to the transit system based on stop frequency; they focus on optimizing the balance between service quality and access to the transit system via stop placement. While such formulations address the trade-off between public transit service quality and access to the system, they do not explicitly consider access to basic amenities nor do they address equity in the levels of access provided.

Recent and expected increases in the cost of transportation and energy, enhanced awareness of transportation as a social equity issue, and the review of transit network design problems, collectively illustrate an opportunity and need to introduce equity and access into transit network design. As a result, this research proposes a methodology incorporating access equity into transit network service design. Routes can be current transit routes in operation, generated by one of the formulations noted above, or the proposed formulation could be integrated into a multiobjective optimization transit design problem as an objective (e.g., maximizing access equity) or design constraint (e.g., specifying a minimum level of equity that must be met). The proposed formulation also accounts for demand uncertainty. Research by Waller et al. (2001) and Lam and Tam (1998) have illustrated accounting for demand uncertainty can influence the solutions of network design problems. Given these findings and the construct of the proposed formulation, demand uncertainty is incorporated by applying a uniform distribution to the number of protected individuals and concentration of amenities. This helps capture the variability in the number of potential transit riders and locations of amenities (e.g., employers, grocery stores), which is likely to occur due to circumstances unobservable to the analyst and infeasible to capture in a single data point. The overarching idea of the formulation is to improve service frequency to achieve equitable access to the given destination compared to travel by personal auto. The following section explains the formulation in additional detail.

3.3 Formulation

Starting from a study area divided into sub-areas (e.g., traffic analysis zone, Census Block), the formulation in this section can be applied to find the frequencies for each bus route that maximize equity given fixed capital and operating budgets. For each sub-area pair, the difference between measures of accessibility by transit and accessibility by automobile is calculated. The difference between this accessibility difference and the mean of all accessibility differences, across all sub-area pairs, is then calculated for each sub-area. The goal of the model is to make these differences as close to equal as possible for all sampled realizations of uncertain attractiveness and number of protected persons, with more weight given to the sub-area pairs with higher numbers of protected persons living in the origin sub-area.

The objective function is defined in equation 1 and is based on the concept of coefficient of variation, which acts to minimize the standard deviation of the accessibility measures, defined as A_{ij}^{w} for each sub-area pair and each realization of the uncertain parameters, divided by the mean of these measures $\overline{A}^{w} = \frac{1}{|I||J|}\sum_{i \in I}\sum_{j \in J} A_{ij}^{w}$ where I is the set of all origin sub-areas in the study area, J is the set of all destination sub-areas, and w (where $w = 1,..,W$) is used to denote a result based on the w^{th} realization of the uncertain parameters. When calculating the standard deviation, the squared terms are weighted according to P_i^{w}, the proportion of all protected persons in the study area that reside in origin sub-area i $\left(\sum_{i \in I} P_i^{w} = 1\right)$.

$$\min \frac{1}{W}\sum_{w=1}^{W}\left[\frac{1}{\overline{A}^{w}}\left[\frac{1}{|I||J|}\sum_{i\in I}\sum_{j\in J}P_i^w\left(A_{ij}^w-\overline{A}^w\right)^2\right]^{1/2}\right] \tag{1}$$

The accessibility measure, A_{ij}^{w} is calculated as the difference between accessibility by car, $A_{ij}^{w,c}$, and accessibility by bus, $A_{ij}^{w,b}$, which are defined in equations 2 and 3. Both measures are based on the area-wide accessibility measure and network connectivity index defined in Kittelson et al. (2003). Area-wide accessibility measures travel attractiveness based on the concentration of amenities at the destination and the travel time for the trip. The connectivity index is a ratio of the number of bus or car routes to the number of intersections in the origin sub-area. This ratio is a means to measure the ease with which travelers' can reach the bus or car routes for a given origin-destination (O/D) pair.

Accessibility by bus is an increasing function of R^b - the number of bus routes that service a given sub-area pair, F - the total frequency of bus routes between sub-areas (the decision variable in this problem), S - the (uncertain) number of employment opportunities in the destination sub-area; a decreasing function of N - the number of intersections in the origin sub-area and t^b - travel time by bus between sub-areas; and constants α and β. Values of α are one or greater; larger values indicate a great attraction per unit of the amenity, S. Values of β indicate travelers' tolerance for increased travel time. Both α and β can be calibrated to fit local conditions.

$$A_{ij}^{w,b}=\frac{R_{ij}^{b}}{N_i}F_{ij}\left(S_j^w\right)^{\alpha}e^{-\beta t_{ij}^{b}} \qquad \forall i,j\in I, w=1..W \tag{2}$$

Accessibility for automobiles is an increasing function of R^c - the number of car routes between a pair of sub-areas, D - the number of potential departure times, and S defined in the above paragraph; a decreasing function of t^c - travel time by car between sub-areas, and N (defined above); and constants α and β.

$$A_{ij}^{w,c}=\frac{R_{ij}^{c}}{N_i}D_{ij}\left(S_j^w\right)^{\alpha}e^{-\beta t_{ij}^{c}} \qquad \forall i,j\in I, w=1..W \tag{3}$$

The objective is constrained by a fixed capital budget and a fixed operating budget. The two budgets are considered separately since they typically come from different funding sources and also operate on different time scales (i.e., capital investments incur one-time costs whereas operating costs are recurring) (Cambridge Systematics, 2009). Shown in equation 4, the cost of purchasing buses to service all routes *k* in the set of routes *K* is constrained by *CapitalBudget*, where C_{bus} is the cost to purchase one bus and B_k is the number of buses assigned to route *k*.

$$C_{bus} \sum_{k \in K} B_k \leq CapitalBudget \tag{4}$$

Shown in equation 5, the weekly cost of operating the buses assigned to each route is constrained by a constant weekly *OperatingBudget*, where C_{fuel} is the fuel cost per mile, C_{wage} is the hourly wage paid to each bus driver, L_k is the length of route *k*, and *H* is the number of hours per week each bus is operated.

$$C_{fuel} \sum_{k \in K} \frac{L_k B_k H}{t_k} + C_{wage} \sum_{k \in K} B_k H \leq OperatingBudget \tag{5}$$

The number of buses assigned to route k is completely determined by the frequency for that route as shown in equation 6. All routes are assumed to be line routes, meaning they run from endpoint to endpoint, and then back again along the same route, which is why a multiple of two is used in the equation.

$$B_k = 2\left\lceil F_k t^b_k \right\rceil \tag{6}$$

The variables for frequency and travel time indexed by route are related to frequency and travel time indexed by sub-area pair as shown in equations 7 and 8, respectively, where $\delta_{k,ij}$ is an indicator variable for whether or not route *k* connects sub-area *i* to sub-area *j*.

$$F_{ij} = \sum_{k \in K} F_k \delta_{k,ij} \quad \forall i \in I, j \in J \tag{7}$$

$$t_{ij} = \min_{\substack{k \in K \\ s.t. \delta_{k,ij}=1}} t_k \quad \forall i \in I, j \in J \tag{8}$$

3.4 Solution Method

The consideration of uncertainty in the formulation as well as the nonlinear objective function necessitates the use of a heuristic solution method. A genetic algorithm (GA)-based method is proposed due to the successful applications of GA to other related problems (Fan and Machemehl, 2006). Unlike random search heuristics, GA also takes advantage of any existing neighborhood effects, which in this research means considering frequencies similar to those that have been shown to perform well. The complexity of the objective function does not significantly increase the problem difficulty since its fitness is assessed through a simple function evaluation.

The notation used to describe a GA as applied to the current problem is as follows. There is one "chromosome" per bus route. "Chromosome" refers to a vector of length *chrom* such that $\sum_{a=0}^{chrom/R^b-2} 2^a < F^{\max} \leq \sum_{a=0}^{chrom/R^b-1} 2^a$ where F^{max} is the maximum bus frequency allowed and R^b is the total number of bus routes. For example if F^{max} is set to six buses per hour and R^b is set to 4, then *chrom* equals twelve. Each chromosome is composed of sub-strings, one for each bus route (e.g., | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 0 | 1 |, where shading is used to separate the sub-strings of the chromosome). Since each entry (or "gene") in the chromosome takes a binary value, zero or one, a sub-string of length three can be used to represent frequencies from zero to seven. A "population" is a set of *c* chromosomes in the same "generation"; and "fitness" is an equivalent term for objective function value. A thorough discussion of GA approaches is given by Holland (1975) and Goldberg (1989).

Step 1. Initialization of population. Set the index for the current generation, $n = 1$. Randomly set each gene in each of the *c* chromosomes equal to zero or one. Let F_k equal the base 10 transformation of the k^{th} binary sub-string in a given chromosome. This first set of chromosomes represents the initial population, pop_n.

Step 2. Calculate objective. Solve for the objective function value of each chromosome in pop_n. Check for convergence (i.e., whether $n = n_{max}$). If convergence is not reached, go to Step 3.

Step 3. s-tournament selection. For each group or "tournament" of s chromosomes, keep the best chromosome as a parent for generation $n + 1$.

Step 4. Crossover. Let $n = n + 1$. Generate K uniform(0,1) random numbers for each pair of "parent" chromosomes. If the k^{th} random number is less than the probability of crossover, p_c, perform a single-point crossover operation on the k^{th} sub-strings in the pair to create two new "child" chromosomes. The set of children chromosomes is pop_n.

Step 5. Mutation. Mutate each gene of each chromosome in pop_n with probability p_m.

The three basic parameters that can be varied in applying the above solution method is the number of "generations", the GA "population", and the number of realizations used in accounting for uncertainty.

3.5 Numerical Analysis

Numerical analysis was conducted to demonstrate the proposed methodology. The formulation focuses on setting the frequency of bus service working within the context of pre-established routes. The numerical analysis presented here uses a set of existing bus routes in a mid-sized metropolitan area. This application focuses on low-income individuals' access to employment opportunities during the weekday morning commute to work. The data assembly process and numerical analysis results are presented and discussed below.

3.5.1 Data assembly

Nine sets of information need to be gathered and assembled in tabular form to apply the proposed formulation. These sets are: 1) residential locations of the protected population (to serve as trip origins); 2) number of protected persons in each origin area; 3) destinations of interest (e.g., location of supermarkets); 4) concentration of amenities in each destination area; 5) bus routes connecting origins and destinations and corresponding travel time; 6) number of alternative routes by auto connecting origins and destinations and corresponding travel time; 7) number of intersections in the origin area; 8) measures of uncertainty for protected population and concentration of amenities; and 9) transit operation characteristics (e.g., cost of fuel). For the purpose of demonstrating the formulation, the number of sub-areas under consideration was limited to fifteen origins and ten destinations. Focusing on a limited study area allows for a more thorough understanding and discussion of the results produced. The process used to gather data is summarized in Table 3.1.

Table 3.1. Data Assembly

Data Needs	Source	Comments
Location of Protected Population	2000 U.S. Census Bureau Data	Defined low-income as those at or below the poverty line.[a] Used the fifteen census tracts with highest percent of population at or below poverty line as the origin sub-areas.
Number of Protected Persons	2000 U.S. Census Bureau Data	Poverty rates and total population used to obtain number of protected persons per census tract.
Destinations of Interest	2005 Data from Local MPO	Focused on access to employment opportunities; used top ten census tracts based on number of employment opportunities.
Concentration of Amenities	2005 Data from Local MPO	Number of employment opportunities in selected census tracts.
Bus Routes and Transit Travel Time	Existing Bus Routes, Google Maps	Used the transit agency's online Trip Planner tool to identify direct routes (no transfers) for each origin-destination (O/D) pair and transit travel time.[b,c] Identified three new routes to provide direct service for 85 O/D pairs currently without direct service. Google Maps was used to find distance along new routes between these 85 O/D pairs. Distance converted to travel time using assumed average travel speed of 12 miles per hour.[d]
Possible Auto Routes and Auto Travel Time	Google Maps	Identified number of possible routes with similar travel times between each O/D pair. Travel times do not account for traffic congestion.
Number of Intersections in Origin Sub-Area	2000 Census Tract Boundaries and Street Network	Overlaid tract boundaries and street network to determine number of intersections.
Measures of Uncertainty	-	Assumed uniform distribution for protected population and number of employment opportunities.
Transit Operating Constraints	FTA Report[d]	Cost of purchasing a bus set at $350,000. Cost of fuel per mile set at $6. Hourly wage for bus operator set at $10.

Notes:

[a]Poverty line calculated by U.S. Census Bureau based on number of adults and children (under 18 years old) in household. In 2000, poverty threshold for single adult was annual income of $8,795; for a two adult, two children household it was $17,463 (U.S. Census Bureau, 2006).

[b]Tranist agency's online Trip Planner tool used to identify bus route options for weekday morning commutes to work (arriving by 8:00 a.m.).

[c]Cross-streets were selected in each sub-area to serve as specific trip origin and destinations; kept consistent throughout data collection for bus and auto route options as well as bus and auto travel times.

[d]Source: Niget et al. (2007)

For all numerical analyses, the GA parameters (based on initial test results) were set at 60 "generations" and a "population" of 100; and the number of sampled uncertain realizations is set to 20. In the context of this research, *D*, is set to 60 indicating auto travelers can depart any minute of a given hour.

3.5.2 Context

As discussed above, the fifteen census tracts with the highest poverty rates and the ten census tracts with the highest number of employment opportunities were selected as the origins and destinations, respectively. The poverty rates in the origin sub-areas range from 27.8% to 53.5%, well above the 2007 nationwide average of 13.0% (U.S. Census Bureau, 2009). The number of employment opportunities in the selected tracts ranged from 9,023 to 43,636 opportunities; collectively the selected tracts account for approximately 35% of the employment opportunities in the metropolitan area. The highest concentration in a single census tract occurs in a downtown census tract, which contains 8.4% of employment opportunities. Figure 3.1 illustrates the relative locations of the origin and destination sub-areas identified for the numerical analysis.

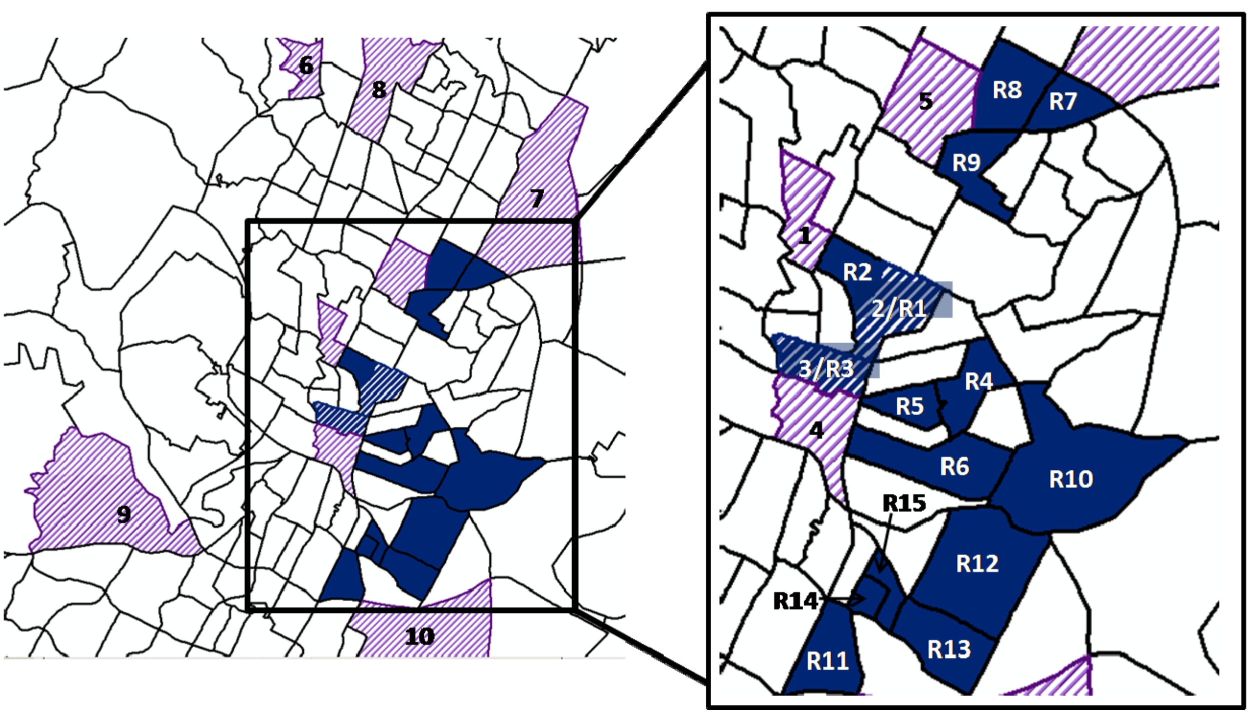

Figure 3.1. Origin and destination sub-areas selected for selected U.S. metropolitan area.

The origin sub-areas are numbered R1 through R15 (seen in a solid dark shade) and the destination sub-areas are numbered 1 through 10 (a lighter hatched pattern). Two census tracts serve as both origin and destinations these are labeled as 3/R3 and 2/R1. The origin sub-areas are relatively close together in spatial proximity. In contrast, the employment sub-areas are dispersed across the metropolitan area. There are 26 existing bus routes providing direct connections between the origins and destination pairs. As noted above, these routes are supplemented with three new routes identified to directly connect 85 O/D pairs currently without direct transit service.

3.5.3 Results

Discussed below are the influence different parameters have on the solution (i.e., set of bus frequencies) as well as the results from comparing existing bus service to the bus service produced by the proposed formulation.

The results of a sensitivity analysis were as expected. Increasing the values of α and/or β increased the gap between transit and auto accessibility, leading to a higher objective value. Increasing the capital and operating budgets results in a lower objective function value indicating that a higher degree of equity is achievable as these constraints are relaxed. Finally, results indicate the degree of uncertainty associated with the protected population and employment opportunities influences the consistency of bus route frequencies provided. Higher values of uncertainty tend to result in more variation in service frequency. An uncertainty of plus or minus 25% of the parameter value appears to be the threshold at which higher uncertainty values result in more variation in the solution set. However, it appears there are 14 bus routes whose frequencies remain the same for uncertainty values ranging from 0 to plus or minus 75%. Figure 3.2 illustrates two bus routes whose frequencies are consistent across levels of uncertainty (see Bus Route 7 and 28) and two bus routes whose frequencies vary based on the level of uncertainty (see Bus Route 4 and 2).

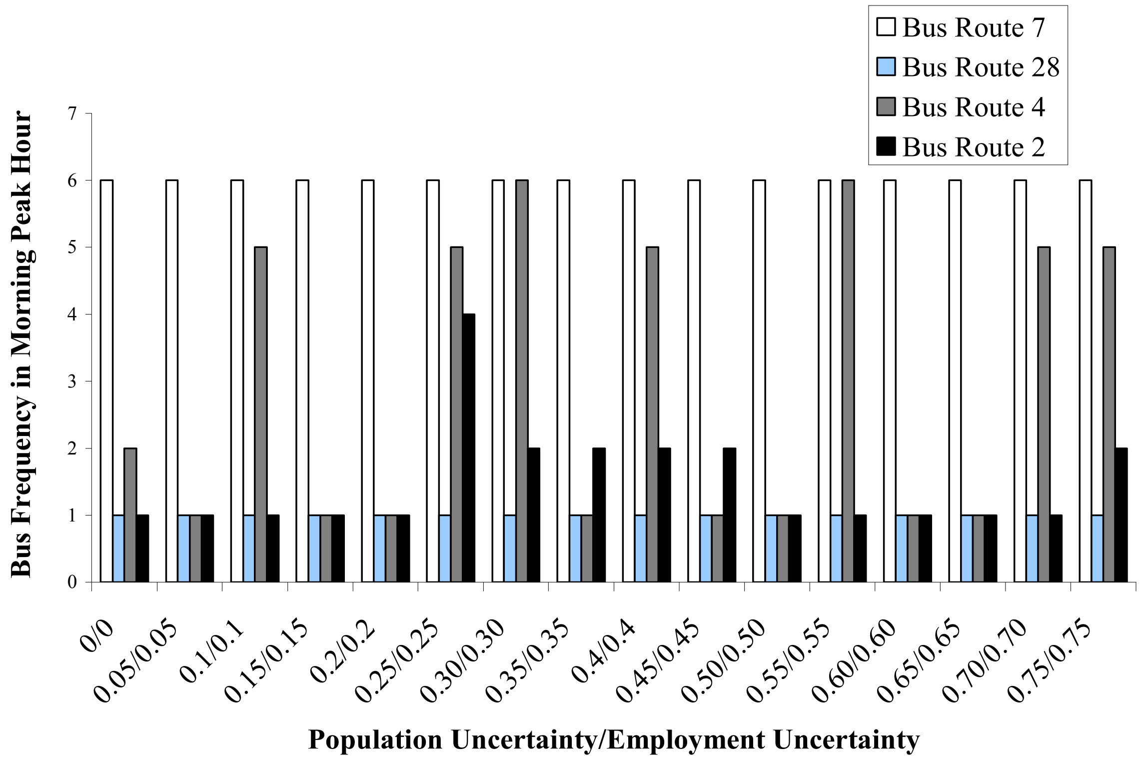

Figure 3.2. Influence of uncertainty parameters on solution set.

The results illustrated in Figure 2 indicate the higher the level of uncertainty, the more important its consideration is in order to achieve robust bus route frequencies.

O/D pair accessibilities for the existing bus systems (calculated using existing service frequencies and routes) were compared to accessibility values resulting from the transit service produced by the proposed formulation. The following parameter values were assumed: $\alpha = 1.00$, $\beta = 0.85$, uncertainty values of plus or minus 15% for the protected population and employment opportunities, a capital budget of \$27,300,000 and an operating budget of \$537,264. The capital and operating budgets were set such that they are approximately equivalent with the costs incurred by the transit agency providing the existing service.

Table 3.2 summarizes a comparison between the transit access provided by existing service (see column "Existing") and the transit access provided by the model (see column "Model Results"). The results shown are for census tracts R12, R8, and R15 to reach each of the employment tracts. (Tracts R12, R8, and R15 have the highest percent of protected population in the metropolitan area with 53.5%, 34.2%, and 37.7% of their respective populations living below the poverty line.)

Table 3.2. Existing Service vs. Results for Access Provided for Residents

Destination Census Tract	Transit Accessibility		Auto Accessibility	Bus Frequency	
				Existing	Results
	Existing	Model Results			
Access Provided for Residents in Tract R12					
1	0.00	66.36	18930.62	0.00	1.00
2	554.94	3107.65	54239.16	2.86	8.00
3	0.00	426.25	54280.52	0.00	2.00
4	4697.93	8833.20	97704.94	5.58	7.00
5	342.04	684.07	32119.84	2.00	2.00
6	0.00	51.10	12643.40	0.00	1.00
7	0.00	61.06	26830.82	0.00	1.00
8	0.00	71.02	30935.97	0.00	1.00
9	0.00	71.11	7206.00	0.00	1.00
10	430.90	186.72	7542.60	4.62	2.00
Access Provided for Residents in Tract R8					
1	0.00	739.19	20569.49	0.00	4.00
2	500.30	6003.61	53370.84	2.00	6.00
3	0.00	2653.92	33646.03	0.00	5.00
4	0.00	915.66	58870.98	0.00	3.00
5	4885.30	5066.24	25692.61	7.71	6.00
6	0.00	569.80	13354.18	0.00	4.00
7	0.00	1277.49	20568.91	0.00	4.00

8	0.00	1583.78	50421.24	0.00	4.00
9	0.00	32.10	18725.52	0.00	1.00
10	0.00	344.42	12002.03	0.00	4.00
Access Provided for Residents in Tract R15					
1	0.00	1003.45	73414.50	0.00	3.00
2	4013.95	26784.35	136311.85	2.86	10.00
3	19010.42	62256.67	146429.04	6.86	16.00
4	35202.15	90785.38	389796.97	6.86	15.00
5	14492.24	13962.68	116045.36	8.00	10.00
6	0.00	337.71	70487.87	0.00	3.00
7	0.00	790.02	99722.54	0.00	3.00
8	0.00	932.05	177426.87	0.00	3.00
9	0.00	341.04	55104.69	0.00	1.00
10	2449.68	2937.83	57678.74	4.62	4.00

Tracts 4, 2, 3, and 8 are the four census tracts with the highest percent of employment opportunities in the metropolitan region at 8.39%, 4.59%, 4.53%, and 4.40%, respectively. Noticeable improvements can be seen for access to these tracts. Among the three residential tracts summarized in Table 3.2, there are two instances in which access slightly decreases. This occurs for residential tract R12's access to employment tract 10 and residential tract R15's access to employment tract 5. These slight decreases reflect the problem's financial constraints. Limited resources are being reallocated to provide equitable access across 150 origin-destination pairs, which means, in some instances, access will be slightly reduced to enable improvements elsewhere. Under the given financial constraints, the decrease in R12's access to employment tract 10 occurs because employment tract 10 has the lowest number of employment opportunities of the 10 tracts considered making it relatively unattractive compared to the other employment tracts. The decrease in R15's access to employment tract 5 occurs because, of the five tracts served under existing service, tract 5 is least attractive to residents in R15 (based on the travel time by transit and number of employment opportunities). The formulation reduces access to these tracts to provide improved access to the remaining employment tracts that are more attractive due to travel time and/or the number of employment opportunities.

The success in the proposed formulation can be seen in the clear access improvements provided to the residents in the most at-risk residential areas. Particularly notable, access for the most at-risk residential tracts to the employment tracts with the highest concentration of employment opportunities is consistently improved.

3.6 Summary

The need to incorporate equitable access in transit service design was demonstrated by documenting current and anticipated continued rise in transportation and energy expenditures, awareness of transportation as social equity issue, and a review of transit network design problems. The purposes of improving equity in transit access to basic amenities are to: 1) reduce the disproportionate burden transit dependent populations' experience; and 2) increase the financial security of low-income households by giving them the option to reduce their dependence on autos. This research contributes to the transit design literature by integrating equitable access into the transit frequency-setting problem. The formulation was shown to be successful at increasing access to employment opportunities for residents in areas with the high percentages of protected persons by applying it to a mid-sized metropolitan area. The example application also reflects the financial constraints and inherent tradeoffs necessary in reaching equitable access across multiple origin-destination pairs. Finally, results in Figure 2 indicate the higher the level of uncertainty, the more important its consideration is in order to achieve robust bus route frequencies.

There are many opportunities for integrating the proposed formulation into different planning contexts and future research. First, the proposed formulation can be used to address access to needs other than employment opportunities such as supermarkets and medical services. Second, it could also be applied in the context of evaluating alternative future land use scenarios for a region. Agencies could consider the levels of access and mobility experienced by transit dependent populations in multiple land use scenarios under a fixed transit budget. Such planning analysis would bring awareness to which land use patterns and transit service designs provide the most equitable levels of access and mobility. This would be particularly valuable for coordinating land use and transit planning that explicitly considers equity in access and mobility. Finally, the current formulation could be incorporated into multiobjective transit design formulations, which would allow analysts to consider access equity in concert with travel demand, user cost and operator costs when designing routes as well as setting frequencies for a transit network.

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